Sequential observer selection for source localization

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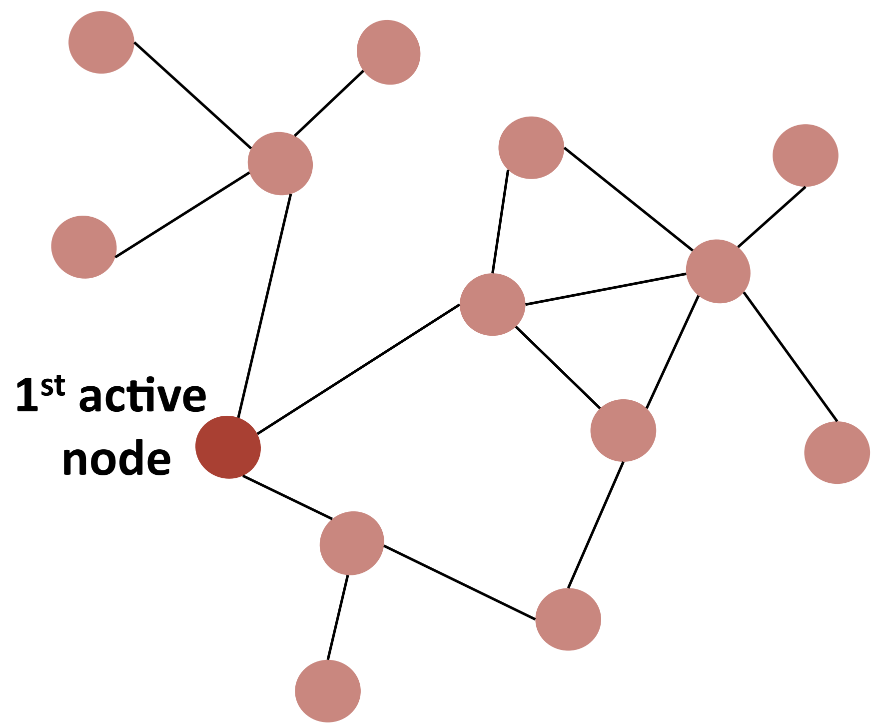
Different phenomena modeled as network diffusion



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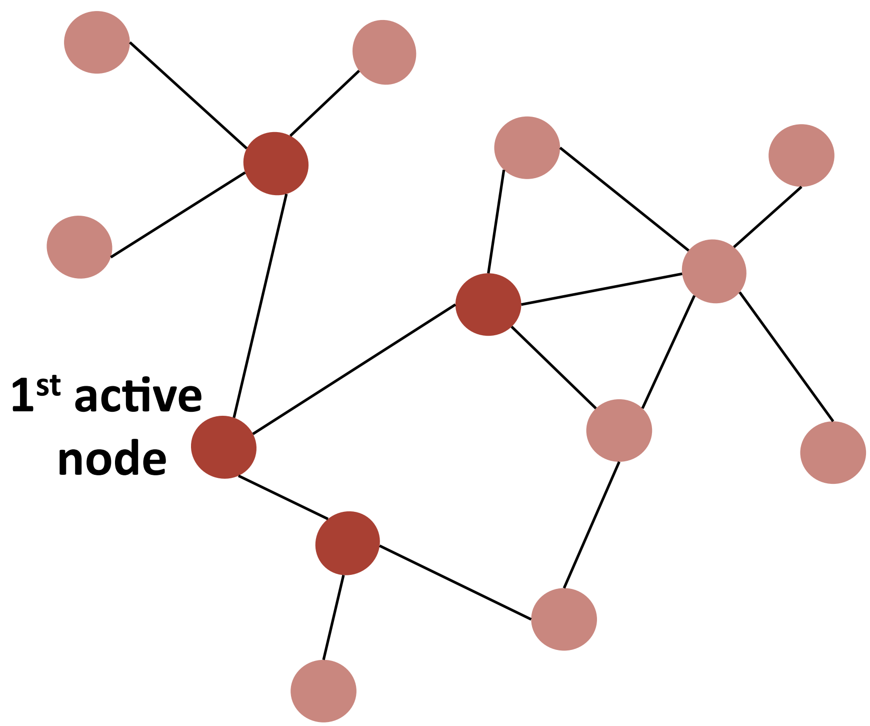

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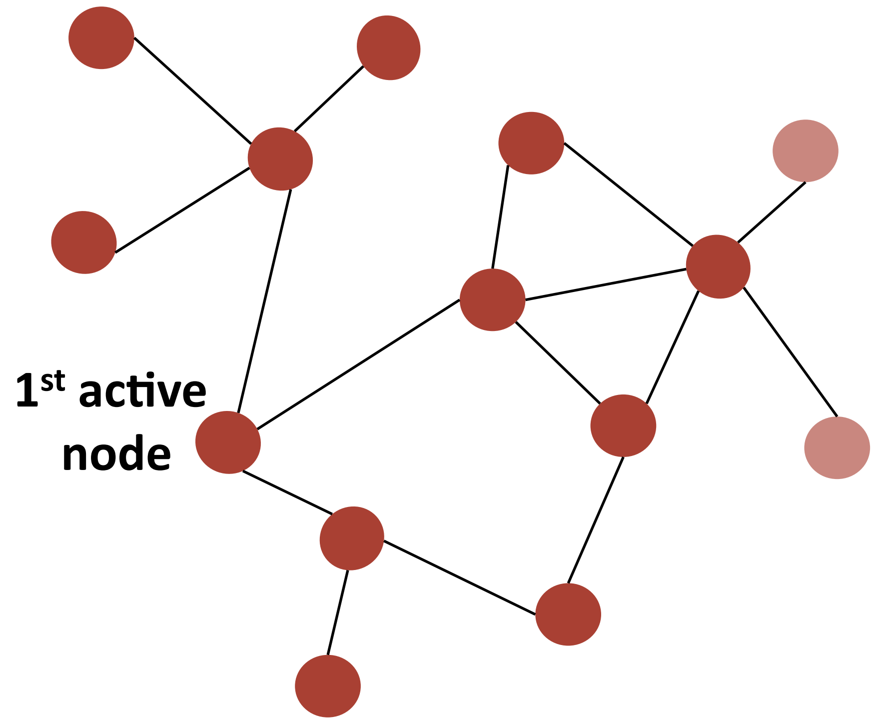
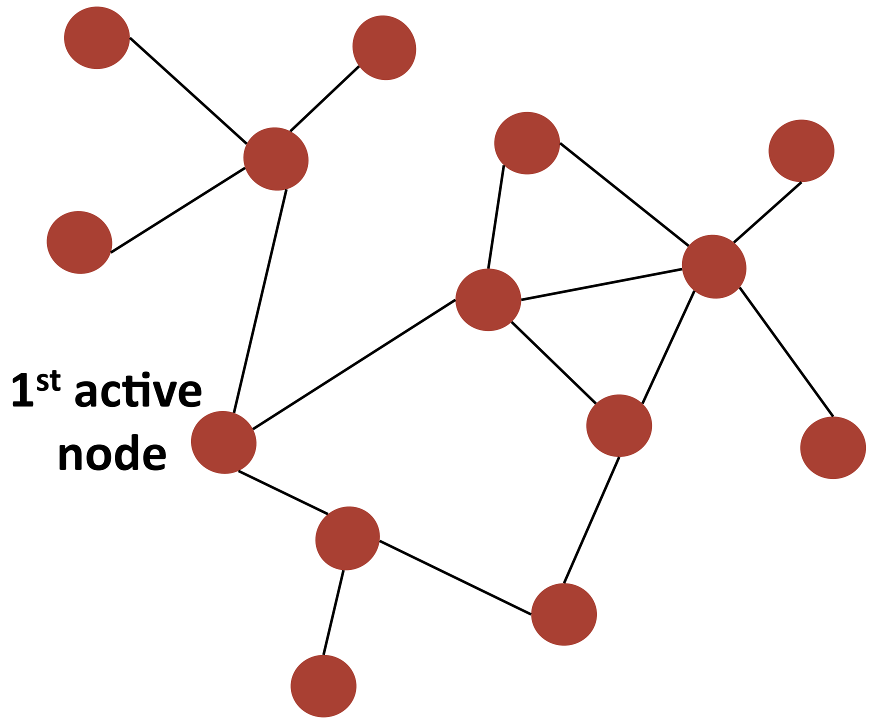


