Determined Blind Source Separation via Proximal Splitting Algorithm

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Introduction

- Many **independence-based blind source separation (BSS)** methods reduces to the following minimization problem:

$$\underset{\{W[f]\}_{f=1}^{F}}{\text{Minimize}} \;\; \mathcal{P}(W[f]\mathbf{x}[t,f]) - \sum_{f=1}^{F} \log|\det(W[f])|$$

➢ Laplace-distribution-based independent component analysis **(FDICA)**

$$\mathcal{P}(\mathbf{y}[t,f]) = C\,\|\mathbf{y}[t,f]\|_1 = C\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{f=1}^{F}|y_m[t,f]|$$

➢ Spherical-Laplace-distribution-based Independent vector analysis **(IVA)**

$$\mathcal{P}(\mathbf{y}[t,f]) = C\,\|\mathbf{y}[t,f]\|_{2,1} = C\sum_{m=1}^{M}\sum_{t=1}^{T}\Big(\sum_{f=1}^{F}|y_m[t,f]|^2\Big)^{\frac{1}{2}}$$

➢ Independent low-rank matrix analysis **(ILRMA) [1]**

$$\mathcal{P}(\mathbf{y}[t,f]) = C\sum_{m=1}^{M}\min_{\varphi_{f,r}^{[m]}\geq 0,\psi_{r,t}^{[m]}\geq 0}\sum_{t=1}^{T}\sum_{f=1}^{F}\left(\frac{|y_m[t,f]|^2}{\sum_{r=1}^{R}\varphi_{f,r}^{[m]}\psi_{r,t}^{[m]}} + \log\sum_{r=1}^{R}\varphi_{f,r}^{[m]}\psi_{r,t}^{[m]}\right)$$

- **State-of-the-art algorithms** based on the majorization-minimization (MM) principle [1–3] require specially designed upper bounds whose **derivation might be complicated and time-consuming** for some $\mathcal{P}$.

Primal-dual Splitting Algorithm

$$\left|\begin{array}{l}
\tilde{\mathbf{w}} = \mathrm{prox}_{\mu_1 g}\big[\,\mathbf{w}^{[k]} - \mu_1\mu_2 L^H\mathbf{y}^{[k]}\,\big] \\
\mathbf{z} = \mathbf{y}^{[k]} + L(2\tilde{\mathbf{w}} - \mathbf{w}^{[k]}) \\
\tilde{\mathbf{y}} = \mathbf{z} - \mathrm{prox}_{h/\mu_2}\big[\,\mathbf{z}\,\big] \\
(\mathbf{w}^{[k+1]},\mathbf{y}^{[k+1]}) = \alpha(\tilde{\mathbf{w}},\tilde{\mathbf{y}}) + (1-\alpha)(\mathbf{w}^{[k]},\mathbf{y}^{[k]})
\end{array}\right.$$

- Primal-dual splitting (PDS) algorithm is one of the **proximal splitting algorithms** which can solve the convex optimization problem.

$$\underset{\mathbf{w}}{\text{Minimize}} \;\; g(\mathbf{w}) + h(L\mathbf{w})$$

- PDS iteratively utilizes the **proximity operator** of each function.

$$\mathrm{prox}_{\mu g}[\mathbf{y}] = \arg\min_{\mathbf{z}}\Big[\,g(\mathbf{z}) + \frac{1}{2\mu}\|\mathbf{y}-\mathbf{z}\|_2^2\,\Big]$$

- Since the **proximity operators are easier subproblems** which can be solved in many ways, the PDS algorithm applied to the independence-based BSS should be **able to handle complicated source model** $\mathcal{P}$ more easily than the state-of-the-art MM-based algorithms.

Proposed Primal-dual Algorithm for Independence-based BSS problem

- By rewriting matrix determinant with singular values and vectorizing variables, the BSS problem can be seen as a **PDS applicable form**.

$$\underset{\mathbf{w}}{\text{Minimize}} \;\; \mathcal{I}(\mathbf{w}) + \mathcal{P}(X\mathbf{w})$$

- **Matrix determinant was rewritten by singular values** because the proximity operators of orthogonally invariant functions are known. This modification allows direct handling of an over-determined case.

$$\mathcal{I}(\mathbf{w}) = -\sum_{f=1}^{F}\sum_{m=1}^{M}\log\sigma_m(\mathcal{M}(\mathbf{w})[f]) \qquad \Big(\mathcal{M}(\mathbf{w})[f] = W[f]\Big)$$

- **Variables are vectorized** so that the role of observed data X in terms of the optimization variable $\mathbf{w}$ becomes apparent.

