Multilevel Illumination Coding for Fourier Transform Interferometry in Fluorescence Spectroscopy

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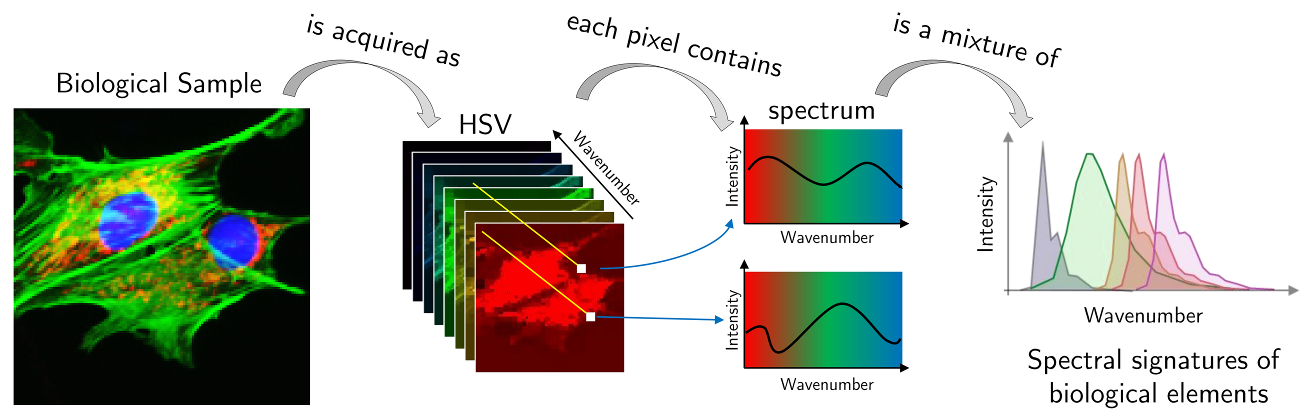
1. Summary

- **Motivations:** Acquisition of biological HyperSpectral Volume (HSV).
  - ✔ FTI reaches high spectral resolution without reducing SNR (see below).
  - ✗ Higher resolution ⇒ longer exposure time ⇒ more photo-bleaching.
- **Contribution:** Resorting to the theory of Compressed Sensing (CS) to
  - design an optimum light coding scheme,
  - reduce light exposure ⇒ less photo-bleaching,
  - provide a robust and stable HS recovery guarantee.

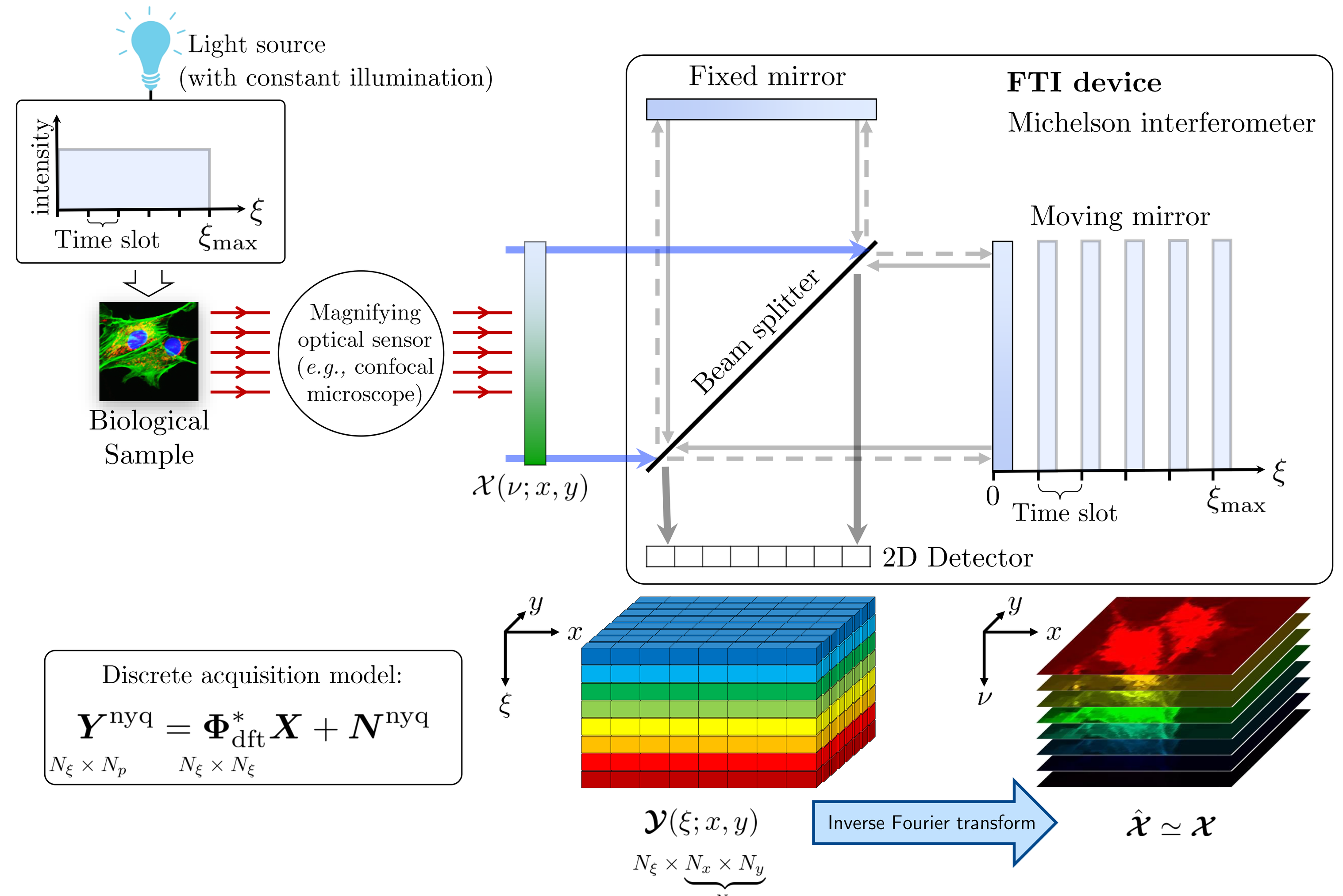
2. What is a Biological Hyperspectral Volume?