Different phenomena modeled as network diffusion

- *Propagation of a disease in human population*
- *Dissemination of information in a social network*
- *Spreading of a computer virus in a communication network*



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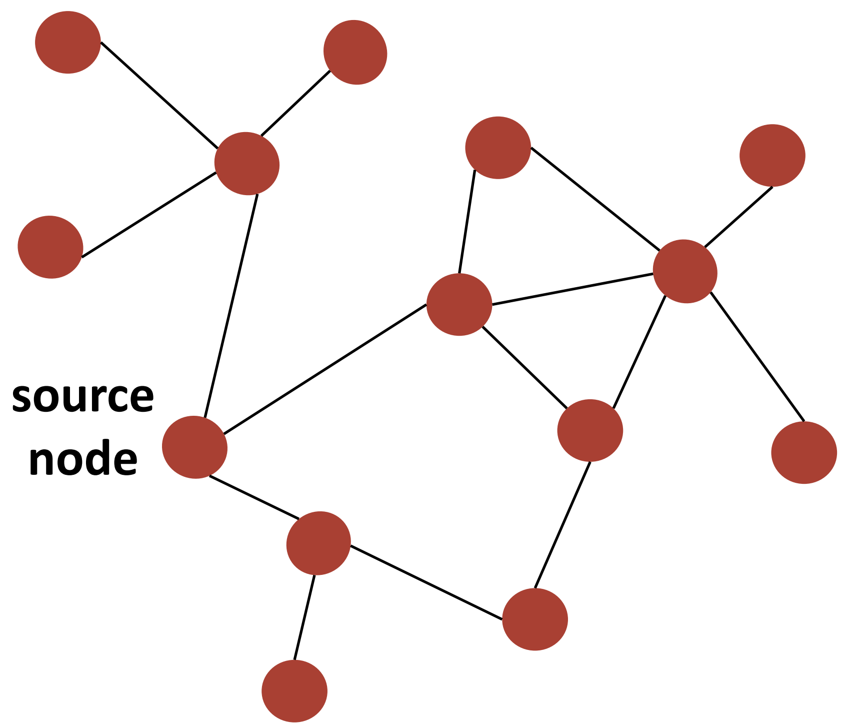
Source of diffusion

- *Patient zero*
- *Trendsetter*
- *1st infected computer*



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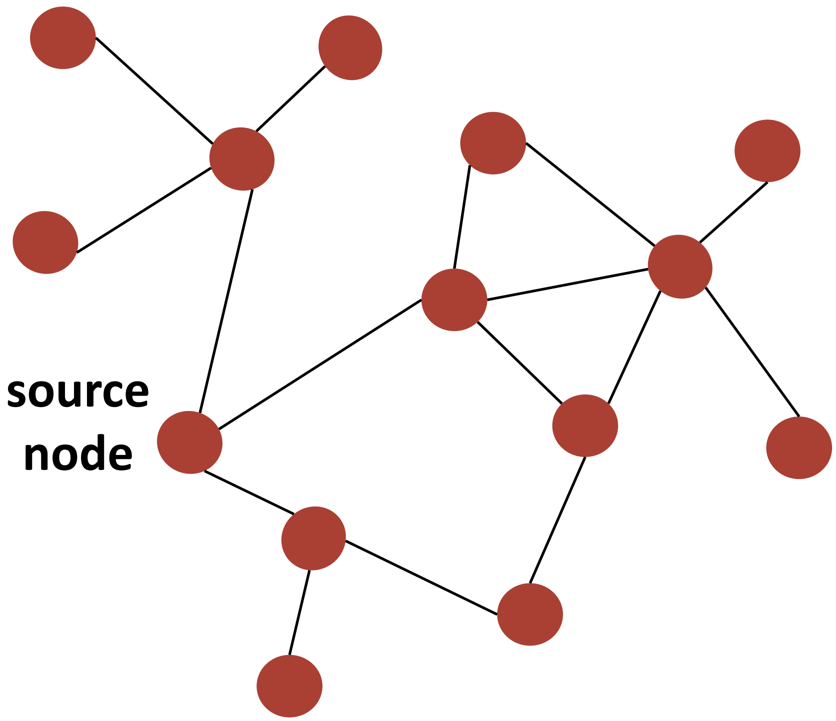


Source of diffusion

- *Patient zero*
- *Trendsetter*
- *1st infected computer*

Limited access to state of the network nodes

- ***Network size***
- ***Privacy issues***
- ***Cost of observation***

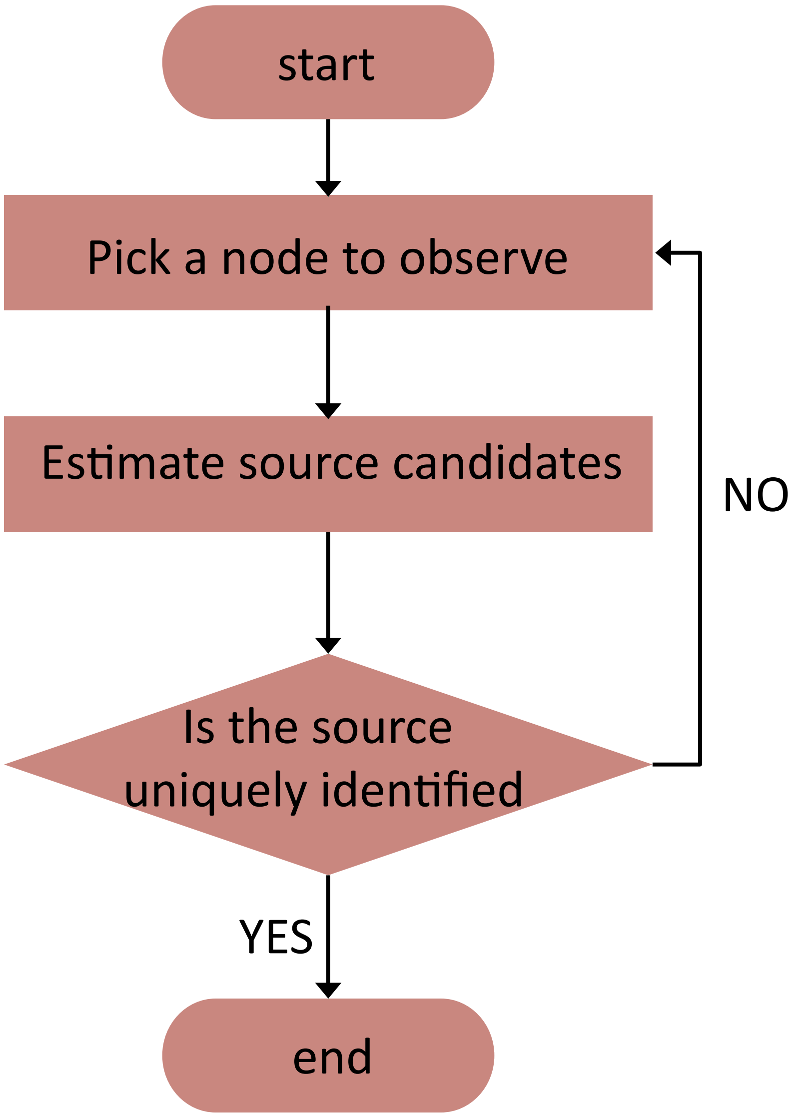
Research on network diffusion

- parameters of network diffusion
- source localization with or without timestamps of infection
- strategies for selection of the nodes that are observed: offline, mostly simulation-based

