# Advanced Hybrid Beamforming Design for MIMO Systems

Bowen Yang[1], Mehdi M. Molu[2], Lei Zhang[1]

## Introduction

In the designing of the large antenna array, most of the works relate to hybrid beamforming algorithm are based on two assumptions: either sparse channel [1] or large antenna array [2][3], which will introduce limits or extra complexities to the system. Hence, it's desirable to exploit an algorithm which could be applied without considering the number of antennas and channel sparsity.

An approximation based algorithm to solve this issue is proposed in this paper. Some of the contributions are summarized as follows:

- The phase shifter values of the analog beamformer are derived analytically, i.e., they are generated by a simple arctan(·) function, which makes it easy to be implemented.
- Analytical work has been done to the noise model, which proves that noise distribution will not be impacted by the RF beamformer.
- The proposed algorithm can be applied without the limitation of antenna array size and channel sparsity.

## System Model

A "fully connected" (i.e., each RF chain connects to all the antennas) Multiple-Input Multiple-Output (MIMO) system as illustrated in Fig.1 is considered in this paper. It consists of a transmitter (Tx) with $\mathbf{n}_T$ antennas and $\mathbf{r}_T$ RF chains, where we assume $\mathbf{n}_T \geq \mathbf{r}_T$, and a receiver (Rx) with $\mathbf{n}_R$ antennas and $\mathbf{r}_R$ RF chains, where we assume $\mathbf{n}_R \geq \mathbf{r}_R$. Besides, in order to avoid introducing too many parameters, it's assumed that the number of data streams is equal to the number of RF chains in both Tx and Rx.

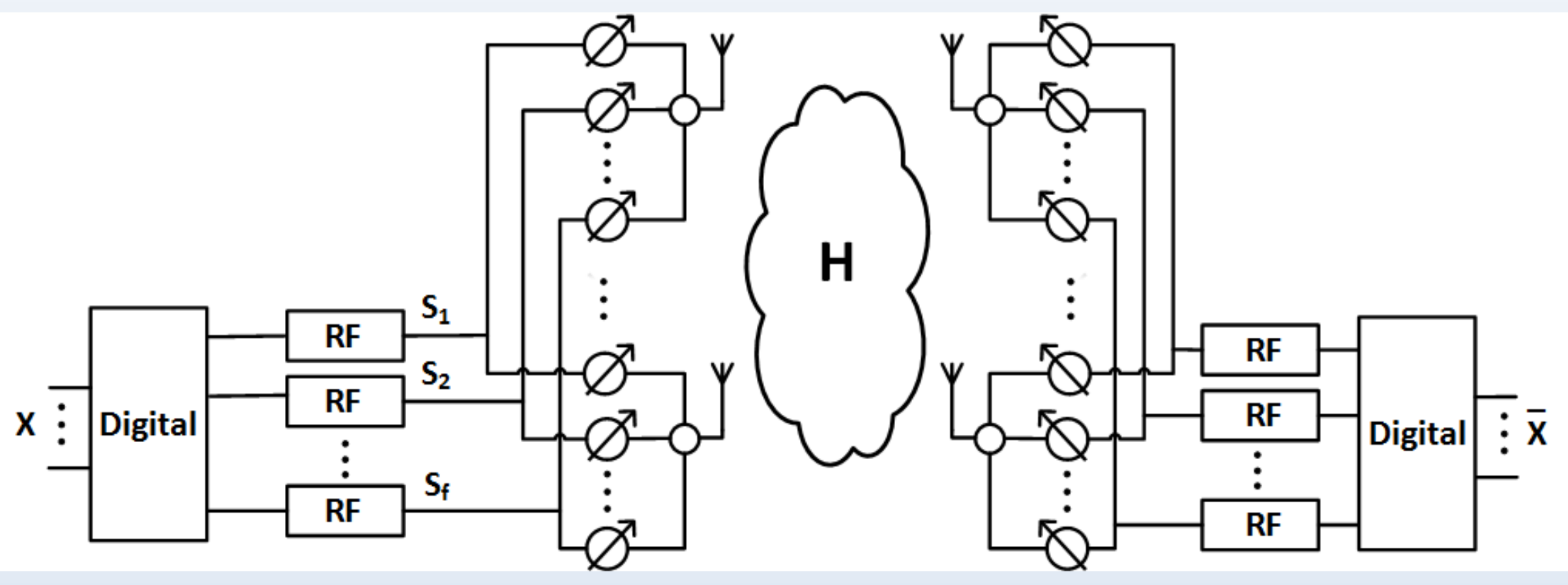

Fig. 1 System Model

The analog beamformers in Tx and Rx are represented as $\mathbf{A}_T$ and $\mathbf{A}_R$, and the digital beamformers in Tx and Rx are represented as $\mathbf{D}_T$ and $\mathbf{D}_R$, respectively. In the Tx side, the following transmit power constraint need to be satisfied:

