Multi-Linear Subspace Estimation and Projection for Efficient RFI Excision in SIMO Systems

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**Agenda**

- Motivation
- State-of-the-art
- System model
  - **MLSEP**: Multi-Linear Subspace Estimation and Projection
    - Problem setup
    - Problem formulation
      - RFI subspace estimation
      - Multi-linear (tensor-based) projection
    - MLSEP algorithm
- Simulation results
- Conclusions
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Motivation

- Radio frequency interference (RFI) is caused by:
  - Out-of-band emissions, jamming, spoofing and meaconing

- Such an RFI is prevalent in
  - Radio astronomy, microwave radiometry and global navigation satellite system (GNSS)

- The congestion of licensed spectrum both in satellite and terrestrial communications and the advent of cognitive radios
  - Call for **efficient RFI excision (signal processing) algorithms**

- On the other hand, tensor-based parameter estimators based on truncated higher-order SVD (HOSVD) have outperformed their matrix-based counterparts
  - Nevertheless, tensors **had never been deployed for RFI excision**
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State-of-the-art

- RFI detection and excision algorithms
  - Spectral
  - Temporal
  - Spectral-temporal
  - Statistical
  - Spatial filtering-based
  - Transformed domain-based

- Apply DFT and compare with the theoretical threshold
- Apply FDAF
- Estimation of mean and variance of sample magnitudes
- RFI detection and excision
- Time-frequency (TF) representation
- RFI estimation and excision
- Apply detection theory
- Assumptions on RFI are needed
- RFI subspace estimation
- Orthogonal projection
- Represent in other domain
- RFI detection
- RFI synthesis and excision
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A SIMO system with $N_R$ number of receive antennas suffering from a **stationary severe broadband RFI** (modeled as a zero mean AWGN with variance of $\sigma_f^2$) [1]

The received signal at time $n$ would be

$$y(n) = \sum_{l=0}^{L} h_l s(n-l) + \sum_{l=0}^{L_f} g_l f(n-l) + z(n), \quad (2)$$

**Assumptions:**

$\Rightarrow$ Uncorrelated $s(n)$, $f(n)$ and $z(n)$

$\Rightarrow$ $z(n)$ is a zero mean AWGN vector with a covariance matrix of $\sigma^2 I_{N_R}$

$\Rightarrow$ Perfect estimates of $L$ and $L_f$

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The proposed MLSEP algorithm comprises two phases

**First phase**: No SOI is transmitted in the first long term interval (LTI)
- The RFI subspace tensor is estimated using HOSVD
- From the estimated RFI subspace tensor, the multi-linear projector is derived

**Second phase**: SOI transmission and RFI excision using the already derived multi-linear projector
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MLSEP: Problem setup

- Stacking the observation vectors of the $N_R$ receive antennas and $W$ data windows into one highly structured vector of size $N_R \times W \times 1$ with respect to the $m$th STI gives

\[
y_m = Hs_m + Gf_m + z_m \in \mathbb{C}^{N_R \times W},
\]

where

\[
s_m = [s(mW), \ldots, s(mW - W - L + 1)]^T \in \mathbb{C}^{(W + L)},
\]

\[
f_m = [f(mW), \ldots, f(mW - W - L_f + 1)]^T \in \mathbb{C}^{(W + L_f)}
\]

and $z_m \in \mathbb{C}^{N_R \times W}$ are the SOI, RFI and zero mean AWGN.

$H \in \mathbb{C}^{N_R \times W \times (W + L)}$ is the SOI filtering matrix as defined in [2].