Contributions

- *sequential* dynamic selection
- *theoretical analysis* of the optimal selection strategy
    - show it is combinatorial problem even with the simplified assumptions
    - provide optimal solution
    - derive efficient approximation, yet with guarantees
    - gain insight for more complex assumptions

Sequential identification of the source



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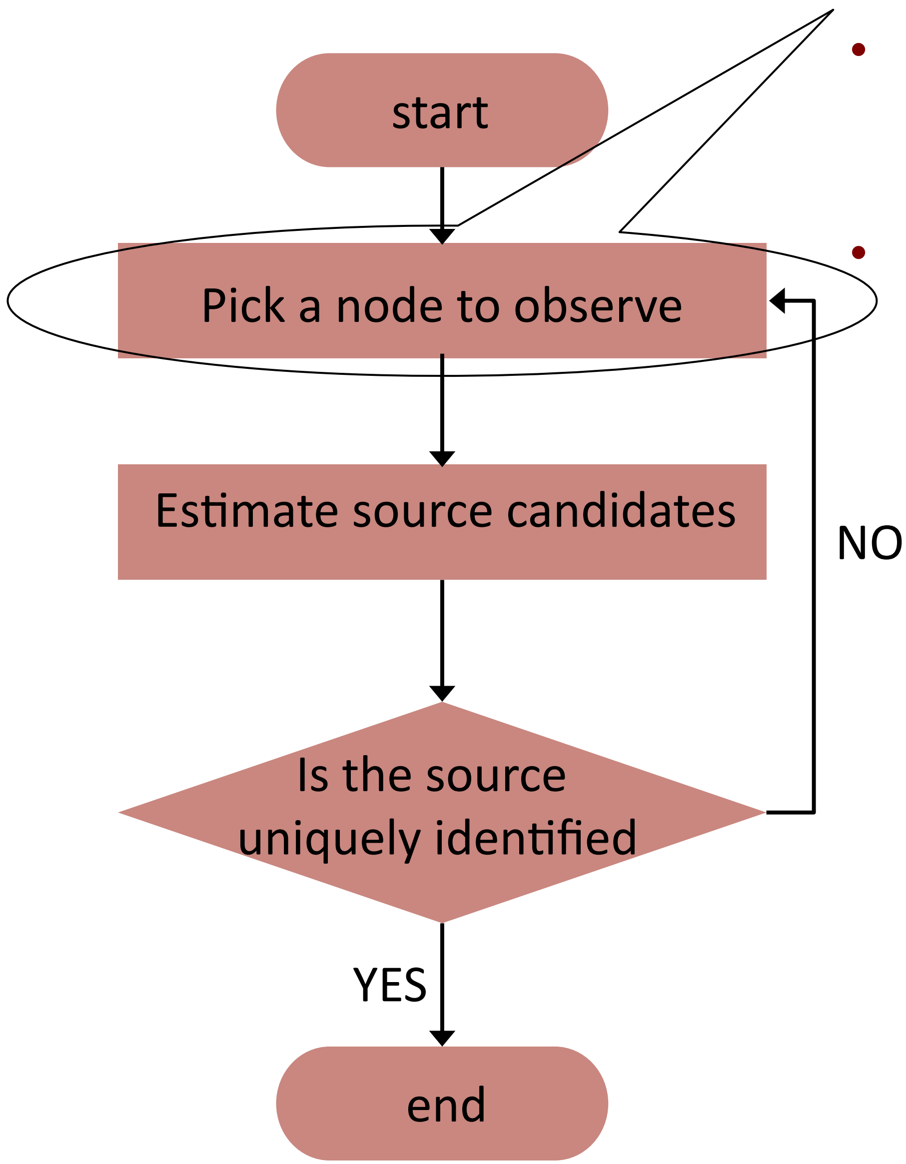
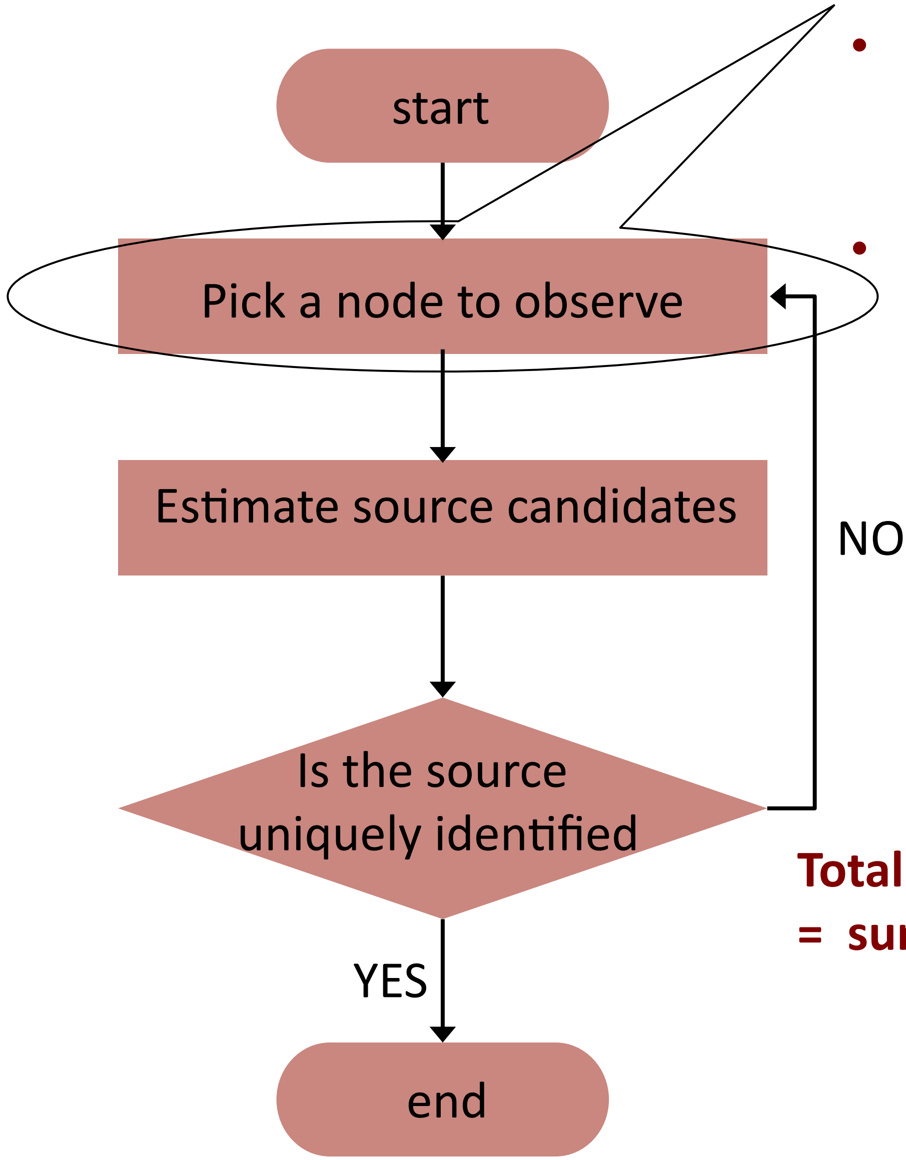


