Terrain-Scattered Jammer Suppression in MIMO Radar Using Space-(Fast) Time Adaptive Processing

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**Terrain-scattered/diffuse jammer suppression** is one of the most important issues in radar signal processing [1, 2].

Significantly increased **degrees of freedom** enable superiorities of MIMO radar over phased-array (PA) radar [3].

**New opportunities** of clutter/jammer suppression have been shown in MIMO radar in recent years [3, 4].

**Space-time adaptive processing (STAP)** techniques play an important role in radar signal processing, especially for clutter/jammer suppression [5].
Contributions

- Problem of terrain-scattered jammer suppression using **space-(fast) time adaptive processing (SFTAP)** is studied in MIMO radar framework.

- **Correlation function** of jamming components after matched filtering (MF) at the receiving end of MIMO radar is derived.

- A minimum variance distortionless response (**MVDR type SFTAP design**) which considers waveform-introduced range sidelobes and cold clutter stationarity over different pulse intervals is proposed.

- **Closed-form solution** to the MVDR type design and a relaxed SFTAP design are provided.
Jamming signals take the form of high-power transmission that aims at impairing the receive system.

Terrain-scattered jamming occurs when the high-power jammer transmits its energy to ground, and it reflects the energy in a dispersive manner.

Pure mutual orthogonality of multiple waveforms does not exist, which leads to necessity of studying the effect of MF on the received jamming signals.

MIMO radar faces the challenge of significantly increased computational burden, therefore, developing computationally affordable STAP techniques is important.
Signal Model

- **Target signal:**
  \[
  y_t(\zeta, \tau) = \sqrt{\frac{E}{M}} \alpha_t D_t(\tau) (R^{T}_{\phi}(\zeta)a(\theta_t)) \otimes b(\theta_t).
  \]

- **Clutter signal:**
  \[
  y_c(\zeta, \tau) = \sqrt{\frac{E}{M}} \sum_{i=1}^{N_c} \xi_i D_i(\tau) (R^{T}_{\phi}(\zeta)a(\theta_i)) \otimes b(\theta_i).
  \]

- **Jamming signal:**
  \[
  y_j(\zeta, \tau) = \sum_{j=1}^{J} \sum_{p=1}^{P} \beta_{j,p} \eta_{j,p}(\zeta, \tau) \otimes b(\nu_{j,p}).
  \]

- **Entire signal:**
  \[
  y_j(\zeta, \tau) \triangleq y_t(\zeta, \tau) + y_c(\zeta, \tau) + y_j(\zeta, \tau) + y_n(\zeta, \tau).
  \]
Signal Model (Cont’d)

**Parameters:**

\( E \): Transmit energy;  
\( M \): Number of transmit antennas.

\( \zeta, \tau \): Fast-time and slow-time indices, respectively.

\( \alpha_t, \xi_i \): Reflection coefficients of target and the \( i \)th clutter patch.

\( \theta_t, \theta_i \): Spatial directions of target and the \( i \)th clutter patch.

\( D_t(\tau), D_i(\tau) \): Doppler shift of target and the \( i \)th clutter patch.

\( a(\theta), b(\theta) \): \( M \times 1 \) transmit and \( N \times 1 \) receive steering vectors.

\( R_\phi(\zeta) \): Correlation matrix of emitted waveforms denoted by \( \phi \).

\( N_c \): Number of clutter patches;

\( J \): Number of jamming sources;

\( P \): Number of diffuse multipath.

\( \beta_{j,p}, \vartheta_{j,p} \): Magnitude and spatial angle of jamming signal associated with the \( j \)th jammer and the \( p \)th propagation path.

\( \eta_{j,p}(\zeta, \tau) \): Match-filtered jamming signal associated with the \( j \)th jammer and the \( p \)th propagation path;

\( (\cdot)^T \): Transpose.
Consider the commonly used **barrage noise jamming signals** \( s_j(t, \tau), \ j = 1, \ldots, J \).

Jamming signals are **mutually independent** and **stationary white** random processes.

**Correlation** between original jamming signals:

\[
\mathbb{E}\left\{ s_j(t, \tau)s_{j'}^*(t', \tau') \right\} = S_j(f_c)\delta_{jj'}\delta(t - t')\delta_{\tau\tau'}
\]

\((\cdot)^*:\) Conjugate transpose operator.
\(t, t':\) Fast-time indices;
\(\tau, \tau':\) Slow-time indices.

\((\cdot)_j\) (or \((\cdot)_{j'}\): W.r.t. \(j\)th (or \(j'\)th) jamming signal.

\(S_j(f_c)\): Jamming power spectral density at carrier frequency \(f_c\).

