Waveform Encoding for Nonlinear Electromagnetic Tomographic Imaging

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Outline

- Background and Motivation
- MIMO Tomographic Imaging Algorithm
- Numerical Simulations
- Conclusion
What is Electromagnetic (EM) Tomography?

- **EM tomography is an inverse scattering problem**
  - Source antenna transmits EM signals into a medium
  - Scattered signals are received
  - Inversion algorithms are applied to reconstruct material properties based upon Maxwell’s equations
  - Applications: Large scale seismic imaging, medical imaging
Electromagnetic Tomography

Mathematically, EM tomography is an inverse problem

- Infer model parameters \( p(r) \) from measured data based upon underlying Maxwell’s equations
- Image to be reconstructed: a spatial distribution of \( p(r), r \in \Omega \)

\[
y_j = A_j(p(r); s_j) + \eta_j
\]

- \( y_j \in \partial\Omega \times [0, T] \): measured data by receivers
- \( p(r), r \in \Omega \): material values (i.e., dielectric const.)
- \( s_j \): \( j \)-th excitation signal
- \( A_j \): Nonlinear operator determined by wave model
- \( \eta_j \): Noise and disturbance
The Challenges of EM Tomography

Challenge 1: The inverse problem is ill-posed

dimension of \( p \) (\# of grids) \( \gg \) dimension \(|\partial \Omega|\) (\# of receivers)

- Classic approach: Regularization is required (e.g., sparsity constraint on \( p \)) to reduce the dimension of solution space

Challenge 2: Nonlinear inversion method

- Classic approach: least squares optimization or the iterative Newton’s method require iterative algorithms
- The cost of computation depends on the size of the data volumes and on the discretization of the wave model
The SIMO Classic Data Collection Process

- **Single-Input Multiple Output (SIMO)**
  - An image is reconstructed from measured data in response to a single excitation antenna source.
  - The reconstruction process continues till all the sources are excited.

![Diagram of Excitation antenna configuration](image)
Our approach: MIMO Excitation and Waveform Encoding

- How to remove cross-talk induced by simultaneous waveform excitation due to wave interference?
  - Time delays
  - Random weights
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MIMO Imaging Problem Formulation

- Imaging Configuration
The 2D transverse magnetic (TM) model

Maxwell’s equations

\[
\frac{\partial E_z}{\partial t}(t) = -\frac{\sigma}{\varepsilon} E_z(t) + \frac{1}{\varepsilon} \left( \frac{\partial H_y}{\partial x} + x \frac{\partial H_x}{\partial y} \right)(t) - \frac{1}{\varepsilon} J_z(t)
\]

\[
\frac{\partial H_y}{\partial t}(t) = \frac{1}{\mu} \frac{\partial E_z}{\partial x}(t)
\]

\[
\frac{\partial H_x}{\partial t}(t) = -\frac{1}{\mu} \frac{\partial E_z}{\partial y}(t)
\]

\(E_z(t) \in \Omega \times [0, T]\) Electric field intensity in z-direction

\(H_y, H_x\) Magnetic field intensity in x-, y-direction

\(p = [\mu, \sigma, \varepsilon]\) Parameter set

\(\varepsilon\) Dielectric constant is to be reconstructed

\(J_z(t)\) Excitation source
Approach to solve

\[ y_j = A_j(p(r); s_j) + \eta_j \]

- Newton’s method: iteration starts from an initial guess \( f^0 \)

\[ p^{k+1} = p^k + \lambda \delta p^k \]

- The increment value \( \delta p^k \) can be solved by adjoint method
Waveform Encoding Schemes

- Random phase encoding: weights $w_j$ are phase coded
  \[
  J_{z,m}^{(1)}(r^t, t) = \sum_{j=1}^{L_m} w_j s_j(t) \delta(r^t - r_j^t)
  \]

- Time-delay encoding: delay $\tau_j$
  \[
  J_{z,m}^{(2)}(r^t, t) = \sum_{j=1}^{L_m} s_j(t - \tau_j) \delta(r^t - r_j^t)
  \]

- Uniform weight encoding
  \[
  J_{z,m}^{(3)}(r^t, t) = \sum_{j=1}^{L_m} s_j(t) \delta(r^t - r_j^t)
  \]
Impact of Excitation Sources

- Re-write the 2D TM Maxwell’s equations

\[
\frac{\partial^2 E_z}{\partial t^2} - c^2 \frac{\partial^2 E_z}{\partial z^2} = f(z, t)
\]

The forcing term

\[f(z, t) = \frac{1}{\varepsilon} \frac{\partial}{\partial t} (J_z + \sigma E_z)\]

- The solution (general solution + particular solution)

\[E_z = E_{z,g} + E_{z,p}\]

\[E_{z,p} = \frac{\sqrt{\mu \varepsilon}}{c} \int_0^t \int_{z-(t-t')/\sqrt{\mu \varepsilon}}^{z+(t-t')/\sqrt{\mu \varepsilon}} f(z, t) \, dz \, dt\]

- Electric field computed in the forward model depends on the excitation source, which also affects the reconstruction procedure implicitly
### Numerical Simulations

- **Simulation configuration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Gaussian modulated pulse</td>
</tr>
<tr>
<td>Target dielectric values</td>
<td>$\epsilon_1 = 1.2\epsilon_0, \epsilon_2 = 1.5\epsilon_0$</td>
</tr>
<tr>
<td>Computational region</td>
<td>12 cm by 12 cm</td>
</tr>
<tr>
<td>Mesh grids</td>
<td>40 by 40</td>
</tr>
<tr>
<td># of antennas</td>
<td>90</td>
</tr>
<tr>
<td>Noise level</td>
<td>5%</td>
</tr>
</tbody>
</table>
Reconstructed Images

(a) Random encoding
(b) Time delay
(c) Uniform weight
Convergence History

\[ \beta = \frac{\left\| \hat{\epsilon}^k - \epsilon_{true} \right\|^2}{\left\| \hat{\epsilon}^0 - \epsilon_{true} \right\|^2} \]
Conclusions

- Proper encoding techniques accelerate convergence of iterative inverse methods for nonlinear EM tomography

- Demonstrate the power of signal processing techniques for improving computational efficiency for solving nonlinear inverse problems

- Suitable for large-scale high resolution imaging applications
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