Rotation-robust beamforming based on sound field interpolation with regularly circular microphone array

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Introduction

- **Background**
  - Array signal processing assumes a time-invariant ATS*.  
  - ATS’s variation makes re-estimation of spatial information (e.g., spatial filter and DOA†) necessary.  
  ⇒ makes online processing difficult.

- **Motivation**
  - We want to follow the ATS’s variation caused by CMA** rotation.  
  - We want to apply interpolation to existing beamformings.

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*: acoustic transfer system  
†: direction of arrival  
**: circular microphone array
Prior work for ATS’s variation

- Case of source movement
  - Blockwise spatial filter estimation using DOA information [Nikunen+, 2018], [Naqvi+, 2011]
  - Sequential covariance estimation every time-frequency bins with a Bayesian tracker [Taseska+, 2018]

- Case of sensor movement
  - Motion compensation method [Tourbabin+, 2015]
    - Formulating circular harmonic domain rotation matrix
    - Applying to DOA estimation using a CMA
Idea of sound field interpolation

- Continuous sound field on circle’s circumference, \( z(\theta) \)
  - \( z(\theta) \) is a periodic function with \( 2\pi \)
  - Discretizing \( z(\theta) \) with \( 2\pi/M \) intervals
    = Observing the sound field using an M-ch CMA
      \[
      z_m = z \left( 2\pi \frac{m}{M} \right), \quad m = 0, \ldots, M - 1.
      \]

- \( z(\theta) \) can be reconstructed by the discrete signal \( z_m \) if the sampling theorem is satisfied*, resulting in sound field interpolation possible.

\*Satisfying the sampling theorem = Band limitation of circular harmonics spectrum
Sound field interpolation using noninteger sample shift

- Relationship b/w CMA rotation and sample shift
  - Sound field observed by a CMA in the reference position, $z_m$
  - Sound field observed by the CMA rotated $\Delta$ (=2$\pi$$\delta$/M) [rad] = $\delta$-sample shifted $z_m$ in the spatial axis

\[
z_{m+\delta} = z \left( 2\pi \frac{m}{M} + \Delta \right)
\]

- The above equation enables estimating $z_{m+\delta}$ from $z_m$, $m=0,\ldots, M-1$. 

![Diagram](image)
Formulation of linear interpolation

- Linear representation of δ-sample shifted sound field using the sample shift theorem in the DFT

\[
z_{m + \delta} = \mathcal{F}_D^{-1} \left[ \mathcal{F}_D [z_m] e^{j\Delta k} \right]
\]

\[
= \frac{1}{M} \sum_{k = -M/2+1}^{M/2} \left( Z_k e^{j\Delta k} \right) e^{-j2\pi \frac{mk}{M}} \overset{\text{def}}{=} \sum_{n=0}^{M-1} z_n u_{m,n,\delta}
\]

- Sound field interpolation using sinc function

(\text{It is derived from the equation above*}.)

\[
u_{m,n,\delta} = \begin{cases} 
\frac{1+(-1)^{n-m}e^{j\delta \pi}}{M} + \frac{\text{sinc} \left( \frac{L}{2} \right)}{\text{sinc} \left( \frac{L}{M} \right)} \cdot \cos \left( \frac{M+2}{2M} L\pi \right), & \text{if } M \text{ is even,} \\
\frac{1}{M} + \frac{M-1}{M} \frac{\text{sinc} \left( \frac{L(M-1)}{2M} \right)}{\text{sinc} \left( \frac{L}{M} \right)} \cdot \cos \left( \frac{M+1}{2M} L\pi \right), & \text{if } M \text{ is odd.}
\end{cases}
\]

\[L = n - m - \delta\]

\(\mathcal{F}_D\): DFT operation

*: The detailed derivation is appended on the last page.
Remark of formulation

- **How to handle Nyquist frequency (NyqF) component**

  \[
  z_{m+\delta} = \sum_{n=0}^{M-1} z_n u_{m,n,\delta} \quad (\ast)
  \]

  \[
  u_{m,n,\delta} = \begin{cases} 
  \frac{1 + (-1)^{n-m}}{M} & \text{if } M \text{ is even}, \\
  \frac{1}{M} + \frac{M-1}{M} \frac{\text{sinc} \left( \frac{L(M-1)/2M}{\text{sinc} \left( \frac{L/M}{2M} \right)} \right)} \cdot \cos \left( \frac{M+1}{2M} L \pi \right), & \text{if } M \text{ is odd}.
  \end{cases}
  \]

- When $M$ is even, a noninteger sample shift results in a complex-valued term even if the sign of the NyqF is positive or negative, and which causes some contradiction; e.g., sample shift of a real-valued even point signal by $(\ast)$ translates to a complex-valued signal.