Network node

- **Information about the source through its observation: timestamp of infection**
- **Cost of observation**

Sequential identification of the source

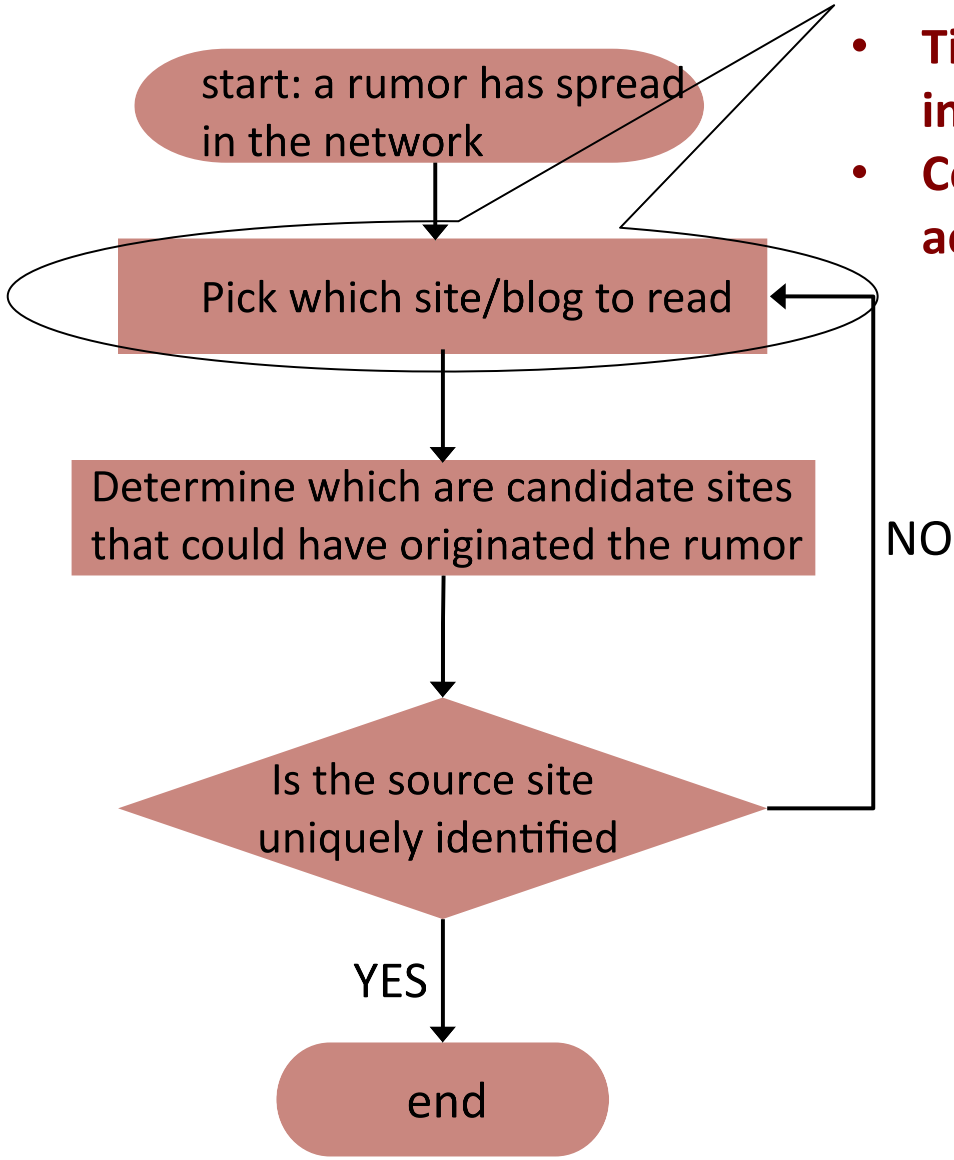


Network node

- **Information about the source through its observation: timestamp of infection**
- **Cost of observation**

Total cost of source identification
= sum of costs of observers

Sequential identification of the rumor source



Each site/blog has

- **Timestamp when it posted information**
- **Cost (time, effort) of accessing the site**

Problem statement

1. **Find a selection strategy such that the source can be unambiguously localized with the smallest total cost.**

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2. **For a fixed number of observer nodes find a selection strategy that would result with the smallest number of source candidates.**

Proposed approaches

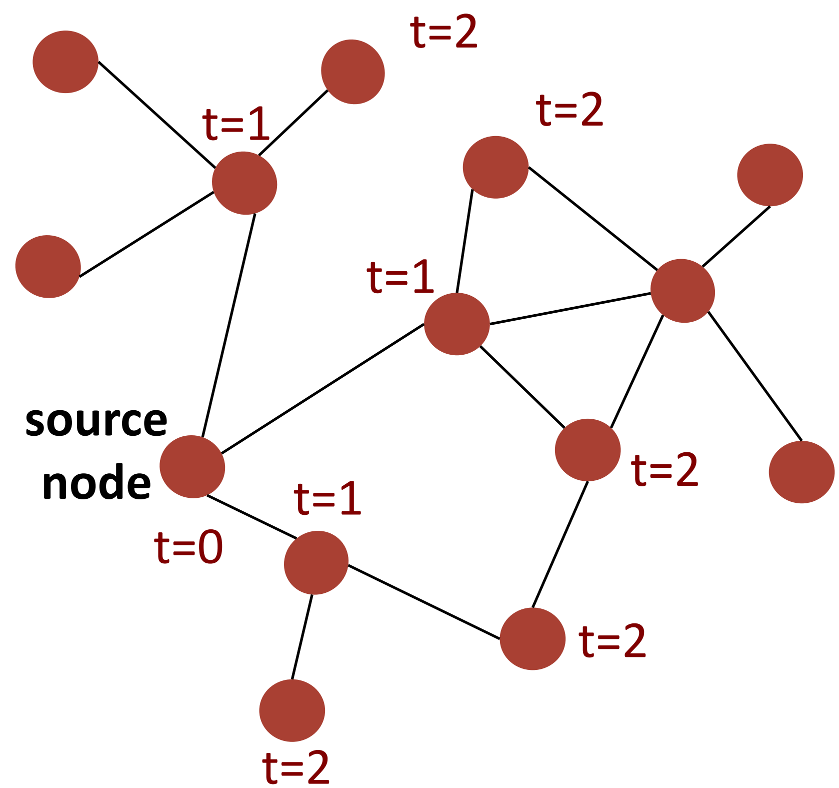
Approach	Optimality	Efficiency
Dynamic programming	✔	
Greedy		✔

