ON THE DETECTION OF NON-STATIONARY SIGNALS IN THE MATCHED SIGNAL TRANSFORM DOMAIN

Andrei ANGHEL¹, Gabriel VASILE², Cornel IOANA², Remus CACOVEANU¹ and Silviu CIOCHINA¹

¹Faculty of Electronics, Telecommunications and Information Technology, University POLITEHNICA of Bucharest
²Grenoble-Image-sPeec-Signal-Automatics Lab, Grenoble INP/CNRS

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andrei.anghel@munde.pub.ro
Motivation

Why study a detection problem in the Matched Signal Transform (MST) domain?

The Matched Signal Transform (MST) can be employed to:

✓ suppress wide-band time-varying interference,
✓ track the Instantaneous Frequency Laws (IFLs) of multi-component non-stationary signals,
✓ process nonlinear beat signals provided by a Frequency Modulated Continuous Wave (FMCW) Radar.

- How can we design a Constant False Alarm Rate (CFAR) detector in the MST domain?
- What is the probability density function (PDF) of the noise samples in this transformed domain?

The Matched Signal Transform (MST)

Definition:

\[ S(\alpha) = \int_{t \in \mathcal{D}} |\theta'(t)|s(t)e^{-j2\pi\alpha\theta(t)} \, dt \]

\( \alpha \): modulation rate
\( \theta(t) \): characteristic (basis) function

An essential property for non-stationary signals:

\[ s(t) = \sum_{m=1}^{M} A_m e^{j2\pi\alpha_m \theta(t)} \quad \text{S}(\alpha) = \sum_{m=1}^{M} A_m \delta(\alpha - \alpha_m) \]

Localizes non-stationary signals described by \( \theta(t) \) at their modulation rates.
Classical MSTs

<table>
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<th>Characteristic function</th>
<th>Transform</th>
<th>Time-Frequency representation of the matched signal</th>
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<tr>
<td>$\theta(t) = t$</td>
<td>Fourier</td>
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<tr>
<td>$\theta(t) = t^2$</td>
<td>Linear MST</td>
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<td>$\theta(t) = e^{kt}$</td>
<td>Exponential</td>
<td><img src="image3" alt="Exponential" /></td>
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MST and Time warping

\[ s(t) = \sum_{m=1}^{M} A_m e^{j2\pi \alpha_m \theta(t)} \]

\[ w(t_w) = \theta^{-1}(t_w) \]

Warping operator

\[ \{ W \mid \frac{dw}{dt} > 0; x(t) \rightarrow Wx(t) = x(w(t)) \} \]

All IFLs become simultaneously complex sinusoids (stationary components).

Direct MST

\[ S(\alpha) = \int_{t \in \mathcal{D}} |\theta'(t)|s(t)e^{-j2\pi \alpha \theta(t)} dt \]

Time warping-based MST

( Fourier transform in the warped time axis)

\[ S(\alpha) = \int_{t_w \in \mathcal{D}_w} s_{\text{warp}}(t_w)e^{-j2\pi \alpha t_w} dt_w \]
MST implementations

Analog MSTs

\[ S(\alpha) = \int_{t \in \mathcal{D}} |\theta'(t)| s(t) e^{-j2\pi\alpha \theta(t)} dt \quad \text{Direct MST: summation for each } \alpha_k \]

\[ S(\alpha) = \int_{t_w \in \mathcal{D}_w} s_{\text{warp}}(t_w) e^{-j2\pi\alpha t_w} dt_w \]

Discretization

\[ s[n] = s(t_n), \quad t_n = nT_S, \quad n = 0, N-1 \]

\[ \alpha_k, k = 0, K-1 \]