$G \in \mathbb{C}^{N_R \times W \times (W + L_f)}$ is the RFI filtering matrix structured as

\[
G = [G_1^T, G_2^T, \ldots, G_{N_R}^T]^T,
\]

\[
G_j = \begin{bmatrix}
g_j^0 & \ldots & g_j^{L_f} & 0 & \ldots & \ldots & 0 \\
0 & g_j^0 & \ldots & g_j^{L_f} & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \ldots & 0 & g_j^0 & \ldots & g_j^{L_f}
\end{bmatrix}
\]
The horizontal concatenation of $N y_m$'s in (3) renders

$$Y = HS + GF + Z \in \mathbb{C}^{NR \cdot W \times N}.$$  \hfill (6)

In the first LTI, no SOI transmission occurs and the received signal would then be

$$Y_I = GF + Z \in \mathbb{C}^{NR \cdot W \times N}.$$  \hfill (7)

From (7), the RFI subspace $\hat{U}_I \in \mathbb{C}^{NR \cdot W \times (W+L_f)}$ estimated using SVD as

$$Y_I = [\hat{U}_I \hat{U}_n] \begin{bmatrix} \hat{\Sigma}_I & 0_{r \times (N-r)} \\ 0_{(NR \cdot W-r) \times r} & \Sigma_n \end{bmatrix} [\hat{V}_I \hat{V}_n]^H, \hfill (8)$$

where $\hat{\Sigma}_I = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r)$ and $r = W + L_f$.

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To deploy the inherent structure of the measurement data, we model the received signal by a 3-way tensor \( \mathbf{Y} \in \mathbb{C}^{N_R \times W \times N} \), where \( N_R \), \( W \) & \( N \) are the number of antennas, samples per a STI and non-overlapping STIs per LTI, respectively.

If \( \mathbf{Y}^T_{(3)} \) should be equal to \( \mathbf{Y} \) in (6), the multi-linear equivalent of (6) would be

\[
\mathbf{Y} = \mathbf{\mathcal{H}} \times_3 \mathbf{S}^T + \mathbf{\mathcal{G}} \times_3 \mathbf{F}^T + \mathbf{Z},
\]

where \( \mathbf{\mathcal{H}} \in \mathbb{C}^{N_R \times W \times (W+L)} \) and \( \mathbf{\mathcal{G}} \in \mathbb{C}^{N_R \times W \times (W+L_f)} \) are constructed as in Fig. 1 and \( \mathbf{Z} \) is the noise tensor.

Besides, \( \mathbf{\mathcal{H}}^T_{(3)} = \mathbf{H} \) and \( \mathbf{\mathcal{G}}^T_{(3)} = \mathbf{G} \).
Fig. 1. Multi-linear formulation from the received signal per LTI $Y$ in (6).
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In the first LTI, no SOI is transmitted and hence the truncated HOSVD of the received signal \( \mathbf{y}_I = \mathcal{G} \times_3 F^T + \mathbf{Z} \) would be

\[
\mathbf{y}_I \approx \hat{\mathbf{S}}^{[I]} \times_1 \hat{\mathbf{U}}_1^{[I]} \times_2 \hat{\mathbf{U}}_2^{[I]} \times_3 \hat{\mathbf{U}}_3^{[I]}, \tag{10}
\]

where \( \hat{\mathbf{S}}^{[I]} \in \mathbb{C}^{r_1 \times r_2 \times r_3} \) is a core-tensor which satisfies the all-orthogonality conditions, \( r_n \) is the \( n \)-rank of the noiseless tensor \( \tilde{\mathbf{y}}_I = \mathcal{G} \times_3 F^T \).

\[
r_1 = \min(N_R, L_f + 1), \quad r_2 = \min(W, N.N_R) \quad \& \quad r_3 = \min(N, W + L_f).
\]
From (10), the estimated RFI signal subspace tensor is defined as [3]

$$\hat{U}^{[I]} = \hat{S}^{[I]} \times_1 \hat{U}_1^{[I]} \times_2 \hat{U}_2^{[I]} \times_3 \hat{\Sigma}_I^{-1}. \quad (11)$$

Note that

$$\left[\hat{U}^{[I]}\right]_{(3)^T} \in \mathbb{C}^{N_R \cdot W \times r_3}$$

span the estimated RFI signal subspace and inspires the underneath theorem

**Theorem 1:** The tensor-based RFI subspace estimator

$$\left[\hat{U}^{[I]}\right]_{(3)^T}$$

and the matrix-based RFI subspace estimator $$\hat{U}_I$$ are related by

$$\left[\hat{U}^{[I]}\right]_{(3)^T} = (\hat{T}_1 \otimes \hat{T}_2) \cdot \hat{U}_I, \quad (12)$$

where $$\hat{T}_r = \hat{U}_r^{[I]} \cdot \hat{U}_r^{[I]}^H$$, $$r = 1, 2$$.

