Mobile Beamforming & Spatially Controlled Relay Communications

Dionysios S. Kalogerias and Athina P. Petropulu

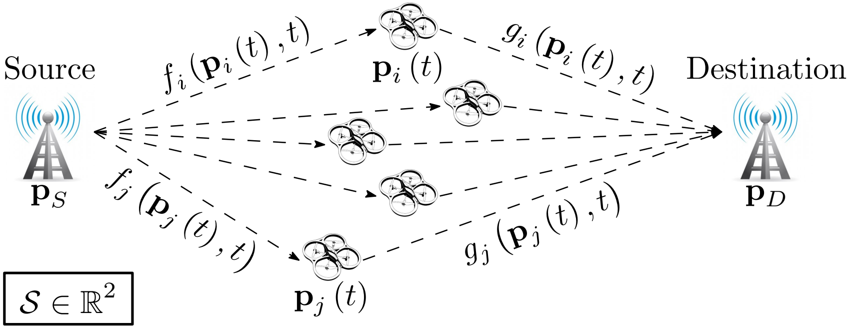
Rutgers, The State University of New Jersey, USA

Work supported by the National Science Foundation (NSF) under Grant CNS-1239188

Overview

- We consider **stochastic motion planning** in **single source/destination AF mobile relay beamforming networks**.
- Assumption: The wireless channel is a **spatiotemporal stochastic field**.
- We present a **2-stage stochastic programming formulation** for spatial relay control, such that
  - the **expected reciprocal of the total power at the relays is maximized**,
  - on the basis of **random causal CSI**.
- We propose a **lower bound relaxation** to the original problem.
  - This is equivalent to a set of simple tractable subproblems.
  - And results in spatial controllers with a predictive character.
- $\Rightarrow$ Under this setting, the optimal control policy is **purely selective**:
  - **Only the "best" relay should move**.

Relay Beamforming (1)



- The network operates over a time horizon of N_T time slots in $[0, T]$.
- For later, let $\mathbf{p}(t) \triangleq \left[\mathbf{p}_1^{\boldsymbol{T}}(t)\ \mathbf{p}_2^{\boldsymbol{T}}(t) \ldots \mathbf{p}_R^{\boldsymbol{T}}(t)\right]^{\boldsymbol{T}}$ (R: # of relays).
- Channels $\{f_i, g_i\}_{i \in \mathbb{N}_R^+}$ are modeled as random variables.
- Relays can exchange "small" messages locally.

Relay Beamforming (2)

- At each time instant, given exact CSI $\{f_i, g_i\}_{i \in \mathbb{N}_R^+}$, we would like to

$$\begin{array}{ll} \underset{\mathbf{w}}{\text{maximize}} & \left(\boldsymbol{w}^{\boldsymbol{H}} \mathbf{D} \boldsymbol{w}\right)^{-1} \\ \text{subject to} & \dfrac{\boldsymbol{w}^{\boldsymbol{H}} \mathbf{R} \boldsymbol{w}}{\sigma_D^2 + \boldsymbol{w}^{\boldsymbol{H}} \mathbf{Q} \boldsymbol{w}} \geq \zeta \end{array},$$

with (P_0 is trans power, σ^2/σ_D^2 are noise powers at relays/destination)

$$\mathbf{D} \triangleq P_0 \text{diag}\left(\left[|f_1|^2 \; |f_2|^2 \; \ldots \; |f_R|^2\right]^{\boldsymbol{T}}\right) + \sigma^2 \mathbf{I}_R,$$

$$\mathbf{R} \triangleq P_0 \mathbf{h}\mathbf{h}^{\boldsymbol{H}} \in \mathbb{S}_+^R, \text{ with } \mathbf{h} \triangleq [f_1 g_1 \; f_2 g_2 \; \ldots \; f_R g_R]^{\boldsymbol{T}} \text{ and}$$

$$\mathbf{Q} \triangleq \sigma^2 \text{diag}\left(\left[|g_1|^2 \; |g_2|^2 \; \ldots \; |g_R|^2\right]^{\boldsymbol{T}}\right).$$

