OPTIMUM DECISION FUSION IN COGNITIVE WIRELESS SENSOR NETWORKS WITH UNKNOWN USERS LOCATION

Andrea Abrardo, Mauro Barni

Department of Information Engineering andy Mathematics
University of Siena
Via Roma 56, 53100 - Siena, Italy

ABSTRACT

We consider a cooperative cognitive wireless network scenario where a primary wireless network is co-located with a cognitive (or secondary) network. In the considered scenario, the nodes of the secondary network make local binary decisions about the presence of a signal emitted by a primary node. Then, they transmit their decisions to a fusion center (FC). The final decision about the channel state is up to the FC by means of a proper fusion rule.

In this scenario, we derive the optimum decision strategy for the FC and the optimum local decision thresholds of the secondary nodes in a Neyman-Pearson setup. In particular, the overall system performance are derived by making the realistic assumption that the position of the primary user is completely unknown to the FC.

1. INTRODUCTION

With the development of wireless communications in the last years, most of the available spectrum has fully been allocated. On the other hand, recent investigations on the actual spectrum utilization have shown that a portion of the licensed spectrum is largely under utilized [1].

In this context, dynamic spectrum access (DSA) and cognitive radio (CR) has the potential to become the solution to the spectrum under utilization problem [2], [3], [4], [5]. One of the main tasks of CR is represented by spectrum sensing (SS), defined as the task of finding spectrum holes [6] i.e., portions of spectrum allocated (licensed) to some primary users but left unused for a certain time. In particular, collaboration of multiple users in SS may highly improve the performance of spectrum sensing by introducing a form of spatial diversity [7], [8]. In cooperative SS, CR users first send the collected data to a combining user or fusion center (FC). Alternatively, each user may independently perform local decisions, and then report binary decisions to the FC. Finally, the FC takes a decision on the presence or absence of the licensed signal based on the information received from the users.

In this paper, we consider a cooperative cognitive wireless network scenario where a primary wireless network is co-located with a cognitive (or secondary) network. Any portion of the spectrum, i.e. a subchannel, can be assigned *freely* by the base station of the primary network to an active user equipment (UE). The nodes of the secondary network, referred to as cognitive UE (C-UE), make local binary decisions about the presence of a signal emitted by a primary node, denoted as P-UE. Then, they transmit a report with their decisions to the FC. The final decision about the channel state is up to the FC, through the adoption of a proper fusion rule.

The main goal of this paper is to derive the optimum decision strategy for the FC and the optimum local decision thresholds of the C-UE nodes in a Neyman-Pearson setup. With respect to previous works, the contribution of our study is twofold: (*i*) we derive the optimum fusion rule without assuming that the position of the P-UE is known. This marks a major difference with respect to previous studies in which the P-UE is assumed to be located at the center of the ROI (see for example [9]); (*ii*) we derive a closed-form solution for the optimum decision rule without resorting to any approximations (e.g, Gaussianity assumption) with regard to the sufficient statistic used by the FC. The simulations we carried out show that the ignorance of P-UE location has a non-negligible impact on the achievable performance, thus confirming the importance of our study.

The rest of the paper is organized as follows. Section 2 introduces the setup analyzed in our study. Section 3 outlines the overall approach that we adopt for our analysis. In Section 4 we present the optimal fusion rule and derive the overall system performance analytically. Section 5 shows simulation results and discusses them. Finally, Section 6 presents our conclusions.

2. THE SETUP

In the setup considered in this paper, both primary and secondary nodes are deployed in a circle of radius R, hereafter referred to as region of interest (ROI). The locations of nodes are unknown to the FC, which, in turn, is assumed to be located at the center of the ROI. Secondary and primary nodes are independent and identically distributed (i.i.d.) within the ROI according to a uniform distribution. We assume that each subchannel is assigned to a P-UE with probability $P(S_1)$ while it is inactive with probability $P(S_0) = 1 - P(S_1)$. In this setting, each C-UE scans all channels in order to detect the presence of a primary signal transmission. In particular, the C-UEs perform a binary hypothesis test on the status of each channel. The channels are idle under hypothesis S_0 and busy under hypothesis S_1.

To be more specific, we assume that, upon channel sensing, k secondary nodes produce k local decision variables $\{w_1, w_2 \dots w_k\}$. Each variable w_q, $q = 1, \dots, k$, is a binary term, i.e., $w_q \in \{0, 1\}$, representing the binary status (idle or busy) of the sensed subchannel, estimated by the q-th node. The secondary nodes transmit the observation variables $\{w_q\}_{q=1}^{k}$ to the FC, which makes a final decision about the status of each subchannel. We further assume that the reporting channel between C-UEs and the FC is not ideal, and that it can be modeled as a BSC channel with common error probability P_e.