Proposed approaches

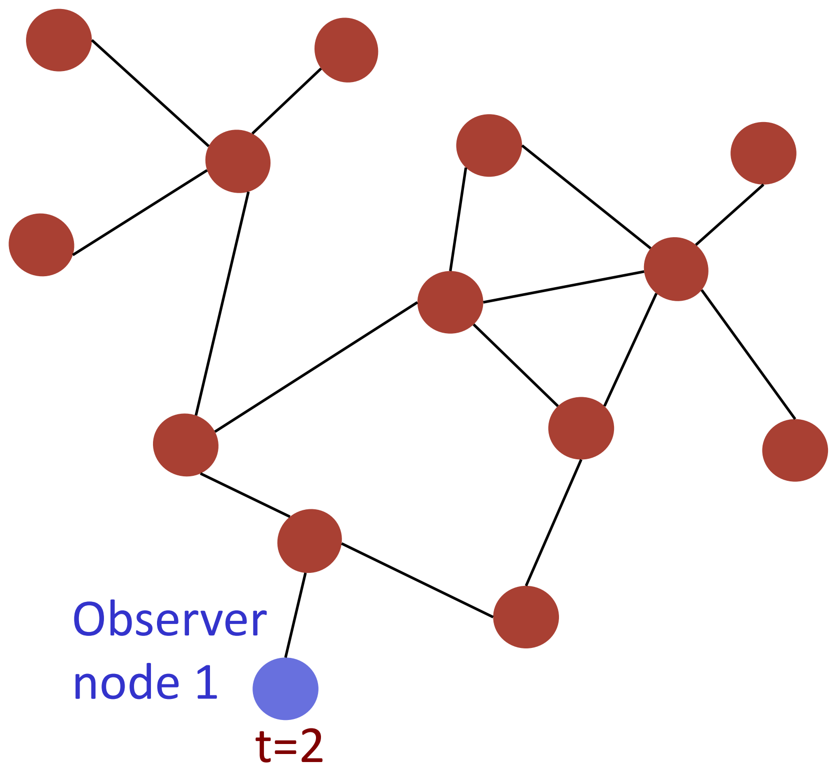
Approach	**Optimality**	**Efficiency**	**Performance guarantees**
Dynamic programming	✔		✔
Greedy (adaptive submodularity)		✔	✔

A simple model of network diffusion

- Single source node
- Nodes either susceptible or infected
- Nodes infected at time *t* infect neighbors with probability 1 at next time step *t+1*
- Times of infections deterministic: distance to the source



Source is localized based on the infection times of observed nodes



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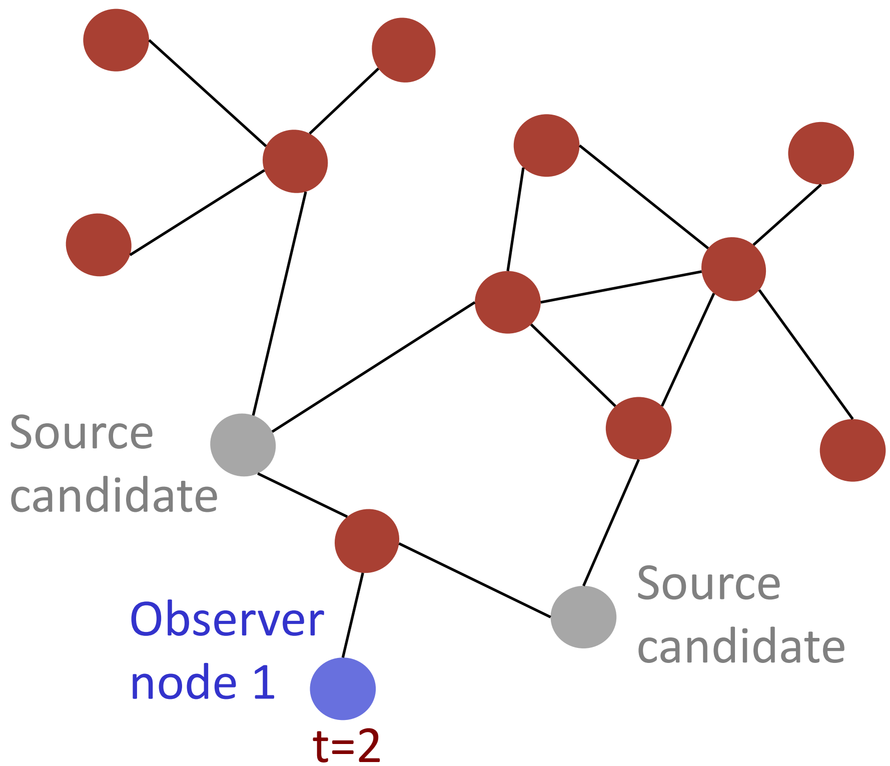

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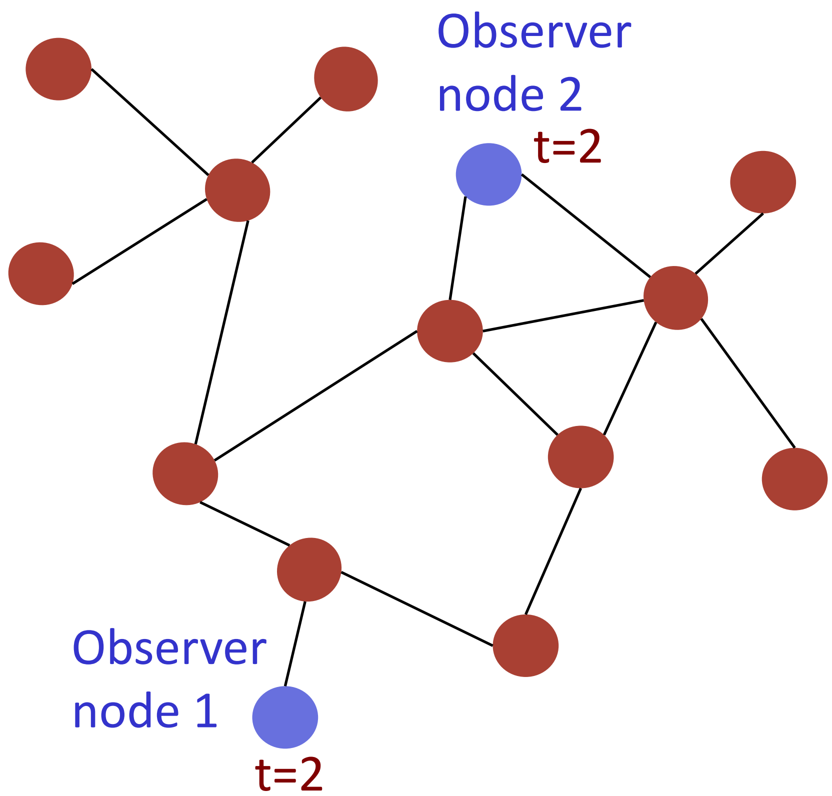