Discrete MSTs

\[ S_{\text{MST}}[k] = \frac{1}{\Theta} \sum_{n=0}^{N-1} |\theta'(t_n)| s[n] e^{-j2\pi\alpha_k \theta(t_n)} \]

\[ \Theta = \sum_{n=0}^{N-1} |\theta'(t_n)| : \text{amplitude normalization} \]

\[ S_{\text{RS}}[k] = \frac{1}{N} \sum_{n=0}^{N-1} s_w[n] e^{-j2\pi\alpha_k t_{w,n}} \]

\[ s_w[n] : \text{resampled version of } s[n] \text{ at the moments } t_{w,n} = nT_{S,w} \]

Time Resampling

+ Fast Fourier Transform (FFT)
Time resampling

\[ s_1(t) = A_1 e^{j2\pi \alpha_1 \theta(t)} \]

\( s_1 \) in the initial time axis (uniform sampling)

\( s_1(t) \quad s_1[n], t_n = nT_S \)

Warping with \( t_w = \theta(t) \)

\( s_1(t_w) \quad s_1[n], \theta(t_n) \)

Resampling (Interpolation)

\( s_1(t_w) \quad s_{w,1}[n], \quad t_{w,n} = nT_{S,w} \)
The MST of noisy signals

\[ x[n] = \sum_{m=1}^{M} A_m e^{j2\pi \alpha_m \theta(nT_s)} + w_R[n] + jw_I[n] = s[n] + w[n] \]

Matched signal + Complex Circular White Gaussian Noise

\[ E\{w_{R,I}[n]\} = 0 \]
\[ E\{|w_{R,I}[n]|^2\} = \sigma^2 \]

Expectation of the MST’s squared magnitude

\[ E\{|X_{RS}[k]|^2\} \]
- Nearest neighbor interpolation
- Linear interpolation

\[ E\{|X_{MST}[k]|^2\} \]

The noise floor in the MST domain depends on:
- Implementation
- Modulation rate.
The MST of noise samples

\[ w[n] = w_R[n] + jw_I[n] \]  \hspace{1cm} \text{Complex Circular White Gaussian Noise} \]

\[ E\{w_R[n]\} = 0 \]
\[ E\{|w_R[n]|^2\} = \sigma^2 \]

Probability Density Function (PDF)
\[ f_1(u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{u^2}{2\sigma^2}} \]

\[ F_1(v) = e^{-\frac{\sigma^2 v^2}{2}} \]  \hspace{1cm} \text{Characteristic Function (CF)}

What is the PDF of the noise samples in the MST domain?

Direct implementation
\[ W_{MST}[k] = \frac{1}{\Theta} \sum_{n=0}^{N-1} |\theta'(t_n)|w[n] e^{-j2\pi\alpha_k \theta(t_n)} \]

Time resampling implementation
\[ W_{RS}[k] = \frac{1}{N} \sum_{n=0}^{N-1} w[n] e^{-j2\pi\alpha_k t_{w,n}} \]

\[ \Re\{W_{MST}[k]\} \quad \Im\{W_{MST}[k]\} \quad \Re\{W_{RS}[k]\} \quad \Im\{W_{RS}[k]\} \]
The MST of noise samples
Direct implementation

\[
Re\{W_{MST}[k]\} = \sum_{n=0}^{N-1} w_R[n] \frac{1}{\Theta} |\theta'(t_n)| \cos(2\pi \alpha_k \theta(t_n)) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{\Theta} |\theta'(t_n)| \sin(2\pi \alpha_k \theta(t_n))
\]

\[
Im\{W_{MST}[k]\} = -\sum_{n=0}^{N-1} w_R[n] \frac{1}{\Theta} |\theta'(t_n)| \sin(2\pi \alpha_k \theta(t_n)) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{\Theta} |\theta'(t_n)| \cos(2\pi \alpha_k \theta(t_n))
\]

The real and imaginary parts of a sample \(W_{MST}[k]\) are a weighted sum of the initial noise samples. The PDF of \(Re/Im\{W_{MST}[k]\}\) can be computed using classical results of random variables theory.