3. METHODOLOGY

The overall approach that we will adopt for our derivations is outlined in the following.

We start with the assumption that the nodes make their decisions by comparing a decision variable (extracted from the received signal) against a local threshold τ_N [10]. In this case, the local false alarm probability (P_{FA}) depends only on the noise power at the C-UE receivers, and, as such, it can be easily calculated. As for the local missed detection probability (P_{MD}), it depends on the propagation conditions between the transmitting and the receiving nodes, which, in turn, depend on the positions of the nodes. We denote in the following by $V_{c,q}$, $q = 1 \dots k$, and V_p, respectively, the position of the secondary users and the primary user within the ROI.

Let now W_q and Z_q be the random variables indicating, respectively, the local decision made by the q-th node and the corresponding report received by the fusion center (they may differ due to the presence of transmission errors between the C-UE and the FC). The optimum strategy of the FC is defined by the Neyman-Pearson criterion and requires the evaluation of the likelihood ratio between the probabilities of observing the reports received from the nodes under the two hypothesis S_1 and S_0, that is:

$$\ell(z_1 \dots z_k) = \frac{P(z_1, z_2 \dots z_k | S_1)}{P(z_1, z_2 \dots z_k | S_0)} \underset{S_0}{\overset{S_1}{\gtrless}} \lambda. \quad (1)$$

A problem with the computation of the above quantity is that under Hypothesis S_1 the rv's Z_q are not independent since they jointly depend on the position of the primary user V_p. To get around this problem, we observe that due to the symmetry of the analyzed setup and to the lack of a-priori information about the position of the secondary users, the probability of the reports received by the FC is permutation invariant, that is $P(z_1, z_2 \dots z_k | S_1) = P(\sigma(z_1, z_2 \dots z_k) | S_1)$ for any permutation σ. Then, it is easy to prove that $\ell(z_1 \dots z_k)$ depends only on the number of zeros (and ones) contained in $(z_1 \dots z_k)$, that is on

$$z_{tot} = \sum_{q=1}^{k} z_q, \quad (2)$$

which, hence, is a sufficient statistic for the problem at hand. Due to the nature of the problem, we also argue that $\ell(z_1 \dots z_k)$ is a monotonically increasing function of z_{tot}, hence permitting us to rewrite the optimal decision rule for the FC as:

$$z_{tot} \underset{S_0}{\overset{S_1}{\gtrless}} T. \quad (3)$$

To complete our analysis, we need to set the threshold T by resorting again to the Neyman-Pearson criterion, i.e., by fixing the desired overall false alarm probability ($P_{FA}^{(f)}$). This can be easily done, since the probability density function of Z_{tot} under S_0 is easy to compute due to the independence of the random variables Z_q when the sensed channel is idle. The computation of the missed detection probability, let us call it $P_{MD}^{(f)}$, is more involved due to the dependence of the Z_q's under S_1. To go on, we observe that, given V_p, Z_q's are conditionally independent and identically distributed according to a Bernoulli distribution for which the probability that $Z_q = 1$ is computable starting from the error probability characterizing the local decisions of the nodes and the error probability of the BSC linking the C-UE and the FC. For this reason $P(Z_{tot}|V_p)$ is a binomial distribution with known parameters, and $P(z_{tot})$ can be derived as:

$$P(z_{tot}) = \int_{\text{ROI}} P(z_{tot}|v_p) f(v_p) dv_p. \quad (4)$$

To conclude, we observe that, for a given τ_N and a desired $P_{FA}^{(f)}$ we have a given T, and, hence, a given $P_{MD}^{(f)}$. Accordingly, we can then optimize the overall system by choosing τ_N in such a way to minimize $P_{MD}^{(f)}$. In the following sections, all the steps outlined above are applied to the particular setup described in Section 2.

4. OPTIMAL FUSION RULE

In order to derive the optimal fusion rule we start by observing that, in wireless scenarios, the path loss depends on the distance between the transmitting and receiving antenna/node. Accordingly, setting $V_c = ye^{2\pi j\theta}$ and $V_p = xe^{2\pi j\phi}$ the generic positions of C-UEs and P-UEs within the ROI, we have that the local missed detection probability is a function of, among other things, the distance $d = \sqrt{x^2 + y^2 - 2xycos(\theta - \phi)}$. Hence, assuming that the ROI is characterized by a known statistical channel model, it is possible to evaluate the average local missed detection probability as a function of d, by averaging with respect to fast and slow fading distributions (e.g., see [11], [12]). In the following we denote by $P_{MD,avg}(d)$ the average missed detection probability for a given d.