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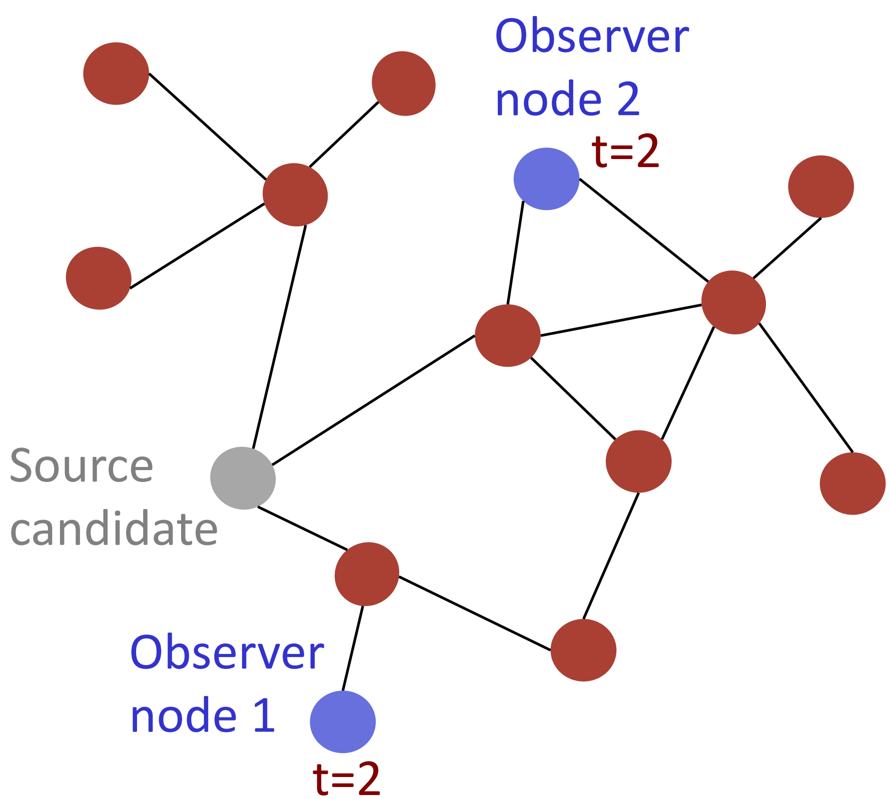

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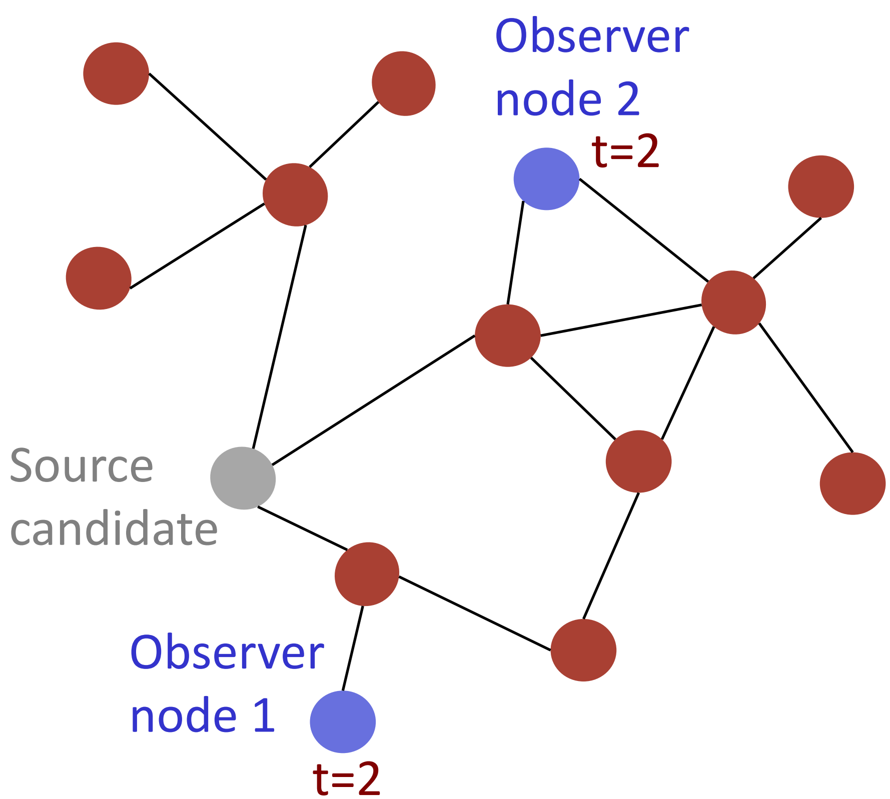

Total cost incurred = cost of observing Observer 1 + cost of observing Observer 2

Problem 1 formulation

Find a selection strategy π such that the source can be unambiguously localized with the smallest cost.

$$\min_{\pi} \mathbb{E}_s \left[c \left(O \left(\pi \right) \right) \right]$$

$$\text{subject to } \boldsymbol{d}(O(\pi), s) \neq \boldsymbol{d}(O(\pi), i), \forall s \in V, s \neq i,$$

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average taken over all possible sources

cost c incurred by observing a subset of nodes O chosen by strategy ***π***

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while each source node has a unique identifier

Problem 2 formulation

For a fixed number of observer nodes T find a selection strategy that would result with the smallest number of source candidates.

$$\min_{\pi} \mathbb{E}_s |S(O(\pi))|$$
$$\text{subject to } |O(\pi)| \leq T$$

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$$\min_{\pi} \mathbb{E}_s |S(O(\pi))|$$
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average taken over all possible sources

number of source candidates given by observing a subset of nodes O chosen by strategy $\boldsymbol{\pi}$

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while the number of observers is no more than T

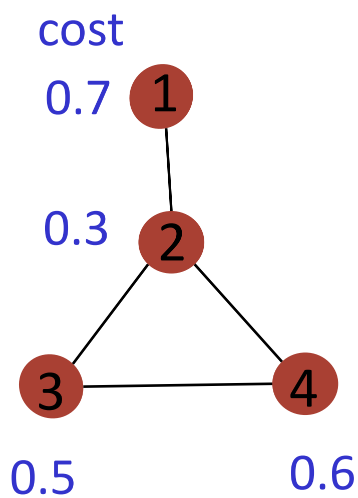
Solve both problems using dynamic programming with imperfect state knowledge

Problem is analyzed

- **backwards:** from the selection of the last observer to the selection of the first observer, one step at the time
- **offline:** considering all the possible sources, deriving what should be the best observer to select for possible observations

Example: Dynamic programming approach for Problem 1

Analysis for cost incurred by selecting node 2 in the first step



distance	node 1	node 2	node 3	node 4
node 2	1	0	1	1

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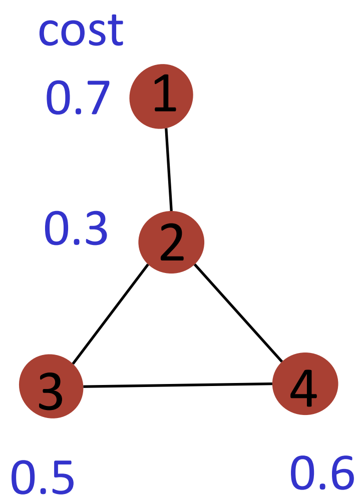

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- *source 1,3,4:* t=1, source candidates {1,3,4}, prob 3/4
- *source 2:* t=0, source candidates {2} , prob 1/4

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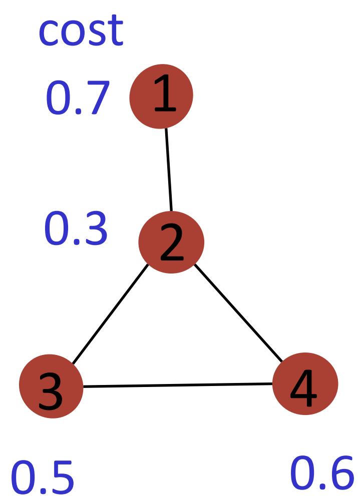

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In the previous step, step 2, we have calculated

- cost-to-go of state {1,3,4} as 0.5
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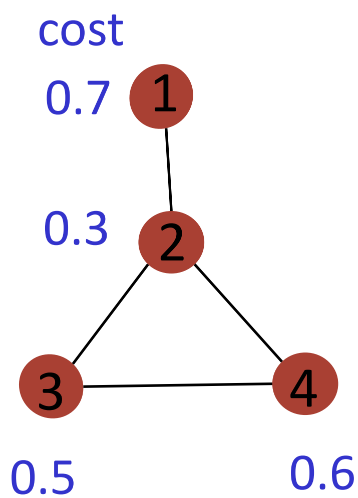

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Cost-to-go of selecting node 2= cost(node 2)+(3/4 *0.5+1/4*0)

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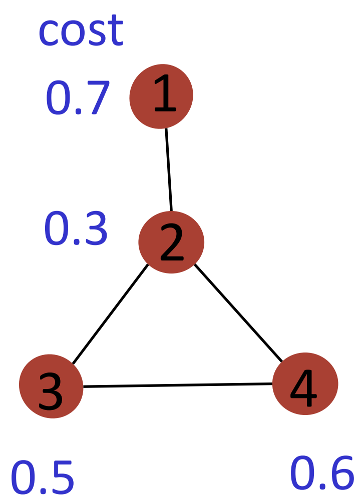

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Optimal node for step 1 is the node with the smallest cost-to-go.