\(\delta(\cdot), \delta_{j,j'}\) (also \(\delta_{\tau\tau'}\)): Dirac and Kronecker delta functions, respectively.
Correlation Analysis

Perform correlation analysis on the MF vector \( \eta_{j,p}(\zeta, \tau) \).

Explicit expression: \( \eta_{j,p}(\zeta, \tau) \triangleq \int_{T_p} s_j(t - \zeta_0 - \zeta_p, \tau) \phi^*(t - \zeta)dt \).

\( \eta_{j,p}(\zeta, \tau) \) is the only term that determines the correlation properties of jamming components.

The \( M \times M \) correlation matrix of \( \eta_{j,p}(\zeta, \tau) \):

Correlation matrix

\[
\begin{align*}
R_{\eta}^{n}(\zeta, \zeta', \tau, \tau') & \triangleq \mathbb{E}\{ \eta_{j,p}(\zeta, \tau) \eta_{j',p'}^{H}(\zeta', \tau') \} \\
& = \mathbb{E}\{ \int_{T_p} \int s_j(t - \zeta_0 - \zeta_p, \tau) s_{j'}^*(u - \zeta_0 - \zeta_{p'}, \tau') \times \phi^*(t - \zeta) \phi^T(u - \zeta')dtdu \} \\
& = S_j(f_c) \delta_{jj'}\delta_{\tau\tau'}R_{\phi}^{T}(\zeta_p - \zeta_{p'} + \zeta' - \zeta + \zeta_0).
\end{align*}
\]
\( R_{j,p,j',p'}^{\eta} \) is guaranteed to be nonzero once \( \zeta_p - \zeta_{p'} + \zeta' - \zeta = 0 \).

The \( MN \times MN \) correlation matrix of the jamming signal:

**Jamming correlation matrix**

\[ R_j(\zeta, \zeta', \tau, \tau') \triangleq \mathbb{E}\{y_j(\zeta, \tau)y_j^H(\zeta', \tau')\} \]

\[
= \sum_{j=1}^{J} \sum_{j'=1}^{J} \sum_{p=1}^{P} \sum_{p'=1}^{P} \beta_{j,p}\beta_{j',p'}^* R_{j,p,j',p'}^{\eta}(\zeta, \zeta', \tau, \tau') \otimes (b(\vartheta_{j,p})b^H(\vartheta_{j',p'})) \\
= S_j(f_c)\delta_{\tau\tau'} \sum_{j=1}^{J} \sum_{p=1}^{P} \sum_{p'=1}^{P} \beta_{j,p}\beta_{j',p'}^* R_{\phi}^T(\zeta_p - \zeta_{p'} + \zeta' - \zeta + \zeta_0) \otimes (b(\vartheta_{j,p})b^H(\vartheta_{j',p'})).
\]
SFTAP Design

- Stack the available $Q$ taps of data vectors associated with the $\tau$th pulse into an $MNQ \times 1$ virtual data vector $y(\tau)$:
  
  \[
  y(\tau) \triangleq \begin{bmatrix} y^T(\zeta_0, \tau), & \ldots, & y^T(\zeta_0 + Q - 1, \tau) \end{bmatrix}^T = y_t(\tau) + y_c(\tau) + y_j(\tau) + y_n(\tau).
  \]

- The $MNQ \times MNQ$ target-free covariance matrix of $y(\tau)$:
  
  \[
  R_y(\tau) \triangleq \mathbb{E}\{y_c(\tau)y_c^H(\tau)\} + \mathbb{E}\{y_j(\tau)y_j^H(\tau)\} + \mathbb{E}\{y_n(\tau)y_n^H(\tau)\} = R_c(\tau) + R_j + R_n \triangleq R_{c-j-n}.
  \]

- For the $\tau$th pulse, SFTAP aims at finding an adaptive filter which **minimizes** the output interference power without attenuating target and meanwhile **maximizes** the output signal-to-jammer-plus-noise ratio (SJNR).
Key issue: Stationarity of cold clutter over different pulse intervals after SFTAP should be maintained.

Proposed SFTAP design:

\[
\begin{align*}
\min_{\mathbf{w}(\tau)} & \quad \mathbf{w}^H(\tau) \mathbf{R}_{jn} \mathbf{w}(\tau) \\
\text{s.t.} & \quad \mathbf{w}^H(\tau) \mathbf{s}_t(\theta_t) = 1 \\
& \quad \frac{\mathbf{w}^H(\tau) \mathbf{R}_c(\tau) \mathbf{w}(\tau)}{\mathbf{w}^H(0) \mathbf{R}_c(\tau) \mathbf{w}(0)} = 1 \\
& \quad \mathbf{w}^H(\tau) \tilde{\mathbf{u}}(\zeta_0, \theta_t) = 0
\end{align*}
\] (1a)-(1d)