- The optimal value can be expressed as [Havary-Nassab et al., 2008]

$$V(\mathbf{p}(t), t) \equiv V \triangleq \frac{\lambda_{max}\left(\mathbf{D}^{-1/2}(\mathbf{R} - \zeta \mathbf{Q})\mathbf{D}^{-1/2}\right)}{\zeta \sigma_D^2} \triangleq \frac{\lambda_{max}(\mathbf{B})}{\zeta \sigma_D^2}.$$

- At each t, V **depends on the positions of the relays**.

Mobile Beamforming? (1)

- Dependence of V on $\mathbf{p}(t)$ immediately generates a basic question:
  - **Given that we are at time** $t-1$, can we **further increase** $V(\mathbf{p}(t), t)$ by exploitting **relay mobility**?
  - What does it mean to "further increase $V(\mathbf{p}(t), t)$"?
  - On the basis of **what information** and **in which sense**?
- First, "further increase $V(\mathbf{p}(t+1), t+1)$" should mean that,
  - **at the new relay positions** $\mathbf{p}(t+1)$,
  - channels $\{f_i, g_i\}_{i \in \mathbb{N}_R^+}$ should be such **that we can beamform with smaller power** (can we do it the opposite way?).
- Then, we may agree on a basic *time division* protocol:
  At each time interval
  1. **Beamforming**.
  2. **Relay steering** to the new positions (to be determined).

Mobile Beamforming? (2)

- Let $\{\mathscr{C}(\mathcal{T}_t)\}_{t\in\mathbb{N}^+_{N_T}}$ denote relay CSI, along the path of their trajectories

$$\mathcal{T}_i \triangleq \{\mathbf{p}(t)\}_{t\in\mathbb{N}^+_i}, i \in \mathbb{N}^+_{N_T}, \quad \text{where} \quad \mathcal{T}_t \equiv \{\mathcal{T}_{t-1}, \mathbf{p}(t)\}.$$

- Suppose that, at time slot $t-1$, **an oracle reveals** $\mathscr{C}(\mathcal{T}_t) \equiv \mathscr{C}(\{\mathcal{T}_{t-1}, \mathbf{p}(t)\})$, *for each choice of* $\mathbf{p}(t)$.
- Then, we could optimize $V(\mathbf{p}(t), t)$ with respect to $\mathbf{p}(t)$ deterministically, for each fixed channel realization the oracle has revealed.
- Such an oracle **does not exist :(** - **Choice of $\mathbf{p}(t)$ cannot be noncausal**!
- *Approach*: **Stochastic Optimization**
  - Maximize $V(\mathbf{p}(t), t)$ **on average** with respect to $\mathbf{p}(t)$,
  - Taking into account **all the history of channel observations** $\{\mathscr{C}(\mathcal{T}_t)\}_{t\in\mathbb{N}^+_{N_T}}$.
  - $\Rightarrow$ Systematically exploit **spatiotemporal statistical dependencies of** $\{f_i, g_i\}_{i\in\mathbb{N}^+_R}$.

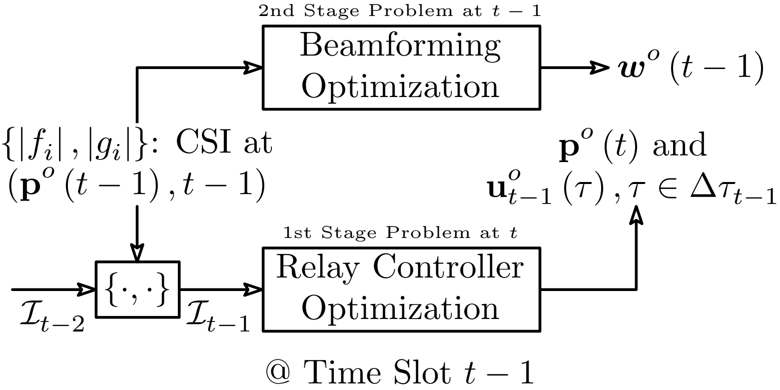
Mobile Beamforming! (1)

