Overcoming Measurement Inconsistency in Deep Learning for Linear Inverse Problems: Applications in Medical Imaging
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Linear Inverse Problems

\[ b \in \mathbb{R}^m \]
\[ x^* \in \mathbb{R}^n \]

measurements
sparse

Reconstruct \( x^* \) from \( b \) containing \( m \) measurements

Deep Learning Methods

Optimization-based Methods

\[ \hat{x} = \arg \min_x \| \Phi x \|_1 \quad \text{s.t.} \quad Ax = b \]
Assume \( \Phi x \) is sparse (DCT, TV, etc.)

Guarantees \( A\hat{x} = b \)

Computationally expensive
Outperformed by deep learning methods

Our Approach

\[ \min_x \| x \|_{TV} + \beta \| x - w \|_{TV} \quad \text{s.t.} \quad Ax = b \]
Theory indicates that \( \beta = 1 \)

Experimental Results

MoDL (Aggarwal, 2019)

CRNN (Qin, 2019)

Ours

<table>
<thead>
<tr>
<th>Method</th>
<th>( | Aw - b |_2 )</th>
<th>( | A\hat{x} - b |_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MoDL (Aggarwal, 2019)</td>
<td>( 3.10 \times 10^{-1} )</td>
<td>( 9.88 \times 10^{-5} )</td>
</tr>
<tr>
<td>CRNN (Qin, 2019)</td>
<td>( 2.06 \times 10^{-6} )</td>
<td>( 7.71 \times 10^{-15} )</td>
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PSNR (SSIM) in the format average ± std

MoDL (Aggarwal, 2019)

CRNN (Qin, 2019)

Ours

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<td>MoDL (Aggarwal, 2019)</td>
<td>( 39.06 \pm 1.58 )</td>
<td>( 0.97 \pm 0.02 )</td>
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<tr>
<td>CRNN (Qin, 2019)</td>
<td>( 45.96 \pm 3.94 )</td>
<td>( 0.98 \pm 0.02 )</td>
</tr>
<tr>
<td>Ours</td>
<td>( 25.45 \pm 0.71 )</td>
<td>( 0.76 \pm 0.02 )</td>
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