\(\mathbf{s}_t(\theta_t)\): \(MNQ \times 1\) target steering vector; \(\mathbf{w}(0)\): Weight vector for the first pulse; \(\tilde{\mathbf{u}}(\zeta_0, \theta_t) \triangleq [0, \mathbf{u}^T(\zeta_0 + 1, \theta_t), \ldots, \mathbf{u}^T(\zeta_0 + Q - 1, \theta_t)]^T\) with \(\mathbf{u}(\zeta, \theta_t) \triangleq (\mathbf{R}_\phi(\zeta) \mathbf{a}(\theta_t)) \otimes \mathbf{b}(\theta_t)\).
Design of (1) deals with SFTAP problem for each transmitted pulse since Doppler information of clutter signals changes over slow-time domain.

Constraint (1c) ensures the cold clutter stationarity over different pulse intervals; (1d) accounts for attenuating side-lobes at range bins other than target direction.

Closed-form solution to (1):

**Closed-Form Solution**

\[
\mathbf{w}(\tau) = (\mathbf{R}_{jn} + \lambda \mathbf{R}_c(\tau))^{-1} \mathbf{v}(\zeta_0, \theta_t) \left( \mathbf{v}^H(\zeta_0, \theta_t) \right)^{-1} e \\
\times (\mathbf{R}_{jn} + \lambda \mathbf{R}_c(\tau))^{-1} \mathbf{v}(\zeta_0, \theta_t)^{-1} e
\]

\[
\mathbf{v}(\zeta_0, \theta_t) \triangleq [s_t(\theta_t), \tilde{u}(\zeta_0, \theta_t)], \quad e \triangleq [1, 0]^T, \quad \text{and} \quad \lambda \text{ is determined by}
\lambda_{\min} \left\{ \mathbf{R}_c^{-1/2}(\tau) \mathbf{R}_{jn} \mathbf{R}_c^{-1/2}(\tau) / (\mathbf{w}^H(0) \mathbf{R}_c(\tau) \mathbf{w}(0)) \right\}.
\]
Relaxed SFT AP Design

Closed-form solution exists when subspace of adaptive weights defined by constraints of (1) is nonempty. In practice, constraints (1c) and (1d) can be relaxed.

Proposed relaxed SFTAP design:

\[
\begin{align*}
\min_{\mathbf{w}(\tau)} & \quad \mathbf{w}^H(\tau)\mathbf{R}_{jn}\mathbf{w}(\tau) \\
\text{s.t.} & \quad \mathbf{w}^H(\tau)\mathbf{s}_t(\theta_t) = 1
\end{align*}
\]

\[\|\mathbf{w}^H(\tau)\mathbf{R}_c^{1/2}(\tau) - \mathbf{w}^H(0)\mathbf{R}_c^{1/2}(\tau)\| \leq \epsilon \tag{2c}\]

\[\|\mathbf{w}^H(\tau)\tilde{\mathbf{u}}(\zeta_0, \theta_t)\| \leq \gamma \tag{2d}\]

\[\epsilon \geq 0: \text{Bounding the output clutter distortion caused by } \mathbf{w}(\tau);\]

\[\gamma \geq 0: \text{Characterizing the worst range sidelobes towards target direction. For given } \gamma, \text{ the feasibility of (2) is guaranteed if }\]

\[\epsilon \geq \epsilon_{\text{min}} \text{ (minimum output clutter distortion w.r.t. (2b) and (2d)).}\]
Simulations

- $M = 8$ transmit and $N = 8$ receive antennas spaced half wavelength apart from each other.
- Transmit energy $E = M$.
- 4 sets of unimodular waveforms: Polyphase-coded (PC), CA, CAN, and WeCAN-based waveforms.
- One CPI contains 10 pulses.
- $P = 19$ diffuse multipath uniformly distributed within $[-9^\circ, 9^\circ]$, in the presence of $J = 1$ jamming source.
- Target parameter: $\theta_t = 0^\circ$.
- $\text{SNR} = 0 \text{ dB}$, $\text{CNR} = 30 \text{ dB}$, and $\text{JNR} = 30 \text{ dB}$. 
Simulation Results (Cont’d)

Figure 1: SJNR performance versus employed data taps.
Figure 2: SCNR performance versus normalized Doppler frequencies.
Conclusions

- Problem of **terrain-scattered jammer suppression** using **SFTAP** has been addressed for MIMO radar.

- The **effect of matched filtering** at the receiving end on barrage noise type jamming has been derived by establishing connections with waveform correlation matrix.

- Proposed **MVDR type SFTAP** and **relaxed SFTAP** designs have been shown able to reduce waveform-introduced range sidelobes and maintain cold clutter stationarity over different pulse intervals.

- **Closed-form solution** to the MVDR type SFTAP design has been obtained.
References