- In this study, we neglect the NyqF component by setting $\delta=0$. 
Matrix representation

Formulation of the relationship b/w the observation by a \(\Delta\)-rotated CMA, \(x\), and by the CMA w/o rotation, \(x_0\)

\[
x = \begin{bmatrix} x_1 & \cdots & x_M \end{bmatrix}^T
\]

\[
= \begin{bmatrix} z\left(2\pi(0 + \delta)/M\right) & \cdots & z\left(2\pi(M - 1 + \delta)/M\right) \end{bmatrix}^T
\]

\[
= \begin{bmatrix}
    u_{0,0,\delta} & u_{0,1,\delta} & \cdots & u_{0,M-1,\delta} \\
    u_{1,0,\delta} & u_{1,1,\delta} & \cdots & u_{1,M-1,\delta} \\
    \vdots & \vdots & \ddots & \vdots \\
    u_{M-1,0,\delta} & u_{M-1,1,\delta} & \cdots & u_{M-1,M-1,\delta}
\end{bmatrix}
\begin{bmatrix}
    z_0 \\
    z_1 \\
    \vdots \\
    z_{M-1}
\end{bmatrix}
\]

\[
= U_{\Delta} x_0.
\]

Note: \(U_{\Delta}\) is a cyclic matrix & does not depend on frequencies

[cf.] Our method has a tight relationship to the circular harmonics domain rotation matrix used by the motion compensation method [Tourbabin+, 2015].
Applying interpolation to beamforming

- Situation: A user or humanoid robot
  - wears a CMA on the head.
  - rotates the head to listen to ambient conversations attentively.

- Problem:
  - The rotation angle $\Delta$ is given.
  - We estimate the observation before CMA rotation.
  - We do beamforming using interpolated M-ch signals.
Evaluation condition

- **Mixture**: 2 sources from SiSEC [Araki+, 2012], fs=16 kHz, 12 envs.
- **RIR**: Simulator [Habets, 2008], RT60 $\approx$ 100 ms
- **STFT**: 64 ms Hamming window with 1/8 shifts

- **Exp. 1**: interpolation performance
  - Array rot. $\Delta$: 10, 20, and 30 deg
  - Measure: SER (signal-to-error ratio)
    \[ \text{SER}_{m,k} = 10 \log_{10} \left( \frac{\sum_t |x_{m,t,k}|^2}{\sum_t |\hat{x}_{m,t,k} - x_{m,t,k}|^2} \right) \]
  - $x_{m,t,k} \in \mathbb{C}$

- **Exp. 2**: signal enhancement
  - Array rot. $\Delta$: 10, 20, and 36 deg
  - Beamformer: MPDR* + RTF** [Doclo+, 2015]
    - 2 methods of applying beamformer.
  - Measure: SDR, SIR [Vincent+, 2006]

*minimum power distortionless response
**relative transfer function
How to apply to beamforming

- **Method 1:** Constantly use of pre-estimated spatial filter
  - Pre-estimation of spatial filter when the CMA does not rotate
    \[ \mathbf{w} = \text{MPDR}(\mathbf{V}, \alpha), \quad \mathbf{V} = \mathbb{E}\left[ \mathbf{x}_0 \mathbf{x}_0^H \right] \]
  - Interpolation & beamforming
    \[ y = \mathbf{w}^H \hat{\mathbf{x}}_0 = \mathbf{w}^H \left( \mathbf{U}_{-\Delta} \mathbf{x} \right) \] \[ \Delta: \text{rotation angle} \]
  \[ \alpha: \text{transfer function} \]
  \[ \mathbf{x}_0: \text{observation in the reference position} \]

- **Method 2:** Re-estimation of spatial filter
  \[ \tilde{\mathbf{w}} = \text{MPDR}(\hat{\mathbf{V}}, \alpha), \quad \hat{\mathbf{V}} = \mathbb{E}\left[ \hat{\mathbf{x}}_0 \hat{\mathbf{x}}_0^H \right], \quad \hat{\mathbf{x}}_0 = \mathbf{U}_{-\Delta} \mathbf{x} \]
  \[ y = \tilde{\mathbf{w}}^H \hat{\mathbf{x}}_0 \]
Exp. 1: Interpolation performance 1/2

- Examples of SER when 2 sources are active
  - Lower band interpolation worked well, but not higher one.

![Graphs showing interpolation performance](image-url)
Exp. 1: Interpolation performance 2/2

- Averaged SER improvement (up to 3 kHz)
  - More the number of microphones improves performance. $\uparrow$
  - NyqF components decrease performance when $M=4, 6$ $\downarrow$
    - e.g., when $M=4$, only 3 components contribute to interpolation.