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The multi-linear projector for **perfect excision** is stated in the underneath theorem

*Theorem 2:* For a perfect $\mathcal{U}^{i-1}$, the multi-linear projector

$\mathcal{P} \in \mathbb{C}^{N_R \times W \times N_R \cdot W}$ which evokes perfect RFI excision is given by

\[
\mathcal{P} = \mathcal{I}_3 - \hat{\mathcal{U}}^{[I]} \times_3 (\hat{\mathcal{U}}^{[I]})^{+3},
\]

where $\mathcal{I}_3 \in \mathbb{C}^{N_R \times W \times N_R \cdot W}$ is the 3-mode identity tensor,

$\hat{\mathcal{U}}^{[I]}$ is the 3-mode pseudo-inverse tensor,

$\mathcal{I}_3^{(3)} = I_{N_R \cdot W}$ and

$\left(\hat{\mathcal{U}}^{[I]}\right)^{+3}_{(3)} = \left(\hat{\mathcal{U}}^{[I]}\right)^{+}_{(3)}$.  

However, perfect excision is impossible and we define the root mean square excision error (RMSEE) as

\[
\text{RMSEE} = \sqrt{\mathbb{E}\left\{\left\|P\mathcal{G}\right\|^2_F\right\}}
\]

\[
\text{RMSEE} = \sqrt{\mathbb{E}\left\{\left\|\mathcal{P} \times_3 \mathcal{G}\right\|_{(3)}^2 F\right\}}.
\]
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### Algorithm I: MLSEP for efficient RFI excision in SIMO systems

**Input:** $Y_I$, $Y$, $N_R$, $W$, $L$, $L_f$, $N$

**Assumptions:** $N \geq \{W + L, W + L_f\}$, $W > \{L, L_f\}$

**Initialization:** $r_1 = \min(N_R, L_f + 1)$, $r_2 = \min(W, N \cdot N_R)$

1. $\mathbf{Y}_I =$ the tensorization of $[\mathbf{Y}_I]_{(3)} = Y_I^T$
2. $[\mathbf{Y}_I]_{(1)} = U_1 \Sigma_1 V_1^H$; $\hat{U}_1^{[I]} = U_1(:, 1: r_1)$
3. $[\mathbf{Y}_I]_{(2)} = U_2 \Sigma_2 V_2^H$; $\hat{U}_2^{[I]} = U_2(:, 1: r_2)$
4. $Y_I = U \Sigma V^H$; $\hat{U}_I = U(:, 1: W + L_f)$
5. $[\hat{\mathbf{u}}^{[I]}]^T = (\hat{T}_1 \otimes \hat{T}_2) \cdot \hat{U}_I$; $\hat{T}_r = \hat{U}_r^{[I]} \cdot \hat{U}_r^{[I]H}$, $r = 1, 2$
6. $\hat{u}^{[I]} =$ the tensorization of $[\hat{\mathbf{u}}^{[I]}]_{(3)}$
7. $\mathbf{P} = \mathbf{I}_3 - \hat{u}^{[I]} \times_3 \left(\hat{u}^{[I]}\right)^{+3} \in \mathbb{C}^{N_R \times W \times N_R \times W}$
8. Repeat
9. $\mathbf{Y} =$ the tensorization of $[\mathbf{Y}]_{(3)} = Y^T$
10. return $[\mathbf{P} \times_3 \mathbf{Y}]^T_{(3)}$
11. Until no SOI transmission

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**Multi-linear RFI excision**

**Multi-linear projection**

**Multi-linear RFI subspace estimation**
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Fig. 2. Average RMSEE for an RFI excision using SP, CSP and MLSEP during $N_{SOI} = 200$ LTIs and 40 observed symbols per LTI at $W = 4$, $N = 10$, $\alpha = 100$ and a pre-excision SINR of 0 dB.
Simulation results (cont.)