Dynamic programming approach for Problem 1:

Selecting an optimal observer for an arbitrary time step k

$$\underset{\text{(observations)}}{\underset{\textbf{step k}}{\textbf{cost-to-go}}} = \underset{\text{observer o}}{\textbf{min}}\ \textbf{E}_{\textit{sources}} \left[\text{cost(o)} + \underset{\text{step k+1}}{\text{cost-to-go}}\ \text{(observations)} \right]$$

Dynamic programming approach for Problem 1:
Selecting an optimal observer for an arbitrary time step k

cost-to-go step k (observations) **=** **min** observer o **E** ***sources*** [cost(o) + cost-to-go (observations) step k+1]

Optimal observer step k (observations) **=** **arg min** *observer o* **cost-to-go step k** (observations)

Dynamic programming approach for Problem 1:
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$$\underset{\text{(observations)}}{\textbf{Optimal observer step k}} = \underset{\textit{observer } o}{\textbf{arg min}} \underset{\text{(observations)}}{\overset{\textbf{cost-to-go}}{\textbf{step k}}}$$

Dynamic programming is optimal, but generally intractable

- combinatorial nature of the problem

Efficient approximation, but with performance guarantees

- In order to obtain guarantees we resort to *adaptive submodularity** :
  if an optimization problem has this property, greedy approach has guarantees

*D. Golovin, A. Krause, "*Adaptive Submodularity: Theory and Applications in Active Learning and Stochastic Optimization*", Journal of Artificial Intelligence Research, 2011

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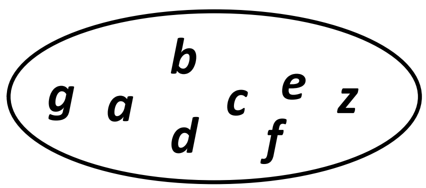
Adaptive submodularity – generalization of diminishing returns

set of items,
unknown states



pick an item

b

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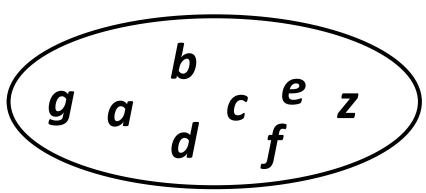

b

↓

learn the state
of the item

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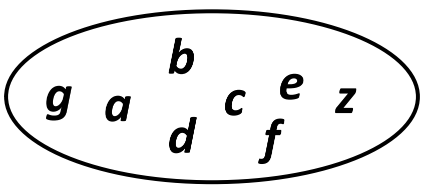

evaluate utility
function

b

↓

learn the state
of the item

↓

update our posterior
belief about the state
of the world

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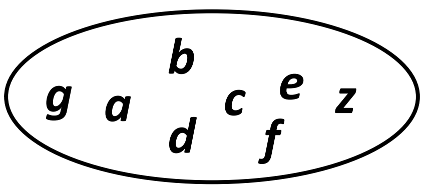
Adaptive submodularity

set of items,
unknown states

b, f



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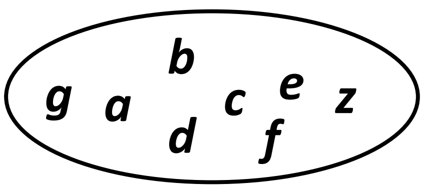

evaluate utility
function

b, f

↓

learn the state
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↓

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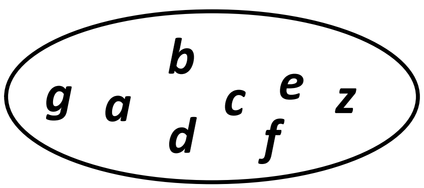

evaluate utility
function

b, f, z

↓

learn the state
of the item

↓

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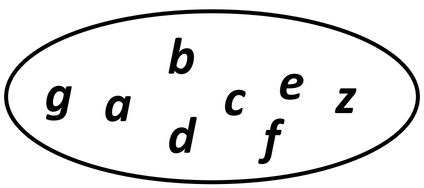

b, f, z

Shorter sequence

b, f, g, e, z

Longer sequence

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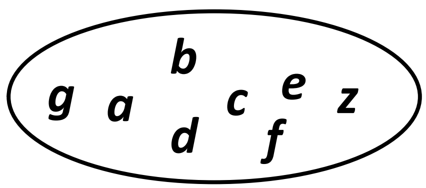

expected increase in utility function after adding z ≥ *expected increase in utility function after adding z*

b, f, z

Shorter sequence

b, f, g, e, z

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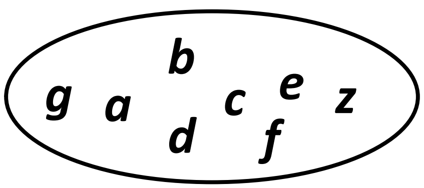

expected increase in utility function after adding z	≥	*expected increase in utility function after adding z*
b, f, z		***b, f, g, e, z***
Shorter sequence		Longer sequence

Expectation is taken with different posterior probability distributions

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We can reformulate problems 1 and 2 such that they have adaptive submodularity property

- ***Introduce utility function f=N-|S(O)|: number of nodes that are not source candidates after observing O***
- ***Prove f=N-|S(O)| is adaptive monotone and adaptive submodular for uniform source prior***
- ***Obtain performance guarantees for greedy selection***

Initial formulation of problem 1

$$\min_{\pi} \mathbb{E}_s \left[c \left(O \left(\pi \right) \right) \right]$$

$$\text{subject to } \boldsymbol{d}(O(\pi), s) \neq \boldsymbol{d}(O(\pi), i), \forall s \in V, s \neq i,$$

Reformulation of problem 1 as Adaptive Stochastic Minimum Cost Cover

$$\min_{\pi} \; \mathbb{E}_s \left[c \left(O \left(\pi \right) \right) \right]$$

$$\text{subject to } N - \left| S \left(O \left(\pi \right) \right) \right| \geq N - 1, \forall s \in V, s \neq i$$

Initial formulation of problem 2

$$\min_{\pi} \mathbb{E}_s |S(O(\pi))|$$
$$\text{subject to } |O(\pi)| \leq T$$

Reformulate problem 2 as Adaptive Stochastic Maximization

$$\max_{\pi} \mathbb{E}_s [N - |S(O(\pi))|]$$
$$\text{subject to } |O(\pi)| \leq T.$$