If \(x_0, x_1, ..., x_{N-1}\) are random variables having the CFs \(F_{x_0}(v), F_{x_1}(v), ..., F_{x_{N-1}}(v)\), then the CF of \(S = \sum_{n=0}^{N-1} a_n x_n\) is \(F_S(v) = \prod_{n=0}^{N-1} F_{x_n}(a_n v)\).

\[
F_{Re/Im\{W_{MST}[k]\}}(v, k) = \exp \left\{ - \left( \frac{\sigma^2}{\Theta^2} \sum_{n=0}^{N-1} |\theta'(t_n)|^2 \right) \frac{v^2}{2} \right\}
\]

The CF of a Gaussian noise, independent of \(\alpha_k\).
The MST of noise samples
Resampling-based implementation (1)

Interpolation method: Nearest neighbor

After nearest neighbor resampling:
- Some samples may not appear anymore,
- While others may be repeated.

We can define an index function
\( \beta[n, l] \)
to link \( N \) samples from the initial signal to \( N \) samples of the resampled one, in the following way:

Sample \( n \) from the initial signal appears in the resampled signal at the indices:
\( \beta[n, 1], \beta[n, 2], \ldots, \beta[n, \nu(n)] \),
where \( \nu(n) \) is the number of repetitions of sample \( n \).
The MST of noise samples
Resampling-based implementation (2)

Interpolation method: Nearest neighbor

\[
\text{Re}\{W_{RS}[k]\} = \sum_{n=0}^{N-1} w_R[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \cos\left(2\pi \alpha_k t_{w,\beta[n,l]}\right) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \sin\left(2\pi \alpha_k t_{w,\beta[n,l]}\right)
\]

\[
\text{Im}\{W_{RS}[k]\} = -\sum_{n=0}^{N-1} w_R[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \sin\left(2\pi \alpha_k t_{w,\beta[n,l]}\right) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \cos\left(2\pi \alpha_k t_{w,\beta[n,l]}\right)
\]

✓ The real and imaginary parts of a sample \(W_{RS}[k]\) are a weighted sum of the initial noise samples that takes into account the index function \(\beta[n, l]\).

\[
F_{\text{Re/Im}\{W_{RS}[k]\}}(v, k) = \exp\left\{-\left(\frac{\sigma^2}{N^2} \sum_{n=0}^{N-1} \left(\sum_{l=1}^{\nu(l)} \cos\left(2\pi \alpha_k t_{w,\beta[n,l]}\right)\right)^2 + \left(\frac{1}{N} \sum_{l=1}^{\nu(l)} \sin\left(2\pi \alpha_k t_{w,\beta[n,l]}\right)\right)^2\right\} \frac{v^2}{2}\right\}
\]

The CF of a Gaussian noise, whose variance \(\sigma_W^2[k]\) depends on:

- \(\alpha_k\) : the modulation rate,
- \(\beta[n, l]\) : the actual linking between the initial signal and the resampled one.
Detection scheme in the MST domain (1)

\[ |W[k]|^2 = \Re\{W[k]\}^2 + \Im\{W[k]\}^2 \]

Sum of two squared independent Gaussian variables.

\[ PDF_{|W[k]|^2}(u) = \frac{1}{2\sigma^2_W[k]} \exp\left(-\frac{u}{2\sigma^2_W[k]}\right) \]
and
\[ P_F = \int_{-\infty}^{\infty} PDF_{|W[k]|^2}(u) \, du \]
Detection scheme in the MST domain (2)

\[ H_1: E\{|X[k]|^2\} = |\hat{A}_k|^2 + 2\sigma_W^2[k] \]
\[ H_0: E\{|X[k]|^2\} = 2\sigma_W^2[k] \]

\[ x[n] \rightarrow \text{MST} \rightarrow ||^2 \rightarrow \text{Peak picking} \rightarrow \text{Decision} \]

ROC (circular noise)
Contributions:

• We point out the characteristics of white Gaussian noise in the MST domain, and

• Propose a detection scheme for non-stationary signals processed with two implementations of the discrete MST.

In future work:

• The theoretical parameters of the noise in the MST domain will be determined for other interpolation methods.

• The results will be applied to radar and ultrasound applications.
Thanks for your attention !!!