$$\mathrm{Trace}\{\mathbf{A}_T\mathbf{D}_T\mathbf{D}_T^H\mathbf{A}_T^H\} = \mathbf{n}_T \qquad (1)$$

Assuming $\mathbf{\Gamma}$ is a ($\mathbf{r}_T$ x $\mathbf{r}_T$) square diagonal matrix obtained using water-filling algorithm that fulfils the transmit power constraints, i.e.,

$$\mathrm{Trace}\{\mathbf{\Gamma}\mathbf{\Gamma}^H\} = \mathrm{Trace}\{\mathbf{Q}\} = \mathrm{P} \qquad (2)$$

By designing the beamforming matrix $\mathbf{A}_T$, $\mathbf{A}_R$, $\mathbf{D}_T$ and $\mathbf{D}_R$ in an appropriate way, the system capacity C can be achieved as follows:

$$C = \max_{\substack{\mathrm{Trace}\{\mathbf{A}_T\mathbf{D}_T\mathbf{D}_T^H\mathbf{A}_T^H\}=\mathbf{n}_T \\ \mathrm{Trace}\{\mathbf{Q}\} = \mathrm{P}}} \{R\} \text{ bit/s/Hz} \qquad (3)$$

where *R* is the transmission rate.

## Beamformer Design

### Analog beamformer

Given **H** as the channel matrix and its spectral decomposition to be

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{H}} \qquad (4)$$

To facilitate the maximization in (3), a low complexity approximated solution is to make $\mathbf{V}^H\mathbf{A}_T\mathbf{D}_T$ as close to an upper identity matrix as possible, i.e.,

I. restrict $\mathbf{D}_T$ to be a unitary matrix and

**II.** $\mathbf{V}^H\mathbf{A}_T \rightarrow \mathbf{I}_{nT \times rT}$

It can be proved that the above approximation can be achieved if each entry of $\mathbf{A}_T$ is set to be:

$$a_{ki} = e^{j\delta_{1k}^{(i)}} \qquad (5)$$

where $\delta_{mk}^{(i)}$ is defined as:

$$\delta_{mk}^{(i)} = \begin{cases} \arctan\left(\frac{\Re\{v_{mi}^* v_{ki}\}}{\Im\{v_{mi}^* v_{ki}\}}\right), & \Re\{v_{mi}^* v_{ki}\} > 0 \\ \arctan\left(\frac{\Re\{v_{mi}^* v_{ki}\}}{\Im\{v_{mi}^* v_{ki}\}}\right) + \pi, & \Re\{v_{mi}^* v_{ki}\} < 0 \end{cases} \qquad (6)$$

In addition, the same approach can be used to derive $\mathbf{A}_R$ without introducing extra impacts (i.e., colored noised).

### Digital beamformer

The equivalent channel observed by the baseband units can be achieved as follows:

$$\mathbf{H}_{eq} = \mathbf{A}_R\mathbf{H}\mathbf{A}_T \qquad (7)$$

By implementing Singular-value decomposition (SVD), (7) can be rewritten as $\mathbf{H}_{eq} = \mathbf{U}_{eq}\mathbf{\Sigma}_{eq}\mathbf{V}_{eq}^{\mathbf{H}}$. The digital beamformer can be easily set to $\mathbf{D}_T = \mathbf{V}_{eq}$ and $\mathbf{D}_R = \mathbf{U}_{eq}^{\mathbf{H}}$.