The description of a biological sample both by its spatial and spectral content, with the purpose of distinguishing specific biological elements, as in fluorescence spectroscopy.

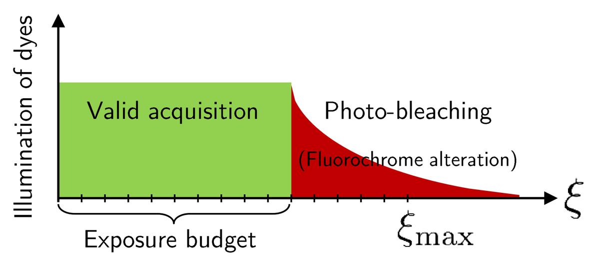


3. Nyquist Fourier Transform Interferometry (FTI)

- Mechanism:
  - For each Optical Path Difference (OPD) 2D focal plane measurements are recorded.
  - An *inverse 1D Fourier transform* is applied along the OPD axis to achieve the HSV.

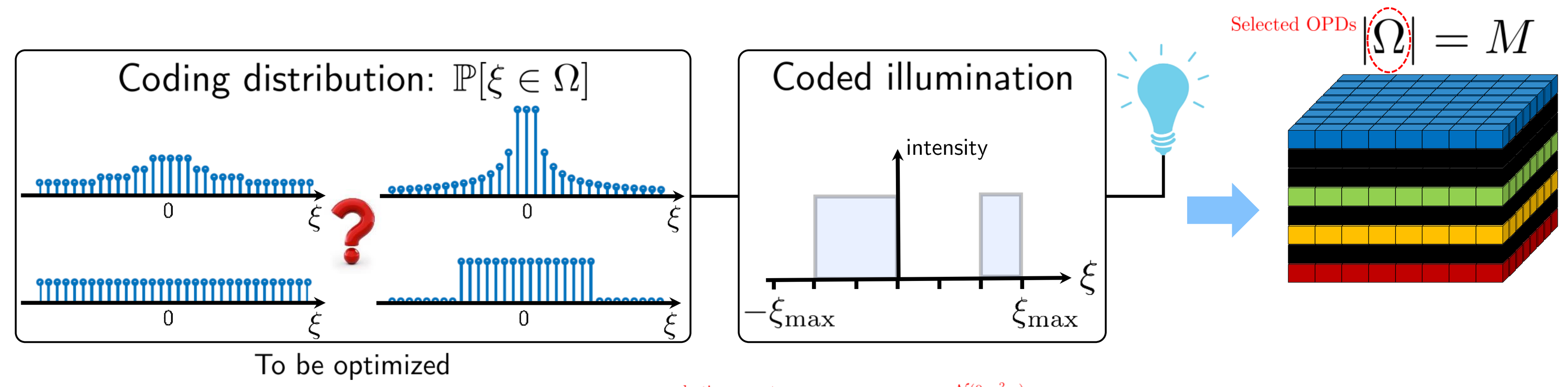


- Pros & cons: As the number of recorded OPD points (N_ξ) increases,
  - ✔ the spectral resolution increases.
  - ✗ the specimen is over-exposed
  - ⇒ more photo-bleaching