Algorithm 1 PDS-BSS

```
Input: X, w^[1], y^[1], μ1, μ2, α
Output: w^[K+1]
for k = 1, ..., K do
    w~ = prox_{μ1 I}[ w^[k] − μ1 μ2 X^H y^[k] ]        ⇐ Modification of singular values
    z = y^[k] + X(2w~ − w^[k])
    y~ = z − prox_{(1/μ2) P}[ z ]                       ⇐ Proximity operator of source term
    y^[k+1] = α y~ + (1 − α) y^[k]
    w^[k+1] = α w~ + (1 − α) w^[k]
end for
```

- The proximity operator of $\mathcal{I}$ is to apply the proximity operator of -log function to each singular value of the un-vectorized variables.

$$\mathrm{prox}_{-\mu\log}[\,\sigma_m] = \Big(\,\sigma_m + \sqrt{\sigma_m^2 + 4\mu}\,\Big)/2$$

- **Source models defined by adding multiple penalty terms** can also be handled by the proposed algorithm with slight modifications. Such multiple penalty terms can be difficult to handle for MM algorithms.

$$\underset{\mathbf{w}}{\text{Minimize}} \;\; \mathcal{I}(\mathbf{w}) + \sum_{q=1}^{Q}\mathcal{P}_q(X\mathbf{w})$$

Algorithm 2 PDS-BSS-multiPenalty

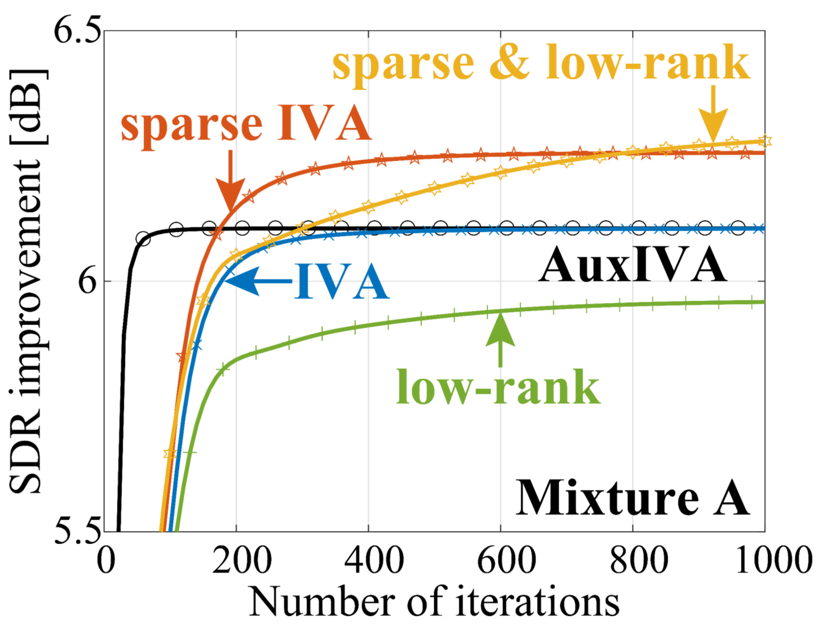
```
Input: X, w^[1], y_1^[1], ..., y_Q^[1], μ1, μ2, α
Output: w^[K+1]
for k = 1, ..., K do
    w~ = prox_{μ1 I}[ w^[k] − μ1 μ2 X^H (Σ_{q=1}^{Q} y_q^[k]) ]   ⇐ Modification of singular values
    for q = 1, ..., Q do
        z_q = y_q^[k] + X(2w~ − w^[k])
        y~_q = z_q − prox_{(1/μ2) P_q}[ z_q ]                      ⇐ Proximity operator of each source term
        y_q^[k+1] = α y~_q + (1 − α) y_q^[k]
    end for
    w^[k+1] = α w~ + (1 − α) w^[k]
end for
```