Selection of the best observer at step k for the greedy approach

observer at step k = **arg max** (observer o) **1/c(o)** **E** *current source candidates* **[** decrease in the number of source candidates after selecting observer o **]**

$$\text{observer at step } k = \underset{\text{observer } o}{\arg\max} \; \frac{1}{c(o)} \; E_{\textit{current source candidates}} \left[\text{decrease in the number of source candidates after selecting observer } o \right]$$

Selection of the best observer at step k for the greedy approach

$$\text{observer at step } k = \underset{\text{observer } o}{\arg\max} \; \frac{1}{c(o)} \; E_{\textit{current source candidates}} \left[\text{decrease in the number of source candidates after selecting observer } o \right]$$

Performance guarantees

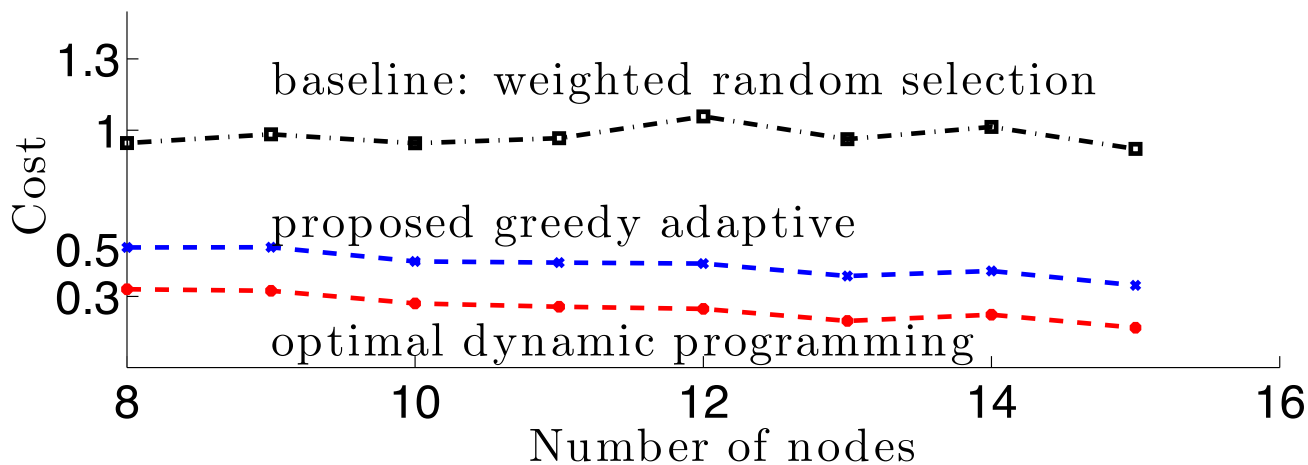
Problem 1. **cost by greedy ≤ optimal cost (log(N(N-1))+1)**

Problem 2. **# candidates by greedy ≤ optimal #candidates (1-1/e) +N/e**

Cost incurred by different approaches for solving problem 1

- Benchmark against the performance of a weighted random selection

- 100 realizations of small-world networks
- uniform source prior
- node cost random uniform [0,1]



Time required by different approaches to solve problem 1

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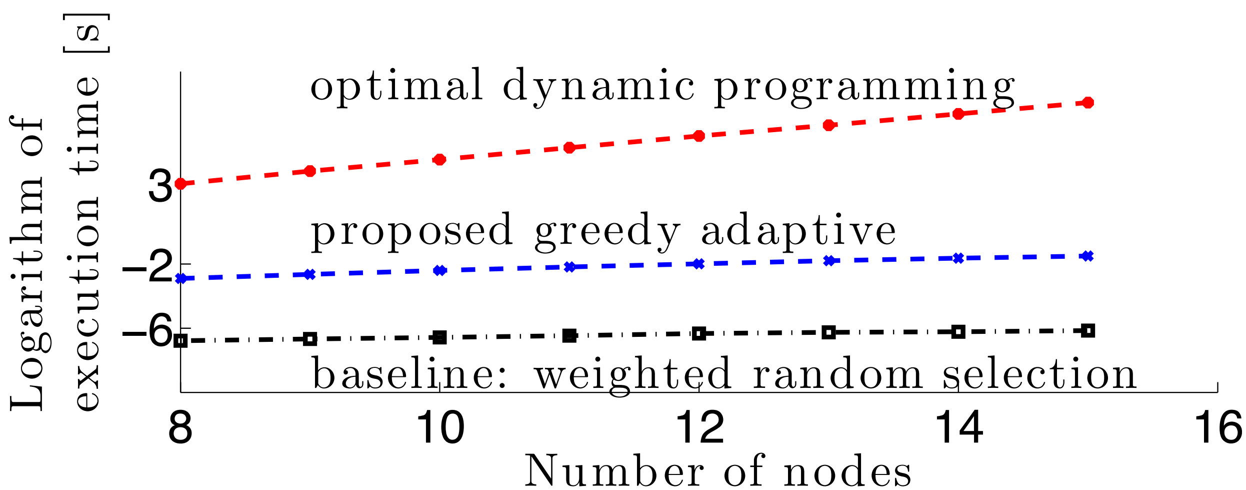
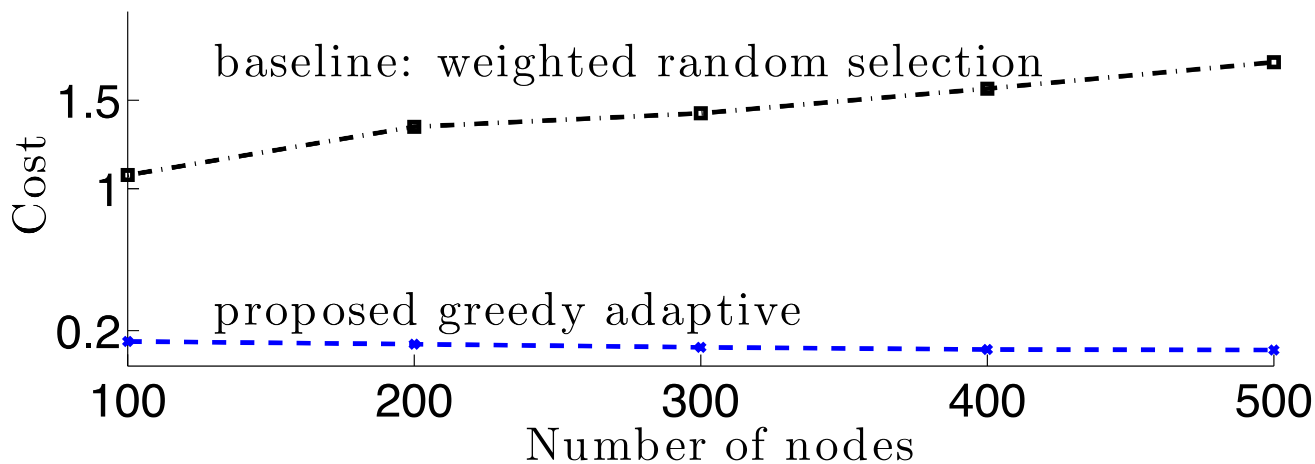


Cost incurred by approximate approaches for solving problem 1

- Benchmark against the performance of a weighted random selection

- 100 realizations of small-world networks
- uniform source prior
- node cost random uniform [0,1]



Conclusions and future work

- Formulated two problems:
  - minimize the cost for unambiguous source localization
  - minimize the number of source candidates after observing a prespecified number of nodes
- Solved problems optimally with stochastic dynamic programming
- Used adaptive submodularity to formulate a greedy algorithm with performance guarantees
- Future work: extend the model to stochastic propagation time