*Image shows box plots for interpolation performance at different rotation angles (10 deg, 20 deg, 30 deg) with number of microphones ranging from 3 to 8.*
Exp. 2: Signal enhancement

- SDR & SIR, M=5
  - In No-Int case, bigger rotation decreases SDR & SIR.
  - Int & Int+Re-est come close to the performance of No-Rot.

**Legend**
- No-Proc: unprocessed
- No-Rot: no rotation
- No-Int: no interpolation
- Int: interpolation
- Int+Re-est: interpolation + filter re-estimation
Conclusion

- **Summary**
  - Sound field interpolation method for rotation-robust beamforming using CMA.
  - Lower band interpolation accuracy was high when M is odd.
  - Higher band interpolation accuracy was low, but applying to beamforming worked well.

- **Future work**
  - Applying to different array processing, e.g., source separation
  - Clarifying relation to circular harmonics domain beamforming
Derivation (1/2)

When $M$ is even

$$ z_{m+\delta} = z \left( 2\pi \frac{m}{M} + \Delta \right) = \mathcal{F}_D^{-1} \left[ \mathcal{F}_D [z_m] e^{j\Delta k} \right] $$

$$ = \frac{1}{M} \sum_{k=-M/2+1}^{M/2} Z_k e^{j\Delta k} W^{-mk} $$

Positive freq.

NyqF component

Negative freq.

$$ = \frac{1}{M} \left\{ Z_0 + \sum_{k=1}^{M/2-1} Z_k e^{j\Delta k} W^{-mk} + Z_M e^{j\Delta M/2} W^{-mM/2} + \sum_{k=-M/2+1}^{-1} Z_k e^{j\Delta k} W^{-mk} \right\} $$

$$ = \frac{1}{M} \left\{ \sum_{n=0}^{M-1} z_n + \sum_{k=1}^{M/2-1} \left( \sum_{n=0}^{M-1} z_n W^{nk} e^{j\Delta k} \right) W^{-mk} + \left( \sum_{n=0}^{M-1} z_n W^{nM/2} e^{j\Delta M/2} \right) W^{-mM/2} \right\} $$

$$ + \sum_{k=1}^{M/2-1} \left( \sum_{n=0}^{M-1} z_n W^{nk} e^{j\Delta k} \right) W^{mk} $$

$$ = \frac{1}{M} \sum_{n=0}^{M-1} z_n \left\{ 1 + \left( \sum_{k=1}^{M/2-1} W^{(n-m-\delta)k} \right) + W^{(n-m-\delta)M/2} + \left( \sum_{k=1}^{M/2-1} W^{-(n-m-\delta)k} \right) \right\} $$

This term is a complex value because $\delta$ is a noninteger.
Derivation (2/2)

Set \( L = n - m - \delta \)

\[
\begin{align*}
    u_{m,n,\delta} &= \frac{1}{M} \left\{ 1 + \frac{W^{L M/2}}{1 - W^L} + \sum_{k=1}^{M/2-1} W^{Lk} + \sum_{k=1}^{M/2-1} W^{-Lk} \right\} \\
    &= \frac{1}{M} \left\{ 1 + W^{L M/2} + W^L \frac{1 - W^{L M/2}}{1 - W^L} + W^{-L} \frac{1 - W^{-L M/2}}{1 - W^{-L}} \right\} \\
    &= \frac{1}{M} \left\{ 1 + W^{L M/2} + W^{L \frac{M}{4} (M+2)} \frac{W^{-L M/4} - W^{L M/4}}{W^{-L/2} - W^{L/2}} + W^{-L \frac{M}{4} (M+2)} \frac{W^{L M/4} - W^{-L M/4}}{W^{L/2} - W^{-L/2}} \right\} \\
    &= \frac{1}{M} \left( 1 + W^{L M/2} + W^{L \frac{M}{4} (M+2)} \frac{\sin \left( \frac{\pi L/2}{\sin (\pi L/M)} \right)}{\sin \left( \frac{\pi L}{M} \right)} + W^{-L \frac{M}{4} (M+2)} \frac{\sin \left( \frac{\pi L/2}{\sin (\pi L/M)} \right)}{\sin \left( \frac{\pi L}{M} \right)} \right) \\
    &= \frac{1}{M} \left( 1 + W^{L M/2} \right) + \frac{1}{2} \cdot \frac{\sin \left( \frac{L}{2} \right)}{\sin \left( \frac{L}{M} \right)} \left( W^{L \frac{M}{4} (M+2)} + W^{-L \frac{M}{4} (M+2)} \right) \\
    &= \frac{1}{M} \left( 1 + e^{-j L \pi} \right) + \frac{\sin \left( \frac{L}{2} \right)}{\sin \left( \frac{L}{M} \right)} \cdot \cos \left( \frac{M + 2}{2M} L \pi \right). 
\end{align*}
\]