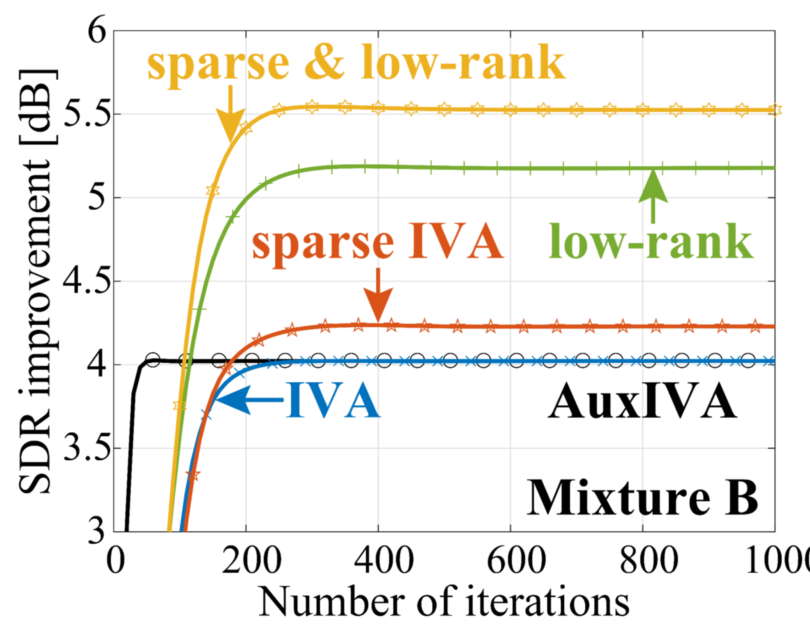
New BSS Models and Experimental Results

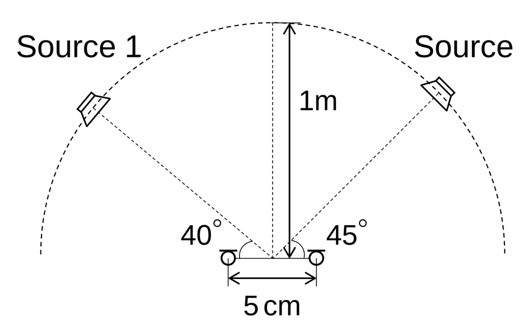
- Four BSS models were investigated as example problems that can be easily handled by the proposed algorithms. ($\lambda = 0.002$)

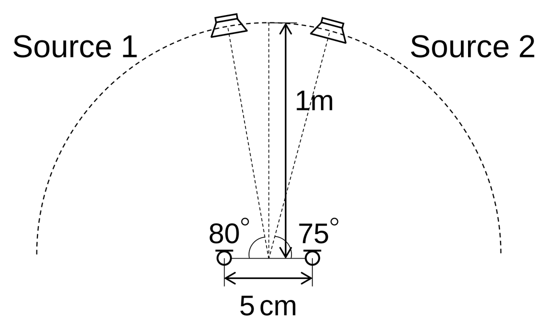
IVA	$\underset{\mathbf{w}}{\text{Minimize}}\;\; \mathcal{I}(\mathbf{w}) + \|X\mathbf{w}\|_{2,1}$
sparse IVA	$\underset{\mathbf{w}}{\text{Minimize}}\;\; \mathcal{I}(\mathbf{w}) + \|X\mathbf{w}\|_{2,1} + \lambda\|X\mathbf{w}\|_1$
low-rank (nuclear)	$\underset{\mathbf{w}}{\text{Minimize}}\;\; \mathcal{I}(\mathbf{w}) + \|X\mathbf{w}\|_*$
sparse & low-rank	$\underset{\mathbf{w}}{\text{Minimize}}\;\; \mathcal{I}(\mathbf{w}) + \|X\mathbf{w}\|_* + \lambda\|X\mathbf{w}\|_1$

- **Live recording** (liverec) of four female speech contained in UND task of the **SiSEC 2011** database was utilized as an test data.
- In the case of IVA, computational time per iteration of the proposed algorithm was **1.7x faster than AuxIVA** (MM algorithm [2]).
- Modification of the codes was able to be done within a few minutes (inside train) that indicate easiness of the proposed algorithms.









Reverberation time: 130ms, Window: 128ms Hann (half-overlap)

[1] D. Kitamura, N. Ono, H. Sawada, H. Kameoka, and H. Saruwatari, "Determined blind source separation unifying independent vector analysis and nonnegative matrix factorization," IEEE/ACM Trans. Audio, Speech, Lang. Process., vol. 24, no. 9, pp. 1626–1641, Sep. 2016.
[2] N. Ono, "Stable and fast update rules for independent vector analysis based on auxiliary function technique," in Proc. IEEE Workshop Appl. Signal Process. Audio Acoust., Oct. 2011, pp. 189–192.
[3] N. Ono and S. Miyabe, "Auxiliary-function-based independent component analysis for super-gaussian sources," in Proc. Int. Conf. Latent Variable Anal. Signal Separation, 2010, pp. 165–172.