Universal Approach for DCT-based Constant-time Gaussian Filter with Moment Preservation

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1. Introduction

Gaussian filter (GF)

- An essential image processing tool, e.g., for scale-space
- The larger window size causes longer computational time.

DCT-based constant-time ($O(1)$) Gaussian filter

- Approximate GF that can run in $O(1)$ time/pixel
- Tradeoff between computational time and accuracy

Ours: comprehensive analysis using a universal model

- Covering all the DCT types and arbitrary moments (see ☞)
- Clear theoretical foundation based on convex optimization

Method	DCT type	Preserved moments
Elboher and Werman (*ICIP 2011*) **[1]**	DCT-1	
Sugimoto and Kamata (*ICIP 2013*) **[2,3]**	DCT-5	μ_0
Charalampidis (*IEEE TSP 2016*) **[4]**	DCT-3	μ_0, μ_2
Ours (*ICASSP 2018*)	All	$\mu_0, \mu_2, \mu_4, \dots$

2. Existing Approaches

Truncated Gaussian filter (1D)

Convolution $(x * h)_t := \sum_{n=-N+1}^{N-1} x_{t+n} h_n$ x_t: input

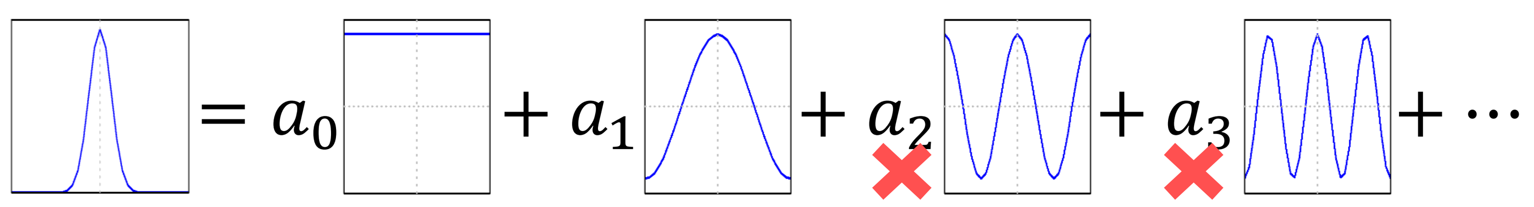
Discrete Gaussian kernel Window size $2N$-1

$$h_n := \eta^{-1} \exp\left(-\frac{n^2}{2\sigma^2}\right),\ \eta := \sum_{n=-R}^{R} \exp\left(-\frac{n^2}{2\sigma^2}\right)$$

Nd Gauss can be decomposed into N of 1D Gauss.

Gaussian kernel approximated by DCT-3

$$h_n = \sum_{k=0}^{R} a_k \cos\left(-\frac{\pi}{N}\left(k+\frac{1}{2}\right)n\right)$$



m-th order moment of kernel

$$\mu_m = \sum_{n=-R}^{R} n^m h_n$$

The 0th, 2nd and 4th moments are named *sum, variance and kurtosis*.

Moments of (continuous) Gaussian kernel

$$\mu_0 = 1,\ \mu_1 = 0,\ \mu_2 = \sigma^2,\ \mu_3 = 0,\ \mu_4 = 3\sigma^4, \dots$$

Which DCT type is the best in approximate $O(1)$ GF? How can we fix moments before-and-after approx?

3. Proposed Universal Approach

Unlike existing methods, we comprehensively analyze all the DCT types using a universal model covering them.

General form of all the DCT types

$$h_n \approx \cos\left(-\frac{2\pi}{T}(k+k_0)(n+n_0)\right)$$

This form can simultaneously represent all the DCT type where T, k_0 and n_0 depend on DCT types.

Only DCT-1,3,5,7 ($n_0 = 0$) are appropriate for approximating Gaussian kernel because it is symmetric at $n = 0$.

How to find out the best approximation

Convex optimization problem

$$\hat{\boldsymbol{h}}_\star := \arg\min_{\hat{\boldsymbol{h}}} f(\hat{\boldsymbol{h}}) \quad \text{s.t.} \quad \boldsymbol{g}(\hat{\boldsymbol{h}}) = \boldsymbol{0}$$

Least-square approximation error ***C*** is a DCT matrix.

$$f(\hat{\boldsymbol{h}}) := \frac{1}{2}(\boldsymbol{h} - \boldsymbol{C}\hat{\boldsymbol{h}})^\top \boldsymbol{W}(\boldsymbol{h} - \boldsymbol{C}\hat{\boldsymbol{h}}) \in \mathbb{R}$$

Constraint for moment preservation *M* indicates number of moments to be preserved.

$$\boldsymbol{g}(\hat{\boldsymbol{h}}) := \boldsymbol{P}^\top \boldsymbol{W}\boldsymbol{C}\hat{\boldsymbol{h}} - \frac{1}{2}\boldsymbol{\mu} \in \mathbb{R}^M$$

The linearly-constrained least square problem can be solved by the Lagrange multiplier method.