4. Coded Illumination-FTI (CI-FT)

- Acquisition model:



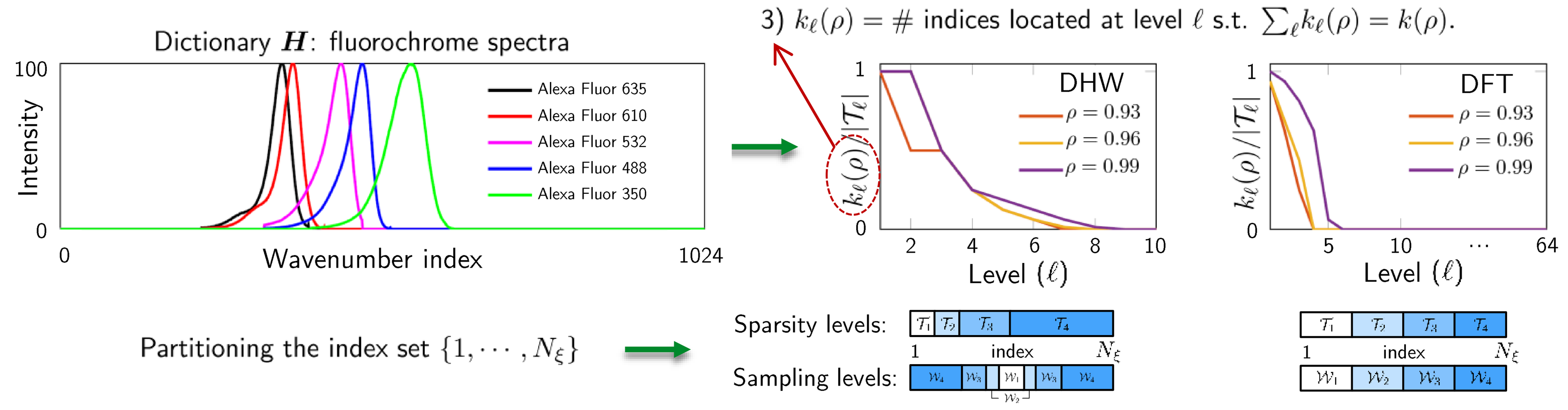
$$Y = R_\Omega \Phi^*_{\rm dft} X + N$$

(selection operator: R_Ω; $n_{ij} \sim_{\rm i.i.d.} \mathcal{N}(0,\sigma^2_{\rm nyq})$)

- Sparsity in levels in different possible bases Ψ:

$\Psi_{\rm dhw}$: 1D discrete Haar wavelet basis
$\Psi_{\rm dft}$: 1D discrete Fourier basis

1) Transform the spectra,
2) Find $k(\rho)$ indices containing ρ portion of the energy,
3) $k_\ell(\rho) = \#$ indices located at level ℓ s.t. $\sum_\ell k_\ell(\rho) = k(\rho)$.



- Optimum coding:

Initial approach [1]: application-independent, based on the Variable Density Sampling scheme [2],

$\Psi = \Psi_{\rm dhw}$: select $M \gtrsim K\ \log^3(K)\log^2(N_\xi)$ w.r.t. $\mathbb{P}[\xi \in \Omega] \propto \frac{1}{|\xi - N_\xi/2|}$.

(with replacement)

New approaches: application-dependent, based on the Multilevel Density Sampling scheme [3],

$\forall$ sampling levels t

$\Psi = \Psi_{\rm dhw}$: select $m_t \gtrsim \sum_\ell 2^{-\frac{|t-\ell|}{2}} k_\ell \log(K)\log(N_\xi)$

$\Psi = \Psi_{\rm dft}$: select $m_t = |\mathcal{W}_t|\ \delta_{k_t}$

w.r.t. $\mathbb{P}[\xi \in \Omega \mid \xi \in \mathcal{W}_t] = \frac{m_t}{|\mathcal{W}_t|}$.

