Motivation and Setup

**Motivation**
- Centralized Approach and Challenges
  - Composite Hypothesis
    \[ H_1: \theta' \neq 0 \]
    \[ H_0: \theta = 0 \]
  - \( \theta' \): M-dimensional parameter, deterministic but unknown
  - Network of \( N \) agents. Observation sequence at each agent
    \[ y_n(t) = h_n(\theta) + \gamma_n(t), \]
  where \( h_n(\cdot) \) is a non-linear function, \( h_n(0) = 0 \), \( h_n(\cdot) : \mathbb{R}^M \rightarrow \mathbb{R}^M \), \( M_a < M \), \( \gamma_n(t) \sim N(0, \Sigma_f) \)

**Centralized Approach and Challenges**
\[ \mathcal{H} = \left\{ \begin{array}{ll} H_1, & \text{if } \max_{\theta} \sum_{n=1}^{N} \log f(y_n(t) | H_1(\theta)) > \eta \bowtie \max_{\theta} \sum_{n=1}^{N} \log f(y_n(t) | H_0(\theta)), \\
H_0, & \text{otherwise,} \end{array} \right. \]

- Key bottleneck
  \[ \max_{\theta} \sum_{n=1}^{N} \log f(y_n(t) | H_1(\theta)) \]
- Depends on the entire sensed data.
- High-dimensional data exchange at all times.
- Involves batch processing; Detection cannot start until the maximizer is obtained

**Distributed Approach**

**Assumptions**
- (A.1) Global Observability: The sensing model is globally observable, i.e., any two distinct values of \( \theta \) and \( \theta' \) in the parameter space satisfy
  \[ \sum_{n=1}^{N} \| h_n(\theta) - h_n(\theta') \|^2 = 0 \]
  if and only if \( \theta = \theta' \).
- (A.2) Network Connectivity: The inter-agent communication graph is connected, i.e., \( \lambda_2(L) > 0 \), where \( L \) denotes the associated graph Laplacian matrix.
- (A.3) Smoothness: For each agent \( n \), \( \forall \theta \neq \theta_1 \), the sensing functions \( h_n \) are continuously differentiable and Lipschitz continuous with constants \( k_n \), i.e.,
  \[ \| h_n(\theta) - h_n(\theta_1) \| \leq k_n \| \theta - \theta_1 \| \]
- (A.4) Monotonicity: For each pair of \( \theta \) and \( \theta' \) with \( \theta \neq \theta' \), there exists a constant \( c' \) such that the following aggregate strict monotonicity condition holds
  \[ \sum_{n=1}^{N} (\theta - \theta')^T (\nabla h_n(\theta) \Sigma_f^{-1} (h_n(\theta) - h_n(\theta'))) \geq c' \| \theta - \theta' \|^2. \]

**CIGLRT: Consensus+Innovations GLRT Algorithm**

**Parameter Estimation Update**
\[ \theta(t+1) = \theta(t) - \frac{1}{t+1} \sum_{\ell=1}^{t} (\theta(\ell) - \theta(t)) + \alpha \nabla h_n(\theta(t)) \Sigma_f^{-1} (y(t) - h_n(\theta(t))) \]

The weight sequences \( \{\alpha_n\}_{n \geq 0} \) and \( \{\beta_n\}_{n \geq 0} \) are given by
\[ \alpha_n = \frac{1}{(1+t)^2} \quad \beta_n = \frac{(t+1)^2}{t} \]

**Decision Statistic Update**
\[ z_n(t+1) = \frac{1}{t+1} \left( z_n(t) - \delta \sum_{\ell=1}^{t} (z_n(\ell) - z(t)) \right) + \frac{1}{t+1} \log \frac{f_n(t)(y(t))}{f_n(\bar{y}(t))}, \]
\[ \delta \in \left( 0 \bigg\| \frac{2}{\lambda_2(L)} \right) \]

**Decision Rule**
\[ H_0(t) = \left\{ \begin{array}{ll} H_0, & z_n(t) \leq \eta \\
H_1, & z_n(t) > \eta \end{array} \right. \]

**Probability of Errors**
\[ P_{H_0}(t) = P_{H_0}(z_n(t) \leq \eta), \quad P_{H_1}(t) = P_{H_1}(z_n(t) > \eta) \]

**Simulation Results**
- Linear Scalar Model: \( h_n(\theta) = h_n(\theta_1) \)
- Network of 5 agents: 5 are defective, only observe noise.
- Scaling factors of the other 5 agents: 1, 1.5, 0.8, 2, 0.9
- \( \sigma^2 = 7.8 \), noise power = 3.
- Random geometric network: \( \| L - \frac{1}{5} I \|_2 = 0.9161 \)

**Main Results**

**Asymptotic Normality and Error Analysis**
- Theorem 1: Asymptotic Normality Consider the CIGLRT algorithm under Assumptions A.1-A.4, and the sequence \( \{a(t)\} \). We then have under \( P_{H_0} \), for all \( \| \theta' \| > 0 \),
  \[ \sqrt{N} \left( z_n(t) - h^T(\theta') \Sigma_f^{-1} h(\theta') \right) \overset{D}{\rightarrow} N \left( 0, \frac{h^T(\theta') \Sigma_f^{-1} h(\theta')}{N^2} \right), \]
  where \( \Sigma_f = I_N \otimes \theta' \), \( h(\theta') = [h_1(\theta'), \ldots, h_N(\theta')]^T \) and \( \overset{D}{\rightarrow} \) refers to convergence in distribution (weak convergence).

- Theorem 2: Error Analysis Consider the decision rule of CIGLRT. For all \( \theta' \) which satisfy \( h^T(\theta') \Sigma_f^{-1} h(\theta') < \frac{1}{k} \), we have the following choice of the thresholds \( \delta \approx 0 \) and \( \delta \approx \sqrt{N} \Sigma_f^{-1} h(\theta') \) for which we have that \( P_{H_0}(t) \rightarrow 0 \) and \( P_{H_1}(t) \rightarrow 0 \) as \( t \rightarrow \infty \). Specifically, \( P_{H_1}(t) \) decays to zero exponentially with the following large deviations exponent
  \[ \limsup_{t \rightarrow \infty} \frac{1}{t} \log P_{H_1}(z_n(t) > \eta) \leq - LE(\min \{\Lambda^*, \lambda_2(L)\}), \]
  where \( LE(\lambda) = \frac{\lambda}{1+\sqrt{K}+\sqrt{N}} + \frac{\frac{1}{2}M_a^2 M_0}{1+\sqrt{K}+\sqrt{N}} \log \left( 1 - \frac{\frac{1}{2}+\sqrt{K}+\sqrt{N}}{1+\sqrt{K}+\sqrt{N}} \right) \),
  \[ \lambda^* = \frac{1}{2+\sqrt{K}+\sqrt{N}}, \quad \lambda = \frac{1}{2+\sqrt{K}+\sqrt{N}} \]