- We are then led to the 2-**stage stochastic program** [Shapiro et al., 2009]

$$\begin{array}{ll} \underset{\mathbf{p}(t)}{\text{maximize}} & \mathbb{E}\left\{ V\left(\mathbf{p}\left(t\right), t\right)\right\} \\ \text{subject to} & \mathcal{C}\left(\mathbf{p}\left(t-1\right)\right) \ni \mathbf{p}\left(t\right) \equiv \mathbb{E}\left\{\mathbf{p}\left(t\right) \middle| \mathscr{C}\left(\mathcal{T}_{t-1}\right)\right\} \end{array},$$

  where $\mathcal{C}\left(\mathbf{p}\left(t-1\right)\right)$ denotes a spatially feasible convex set.
- The outer one is called the first stage problem.
  - Take a predictive action.
- The inner problem whose value is V is called the second stage problem.
  - Uncertainty is revealed.
  - Take a "recourse" (in a sense) action.

Mobile Beamforming! (2)



Mobile Beamforming! (3)

- Using a generalized version of the *Fundamental Lemma of Stochastic Control*, the original 2-stage program stated above is equivalent to the **non-variational** problem

$$\underset{\mathbf{p}(t)\in\mathcal{C}(\mathbf{p}(t-1))}{\text{maximize}} \ \mathbb{E}\left\{ \lambda_{max}\left(\mathbf{B}\left(\mathbf{p}\left(t\right),t\right)\right) \middle| \mathscr{C}\left(\mathcal{T}_{t-1}\right)\right\}$$

- However, $\mathbb{E}\left\{ \lambda_{max}\left(\mathbf{B}\left(\mathbf{p}\left(t\right),t\right)\right) \middle| \mathscr{C}\left(\mathcal{T}_{t-1}\right)\right\}$ is impossible to evaluate **:(**
- $\Rightarrow$ Therefore, we have to resort to some reasonable approximation.

Mobile Beamforming! (4)

- By Jensen's Inequality, we may consider the **lower bound relaxation**

$$\underset{\mathbf{p}(t)\in\mathcal{C}(\mathbf{p}(t-1))}{\text{maximize}}\ \lambda_{max}\left(\mathbb{E}\left\{\mathbf{B}\left(\mathbf{p}\left(t\right),t\right)\middle|\mathscr{C}\left(\mathcal{T}_{t-1}\right)\right\}\right)\triangleq\lambda_{max}\left(\boldsymbol{E}\left(\mathbf{p}\left(t\right)\right)\right).$$

- Under usual modeling assumptions concerning the small scale fading components of $\{f_i, g_i\}_{i\in\mathbb{N}_R^+}$, it is true that (drop the dependence on $(\mathbf{p}(t), t)$)

$$\boldsymbol{E}\equiv\text{diag}\left(\boldsymbol{E}_1\ \boldsymbol{E}_2\ \ldots\ \boldsymbol{E}_R\right),\quad\text{with}$$

$$\boldsymbol{E}_i\triangleq\mathbb{E}\left\{\left.\frac{P_0\left|f_i\right|^2\left|g_i\right|^2-\gamma\sigma^2\left|g_i\right|^2}{P_0\left|f_i\right|^2+\sigma^2}\right|\mathscr{C}\left(\mathcal{T}_{t-1}\right)\right\},\ i\in\mathbb{N}_R^+.$$

- Therefore, the relaxed problem is equivalent to

$$\max_{\mathbf{p}_i(t)\in\mathcal{C}(\mathbf{p}_i(t-1))}\left[\max_{i\in\mathbb{N}_R^+}\boldsymbol{E}_i\left(\mathbf{p}_i\left(t\right)\right)\right]\equiv\max_{i\in\mathbb{N}_R^+}\left[\max_{\mathbf{p}_i(t)\in\mathcal{C}(\mathbf{p}_i(t-1))}\boldsymbol{E}_i\left(\mathbf{p}_i\left(t\right)\right)\right]$$

and the challenge now is the evaluation of $\boldsymbol{E}_i\left(\mathbf{p}_i\left(t\right)\right)$.