(without replacement)

$K = \sum_t k_t$ and $M = \sum_t m_t$

- Recovery guarantee: for every pixel $j = 1, \cdots, N_p$, solve

$$\hat{x}_j = \arg\min_{u \in \mathbb{C}^N} \|\Psi^T u\|_1 \ \text{s.t.}\ \|D(y_j - R_\Omega \Phi^*_{\rm dft} u)\|_2 \le \varepsilon\sqrt{M}$$

($\varepsilon \approx \sigma_{\rm nyq}\sqrt{N_\xi}$)

$$\longrightarrow \|x - \hat{x}\| \lesssim \Big(\sum_{j=1}^{N_p} \alpha^2(\Psi^\top x_j)\Big)^{1/2} + \varepsilon\sqrt{N_p}$$

Initial approach: $\Psi = \Psi_{\rm dhw}$, $d_{ii} = \mathbb{P}[\Omega_i \in \Omega]^{-1/2}$, $\alpha(u) = \frac{\|u - \mathcal{H}_K(u)\|_1}{\sqrt{K}}$ (hard thresholding: $\mathcal{H}_K$)

New approach 1: $\Psi = \Psi_{\rm dhw}$,

New approach 2: $\Psi = \Psi_{\rm dft}$,

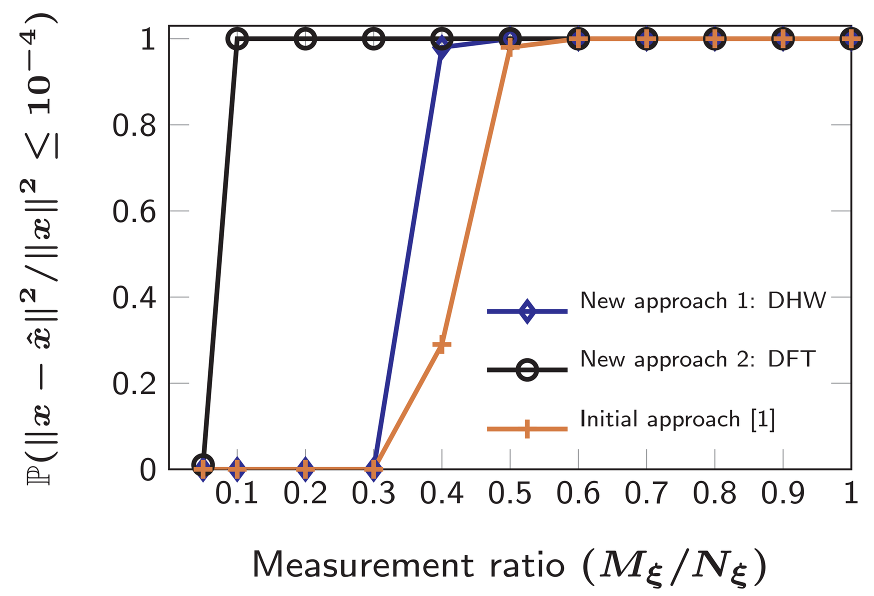
$d_{ii} = N_\xi^{-1/2}$, $\alpha(u) = \sum_\ell \|R_{\mathcal{T}_\ell} u - \mathcal{H}_{k_\ell}(R_{\mathcal{T}_\ell} u)\|_1$