Let now denote by $P(z_q|S_j)$ the pmf of observations z_q under hypothesis S_j, $j = 0, 1$. We can write the pmf terms as:

$$\begin{array}{rcl} P(z_q|S_1) & = & P(z_q|w_q = 1) P(w_q = 1|S_1) \\ & + & P(z_q|w_q = 0) P(w_q = 0|S_1) \\ P(z_q|S_0) & = & P(z_q|w_q = 1) P(w_q = 1|S_0) \\ & + & P(z_q|w_q = 0) P(w_q = 0|S_0) \end{array} \tag{5}$$

where in the considered scenario we have:

$$\begin{array}{l} P(w_q = 1|S_1) = (1 - P_{MD,avg}(d_q)) \\ P(w_q = 1|S_0) = P_{FA} \end{array} \tag{6}$$

d_q being the distance between the q-th C-UE and the P-UE, and $P(z_q|w_q) = 1 - P_e$ if $z_q = w_q$ and $P(z_q|w_q) = P_e$ if $z_q \neq w_q$.

In order to give an estimation of the final performance we start by evaluating the average correct detection probability for each sensor conditioned to the position of the primary node $V_p = xe^{2\pi j\phi}$. We first observe that the assumption of a uniform distribution in the ROI entails a constant joint pfd in Cartesian coordinates. Hence, by passing from Cartesian to polar coordinates, we get a uniform distribution for ϕ and a linear distribution for x. Accordingly, owing to the symmetry of the problem, we can consider $\phi = 0$. Moreover, without loss of generality, we assume $R = 1$ so that the generic P-UE position in the ROI can be characterized by a random term x with pdf $f_x(\delta) = 2\delta$ in the interval $[0, 1]$. Denote now by d the distance between a generic C-UE node located at position V_c and the P-UE. The distribution of d, denoted by $f_d(\rho)$, can be derived by observing that for $\rho < 1 - x$ the annulus centered at V_c with inner radius ρ and outer radius $\rho + d\rho$ is fully included into the unit circle, and, as such, we have $f_d(\rho) = 2\rho$, for $\rho < 1 - x$. On the other hand, for $\rho \geq 1 - x$ only a portion of the annulus lies within the unit circle and in this case $f_d(\rho)$ is given by such portion divided by the surface of the unit circle. This situation is illustrated in Figures 1 and 2, where the abscissa $x_0 = \frac{1-\rho^2+y^2}{2y}$ of the intersection points of two circles centered in 0 and x and with radius 1 and ρ, respectively, is shown. We also show the angle θ subtended by half of the portion of circumference with radius ρ lying outside the unit circle, for the two possible cases $x_0 \geq y$ (Fig. 1) and $x_0 < y$ (Fig. 2). Note that θ is a function of ρ and x and it has the same formal expression in both cases, i.e.:

$$\theta(\rho, x) = \cos^{-1}\left(\frac{1-\rho^2-x^2}{2\rho x}\right). \tag{7}$$

Hence, we have:

$$\begin{array}{ll} f_d(\rho|x) = 2\rho & \text{for} \quad 0 \leq \rho \leq 1 - x \\ f_d(\rho|x) = 2\rho\left[1 - \frac{1}{\pi}\theta(\rho, x)\right] & \text{for} \quad 1 - x < \rho \leq 1 + x \\ f_d(\rho|x) = 0 & \text{for} \quad \rho > 1 + x \end{array} \tag{8}$$

Let now denote by $\bar{P}_{CD}(\delta)$ the probability of correct detection of a generic node averaged over all possible distances d,

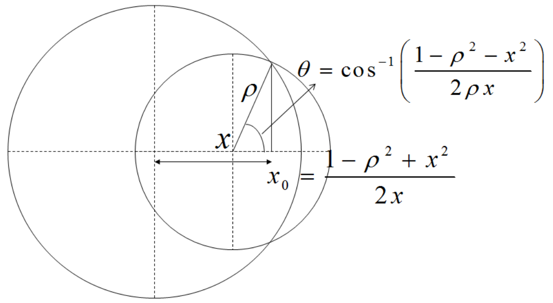

Fig. 1. Calculation of the portion of the annulus in the case $x_0 \geq y$.