Mobile Beamforming! (5)

- Hereafter, we make the **high-SNR assumption**

$$\frac{1}{P_0 \left|f_i\right|^2 + \sigma^2} \approx \frac{1}{P_0 \left|f_i\right|^2}, \quad \forall i \in \mathbb{N}_R^+.$$

- Then,

$$\boldsymbol{E}_i = \mathbb{E}\left\{ \left|g_i\right|^2 \middle| \mathscr{C}\left(\mathcal{T}_{t-1}\right) \right\} - \frac{\gamma \sigma^2}{P_0} \mathbb{E}\left\{ \left|g_i\right|^2 \left|f_i\right|^{-2} \middle| \mathscr{C}\left(\mathcal{T}_{t-1}\right) \right\}.$$

- Final question: **Is it possible to evaluate $\boldsymbol{E}_i$ in a reasonable manner so that we can at least maximize it locally**?
- **This depends heavily on channel modeling**.

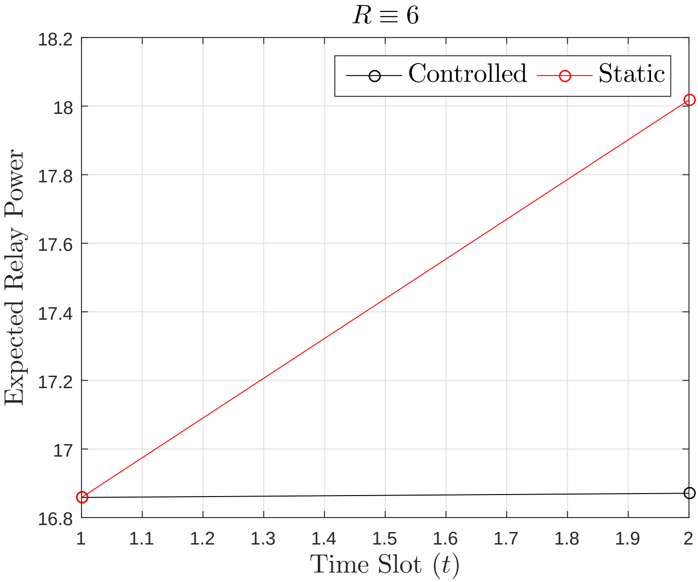
Mobile Beamforming! (6) - On Channel Modeling

- An acceptable flat fading channel model (same for $g_i\left(\mathbf{p}_i(t), t\right)$)

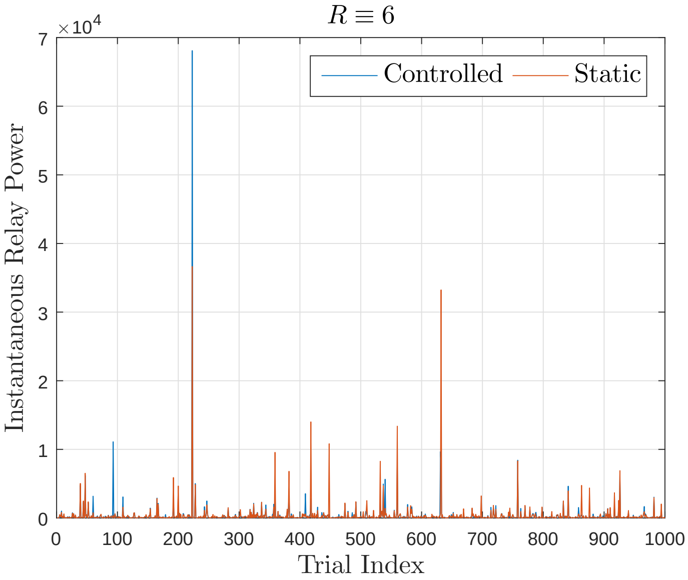
$$f_i\left(\mathbf{p}_i(t), t\right) \equiv \underbrace{f^{PL}(\mathbf{p}_i(t))}_{\text{path loss}} \underbrace{f_i^{SH}(t)}_{\text{shadowing}} \underbrace{f_i^{MF}(t)}_{\text{fading}},$$