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Single-path ((L + 1 = L_f + 1 = 1)) scenario</th>
<th>Multi-path ((L + 1 = L_f + 1 = 2)) scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INR [dB]</td>
<td>INR [dB]</td>
</tr>
</tbody>
</table>

*TABLE I*

Average SINR gain [dB] evoked by perfect excision, SP [1], CSP [2] and MLSEP for both single-path and multi-path scenarios during \(N_{SOI} = 200\) LTIs and 40 observed symbols per LTI at \(W = 4, N = 10, \alpha = 100\) and a pre-excision SINR of 0 dB.
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The paper introduces the multi-linear algebra framework to the RFI excision research.

The MLSEP algorithm outperforms the state-of-the-art projection-based RFI excision algorithms.
Backups
Unfoldings of a 4x5x3 tensor in reverse cyclical column ordering [6]
Tensor algebra [6]

$n$-mode products between $\mathbf{X} \in \mathbb{C}^{M_1 \times M_2 \times M_3}$ and $U_n \in \mathbb{C}^{P_n \times M_n}$

\[
\mathbf{Y} = \mathbf{X} \times_1 U_1 \in \mathbb{C}^{P_1 \times M_2 \times M_3} \\
\mathbf{Y} = \mathbf{X} \times_2 U_2 \in \mathbb{C}^{M_1 \times P_2 \times M_3} \\
\mathbf{Y} = \mathbf{X} \times_3 U_3 \in \mathbb{C}^{M_1 \times M_2 \times P_3}
\]

\[\Leftrightarrow [\mathbf{Y}]_{(n)} = U_n \cdot [\mathbf{X}]_{(n)}\]

i.e., all the $n$-mode vectors multiplied from the left-hand-side by $U_n$.

---

Properties of the HOSVD [4]

**Matrices**

\[ X = U \Sigma V^H \]

- \( U, V \) – unitary
- \( \Sigma \) – diagonal
- \( \Sigma \) – “all orthogonal”

**Tensors**

\[ \chi = U^{(1)} S U^{(2)} \]

- \( U^{(1)}, U^{(2)}, U^{(3)} \) – unitary
- \( S \) – full tensor
- Orthogonal slices (subtensors)

\[ [S]_n \cdot [S]^H_n = \Sigma(n)^2 \]

- “All orthogonality” of \( S \)

The \("n\)-rank” of a tensor [4]

**Matrices**

\[ \text{rank}\{X\} - \text{column rank} \]

\[
X = \begin{pmatrix}
\text{column vectors}
\end{pmatrix}
\]

- \# linearly independent column vectors

**Tensors**

1-rank\{\mathbf{X}\} = \text{rank}\{[\mathbf{X}]_1\}

\[
X = \begin{pmatrix}
\text{1-mode vectors}
\end{pmatrix}
\]

- \# linearly independent 1-mode vectors

2-rank\{\mathbf{X}\} = \text{rank}\{[\mathbf{X}]_2\}

\[
X = \begin{pmatrix}
\text{2-mode vectors}
\end{pmatrix}
\]

- \# linearly independent 2-mode vectors

The “n-rank” of a tensor [4]

Matrices

\[ \text{rank}\{X\} - \text{column rank} \]

\[ X = \begin{pmatrix} \text{column vectors} \end{pmatrix} \]

\[ \Rightarrow \text{\# linearly independent column vectors} \]

\[ \text{rank}\{X^T\} - \text{row rank} \]

\[ X = \begin{pmatrix} \text{row vectors} \end{pmatrix} \]

\[ \Rightarrow \text{\# linearly independent row vectors} \]

\[ \text{rank}\{X\} = \text{rank}\{X^T\} \]

\[ \text{rank}\{X\} \leq \min\{M_1, M_2\} \]

Tensors

\[ 3\text{-rank}\{X\} = \text{rank}\{[X]_{(3)}\} \]

\[ X = \begin{pmatrix} \text{3-mode vectors} \end{pmatrix} \]

\[ \Rightarrow \text{\# linearly independent 3-mode vectors} \]

\[ 1\text{-rank}\{X\} \neq 2\text{-rank}\{X\} \neq 3\text{-rank}\{X\} \]

\[ n\text{-rank}\{X\} \leq M_n \]