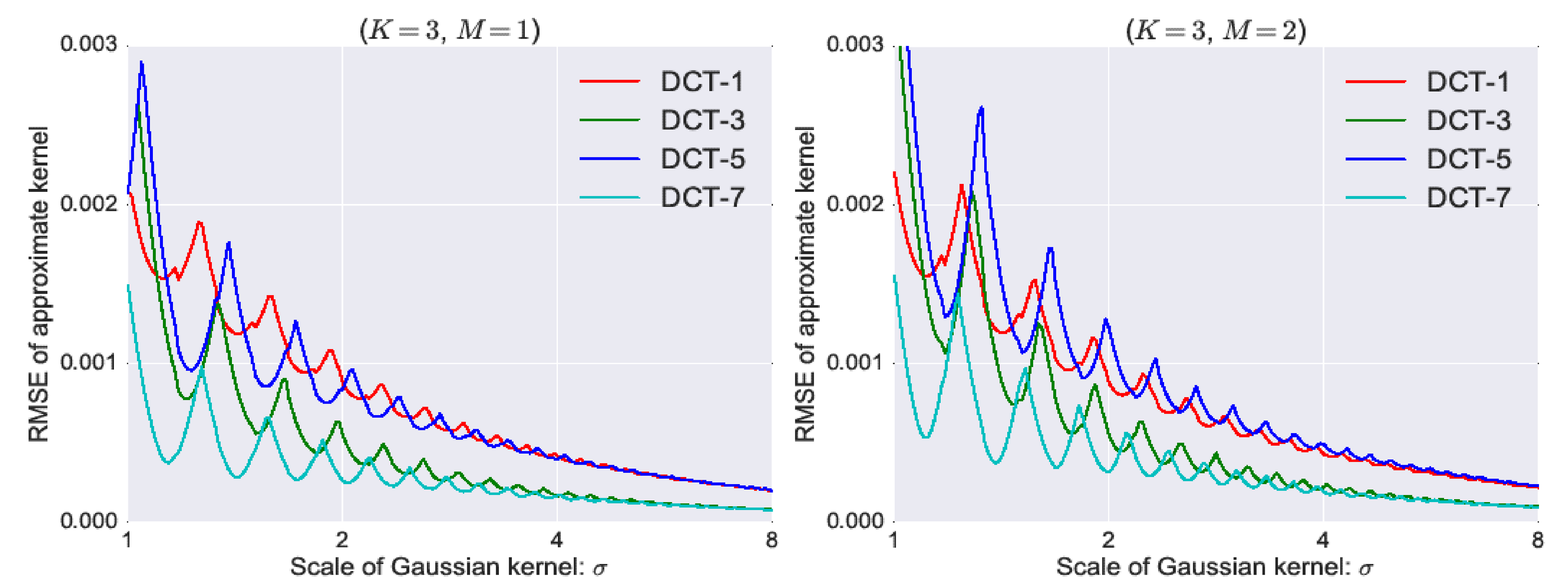
4. Experiments

Moments and accuracy (σ=2, N=7, K=3 & M=2)

Method	Mean μ_0	Var. μ_2	Kurt. μ_4	RMSE
Convol. ($N = \lceil 6\sigma \rceil$)	**1.000**	**4.000**	**48.000**	**0**
Convol. ($N = \lceil 3\sigma \rceil$)	**1.000**	3.808	39.708	7.94 x 10^{-4}
[4] w/ DCT-3 (on paper)	**1.000**	**4.000**	47.470	5.73 x 10^{-4}
[4] w/ DCT-3 (our impl.)	**1.000**	**4.000**	47.429	5.73 x 10^{-4}
Ours w/ DCT-3	**1.000**	**4.000**	47.477	5.73 x 10^{-4}

Ours demonstrably produced almost the same result as the Charalampidis method [4], which only covers DCT-3.

Approximate error (K=3 and M=1,2)



DCT-3,7 showed lower approximate error than DCT-1,5. For smaller σ, DCT-7 evidently outperforms DCT-3.

5. Conclusions

- We discussed DCT-base $O(1)$ GF via a universal model.
- Our model covers all DCT types and arbitrary moments.
- It allowed us to comprehensively compare all DCT types.
- DCT-7 showed the highest approximate accuracy over all.

【References】

1. E. Elboher and M. Werman: **Cosine integral images for fast spatial and range filtering**, *Proc. IEEE ICIP*, 2011/9.
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3. K. Sugimoto and S. Kamata: **Efficient constant-time Gaussian filtering with sliding DCT/DST-5 and dual-domain error minimization**, *ITE Trans. MTA*, 2015/1.
4. D. Charalampidis: **Recursive implementation of the Gaussian filter using truncated cosine functions**, *IEEE Trans. SP*, 2016/7.