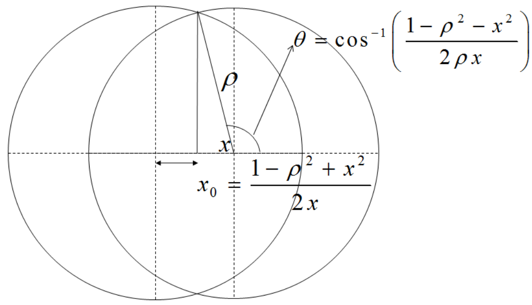

Fig. 2. Calculation of the portion of the annulus in the case $x_0 < y$.

for a given $x = \delta$, we have:

$$\bar{P}_{CD}(\delta) = 1 - \int_{\rho} P_{MD,avg}(\rho) f_d(\rho|\delta) d\rho \tag{9}$$

We are now able to derive the pmf of the observations z_q under hypothesis S_j, $j = 0, 1$ conditioned on $x = \delta$ as:

$$\begin{array}{rcl} P(z_q = 1|S_1, \delta) & = & \bar{P}_{CD}(\delta)(1 - P_e) \\ & + & (1 - \bar{P}_{CD}(\delta))P_e \\ & \triangleq & P_{CD}^{eq}(\delta), \end{array} \tag{10}$$

on the other hand we have:

$$P(z_q = 1|S_0) = P_{FA}(1 - P_e) + (1 - P_{FA})P_e = P_{FA}^{eq}. \tag{11}$$

Let us denote by $P_{CD}^{(f)}(\delta)$ the final correct detection probability conditioned to δ, which is the correct detection probability achieved by fusing all the received reports. By remembering the discussion we made in Section 3, we have:

$$P_{CD}^{(f)}(\delta) = \sum_{q=T}^{k} \binom{k}{q} (P_{CD}^{eq}(\delta))^q (1 - P_{CD}^{eq}(\delta))^{k-q}. \tag{12}$$

Eventually, from (4), we obtain the unconditioned correct detection probability:

$$P_{CD}^{(f)} = 2\int_0^1 P_{CD}^{(f)}(\delta)\delta d\delta. \tag{13}$$

As for $P_{FA}^{(f)}$, since in this case the local reports do not depend on the position of the P-UE, we have:

$$P_{FA}^{(f)} = \sum_{q=T}^{k} \binom{k}{q} (P_{FA}^{eq})^q (1 - P_{FA}^{eq})^{k-q}. \quad (14)$$

It is then possible to determine the system performance for each given threshold pair (τ_N, T). In particular, we introduce the following functions to highlight the dependency of $P_{CD}^{(f)}$ and $P_{FA}^{(f)}$ on τ_N and T:

$$\begin{array}{l} P_{FA}^{(f)} = \mathcal{F}(\tau_N, T) \\ P_{CD}^{(f)} = \mathcal{G}(\tau_N, T). \end{array} \quad (15)$$

The threshold τ_N is evaluated by following a Neyman-Pearson approach. To elaborate, denoting by $P_{FA}^{(tgt)}$ the desired P_{FA}, for each possible value of τ_N we determine the threshold T^* which allows to achieve $P_{FA}^{(tgt)}$, i.e.:

$$T^*(\tau_N) : \mathcal{G}(\tau_N, T^*) = P_{FA}^{(tgt)}. \quad (16)$$

Hence, the optimal τ_N, denoted by τ_N^* is selected as the value which allows to achieve the minimal $P_{CD}^{(f)}$, i.e.:

$$\tau_N^* = \arg\min_{\tau_N} F[\tau_N, T^*(\tau_N)]. \quad (17)$$

5. SIMULATION RESULTS AND DISCUSSION

We have developed a MATLAB simulation environment which allows to place all the involved nodes within the same circular ROI following a uniform distribution. We have then considered classical path-loss and fading models and we have derived the final ROC curves for different parameters setting. Such curves have been compared with the results obtained through the analytical approach discussed in previous Sections. Such comparisons have shown a perfect matching between the two approaches, thus confirming the validity of the proposed analysis.

Moreover, we have evaluated the system performance for two distinct cases: (*i*) the position of the primary user is unknown with uniform distribution in the ROI. This case is denoted by *Uniform P-UE*, ; (*ii*) the primary user is located at the center of the ROI, i.e., border effects are neglected, as assumed in [9]. This case is denoted by *Fixed P-UE*. A comparison between the two cases, obtained again considering different parameters setting, has demonstrated that the uncertainty in the position of the P-UE strongly affects the system performance, and, as such, we cannot in general neglect the border effects. This observation definitely assesses the importance of our analysis.