5. Numerical Results

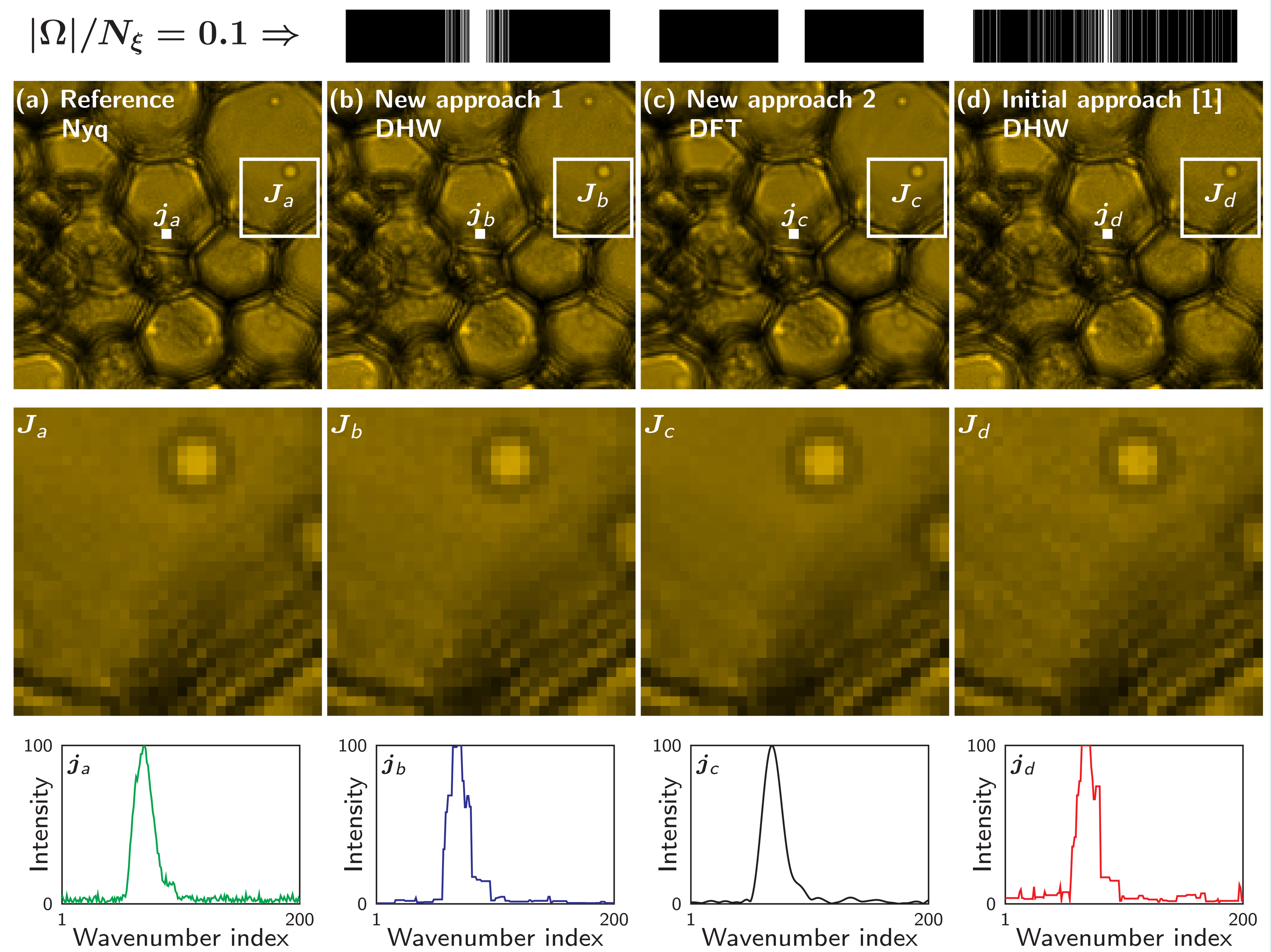
- Synthetic measurements: phase transition curves of successful recovery

$x = Hg \longrightarrow y = R_\Omega \Phi^*_{\rm dft} x$

(fluorochrome dictionary (size 1024 × 39); random weights (size 39 × 1))



- Real FTI measurements: comparison of recovery quality



Illumination coding pattern (first row), reconstructed spatial maps at 594 nm wavelength (second and third row), and reconstructed spectra at the center pixel (bottom).

6. Take Home Messages

- A non-uniform density sampling must be deployed for coherent sampling/sparsity bases.
- The Variable Density Sampling (VDS) scheme [2] is an application-independent optimum strategy that provides uniform recovery guarantee.
- The Multilevel Density Sampling (MDS) scheme [3] is an application-dependent optimum strategy that provides non-uniform recovery guarantee.
- Both VDS and MDS schemes can be leveraged for the realization of CI-FTI in (less photo-bleaching) fluorescent spectroscopy.
- MDS scheme results in superior reconstruction as it takes into account the sparsity structure of the fluorochrome spectra.

7. References

[1] A. Moshtaghpour, V. Cambareri, P. Antoine, M. Roblin, and L. Jacques, "A variable density sampling scheme for compressive Fourier transform interferometry," arxiv preprint arXiv: 1801.10432v2, 2018.

[2] F. Kramher and R. Ward, "Stable and robust sampling strategies for compressive imaging," *IEEE Transactions on Imaging Processing*, vol. 23, no. 2, pp. 612–622, 2013.

[3] B. Adcock,A. C. Hansen, C. Poon, and B. Roman, "Breaking the coherence barrier: A new theory for compressed sensing," *Forum of Mathematics, Sigma*, vol. 5. Cambridge University Press, 2017.

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