  whose **squared** magnitude in dB can be well approximated by a **Gaussian random variable** [Malmirchegini and Mostofi, 2012], where:
  - The path loss component is deterministic (conditioned on $\mathbf{p}_i(t)$).
  - The fading component is *spatiotemporally white, and thus unpredictable*.
  - Useful statistical dependencies are due to shadowing effects.
- Assuming **joint Gaussianity both spatially and temporally**, we end up with a well defined *spatiotemporal Gaussian random field*.
- Then, $\boldsymbol{E_i}$ **can be computed in closed form :)**

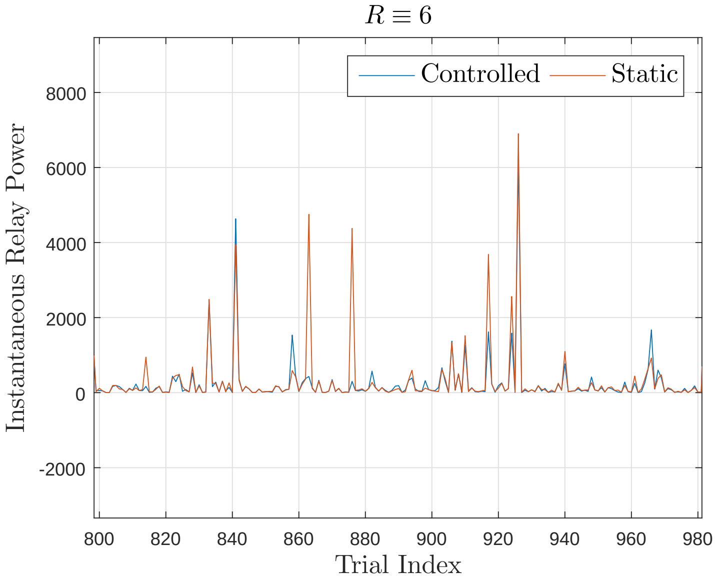
Some Numerical Simulations (1)



Some Numerical Simulations (2)



Some Numerical Simulations (3)



Conclusions

- We considered **stochastic motion planning** in **single source/destination AF mobile relay beamforming networks**.
- We formulated a **2-stage stochastic program** for spatial relay control
- We showed that **beamforming can indeed be benefited by exploitting relay mobility**.
- However, although the proposed solution
  - is **power efficient**,
  - it produces **quite conservative results. :(**
- Topics for future research:
  - Tighter relaxations to the *expected maximum eigenvalue maximization*.
  - Alternative *wireless channel modeling* assumptions.
  - Extensions to *other beamforming problems* (e.g. multiuser networks).
  - And many more!

The End

THANK YOU!

References

V. Havary-Nassab, S. ShahbazPanahi, A. Grami, and Zhi-Quan Luo. Distributed beamforming for relay networks based on second-order statistics of the channel state information. *Signal Processing, IEEE Transactions on*, 56(9):4306–4316, Sept 2008. ISSN 1053-587X. doi: 10.1109/TSP.2008.925945.

M. Malmirchegini and Y. Mostofi. On the spatial predictability of communication channels. *Wireless Communications, IEEE Transactions on*, 11(3):964–978, March 2012. ISSN 1536-1276. doi: 10.1109/TWC.2012.012712.101835.

A. Shapiro, D. Dentcheva, and A. Ruszczynski. *Lectures on Stochastic Programming: Modeling and Theory (MPS-SIAM Series on Optimization)*. SIAM-Society for Industrial and Applied Mathematics, 2009.