As an example, in Fig. 3 we report the ROC curves obtained for the two mentioned cases. In particular, we consider $\tau_N = 1.5$, $R = 1000$ m, a transmitting power from P-UEs of 150 mW, a path-loss with power decaying factor $\alpha = 4$, a log-normal fading with standard deviation of 5 dB, a noise level of -110 dBm, and an energy detector sensing scheme with 10 observations, where the signaling scheme of the P-UE is unknown to the C-UE [10]. In the x-axis we report the $P_{FA}^{(f)}$ that can be achieved for different T, with $T = 1, 2, \ldots, k$. The corresponding $P_{CD}^{(f)}$ is shown in the y-axis. Two different cases are considered in Fig. 3, i.e., $k = 10$ and $k = 20$. It is worth noting that the *Uniform P-UE* case achieves noticeable worse performance than *Fixed P-UE*, and, hence, neglecting the border effects would lead to highly over-estimate the system performance.

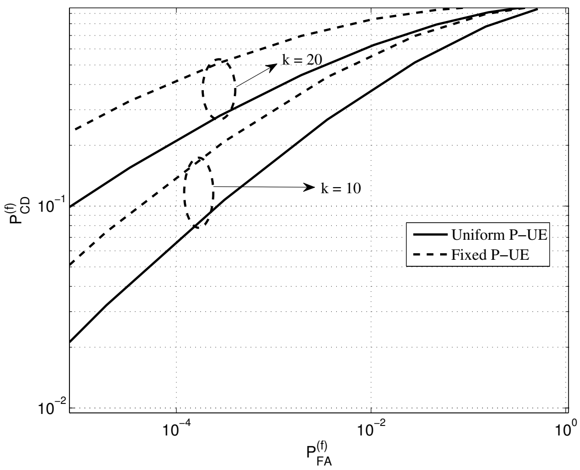

Fig. 3. Comparisons between *Uniform P-UE* and *Fixed P-UE* for $k = 10$ and $k = 20$.

6. CONCLUSIONS

We have analyzed a cooperative cognitive wireless network scenario where a primary wireless network is co-located with a cognitive (or secondary) network. In the considered scenario, the nodes of the secondary network perform local binary decisions about the presence of a signal emitted by a primary node. Such decisions are then transmitted to the FC which adopts a proper fusion rule to derive a final decision. In the considered setting, we make the realistic assumption that the position of the primary user is completely unknown to the FC.

Hence, we have derived a closed-form solution for the optimum decision rule at the FC and, accordingly, we have provided an exact analytical characterization of missed detection and false alarm probabilities. Then, the proposed approach has permitted to determine the optimum local decision thresholds of the secondary nodes in a Neyman-Pearson setup. Finally, we have shown the the uncertainty in the primary node position strongly affects the system performance, thus assessing the importance of the proposed study.

7. REFERENCES

[1] D. Datla, A. Wyglinski, and G. Minden, "A spectrum surveying framework for dynamic spectrum access networks," *IEEE Trans. Veh. Technol.*, vol. 58, no. 2, pp. 4158–4168, October 2009.

[2] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13–18, August 1999.

[3] J. Mitola, *Cognitive radio: An integrated agent architecture for software-defined radio*. Ph.D. dissertation, Roy. Inst. Technol., Stockholm, Sweden, 2000.

[4] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, pp. 201–220, October 2005.

[5] S. Heykin, *Fundamentals issues in cognitive radio*. Springer, 2007.

[6] R. Tandra, M. Mishra, and A. Sahai, "What is a spectrum hole and what does it take to recognize one?" *Proc. IEEE*, vol. 97, pp. 824–848, May 2009.

[7] G. Ganesan and G. Y. Li, "Cooperative spectrum sensing in cognitive radio, part i: Two user networks," *IEEE Trans. Wireless Commununications*, vol. 6, no. 6, pp. 2204–2213, June 2007.

[8] ——, "Cooperative spectrum sensing in cognitive radio, part ii: Multiuser networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2214–2222, June 2007.

[9] Q. C. Ruixin Niu, Pramod K. Varshney, "Distributed detection in a large wireless sensor network," *Information Fusion*, no. 7, pp. 380–394, 2006.

[10] M. N. Alexander M. Wyglinski and Y. T. Hou, *Cognitive Radio Communications and Networks Principles and Practice*. Elsevier, 2010.

[11] M. A. Hossain, M. S. Hossain, and M. I. Abdullah, "Cooperative spectrum sensing over fading channel in cognitive radio," *International Journal of Innovation and Applied Studies*, vol. 1, no. 1, pp. 84–93, November 2012.

[12] S. Nallagonda, S. D. Roy, and S. Kundu, "Performance of cooperative spectrum sensing in log-normal shadowing and fading under fusion rules," *International Journal of Energy, Information and Communications*, vol. 3, no. 3, August 2012.