## Results

In Fig. 2 and Fig. 3, we compare the performance of the proposed algorithm with one state-of-the-art algorithm proposed in [1]. Specifically, sparse scattering channel (based on the model in [4]) is implemented in Fig. 2 and rich scattering channel (based on the Rayleigh fading model) is implemented in Fig. 3. The assumed number of antennas is 100, and the number of RF chains is 5 in both cases. It can be observed that with sparse channel (i.e., Fig. 2), the performance of two algorithms are comparable, however the performance in [1] begins to degrade as the number of scatters increase (i.e., Fig. 3), while the performance of the proposed algorithm remains unchanged.

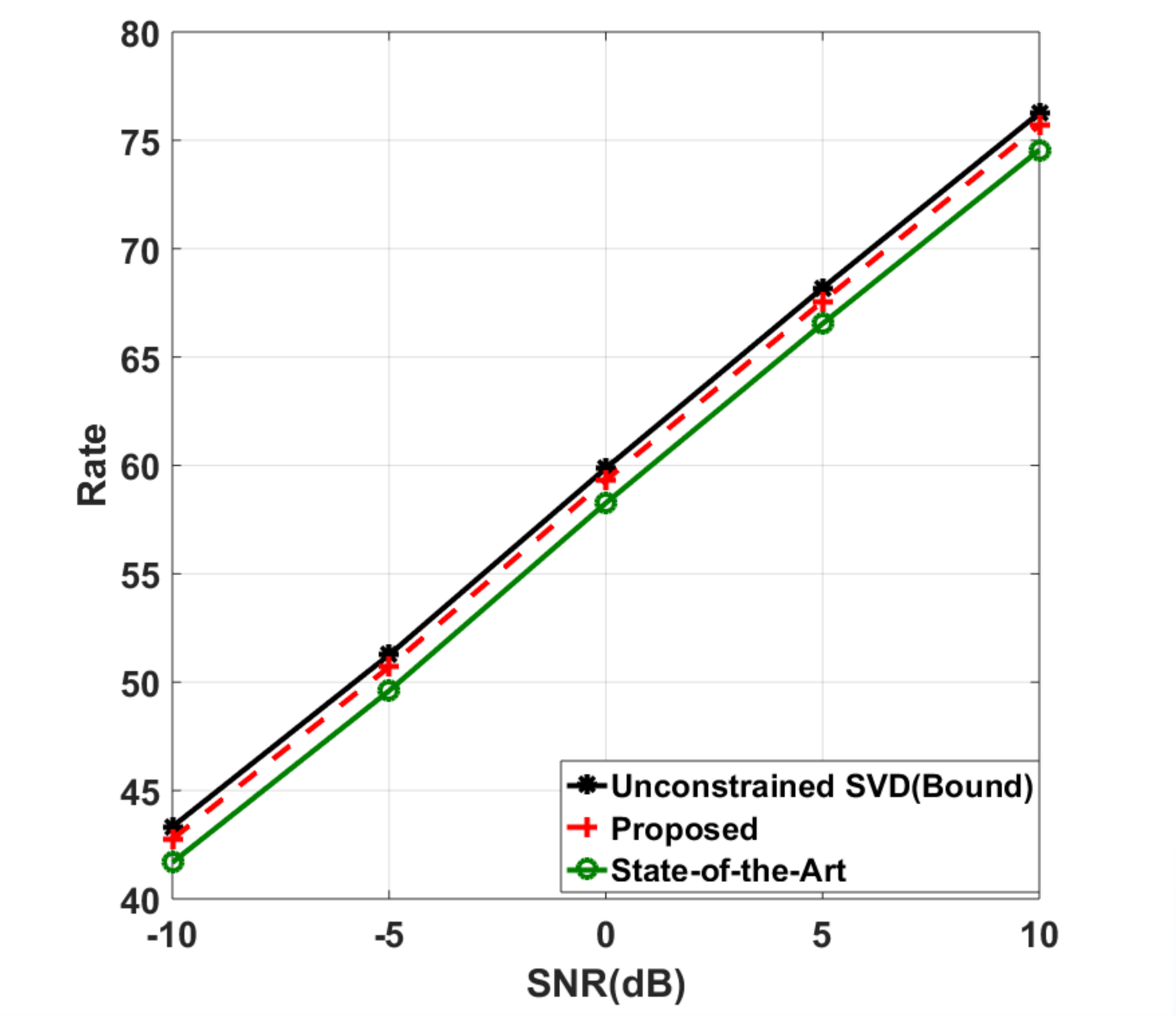

Fig. 2 Transmission Rate vs. SNR in a sparse channel

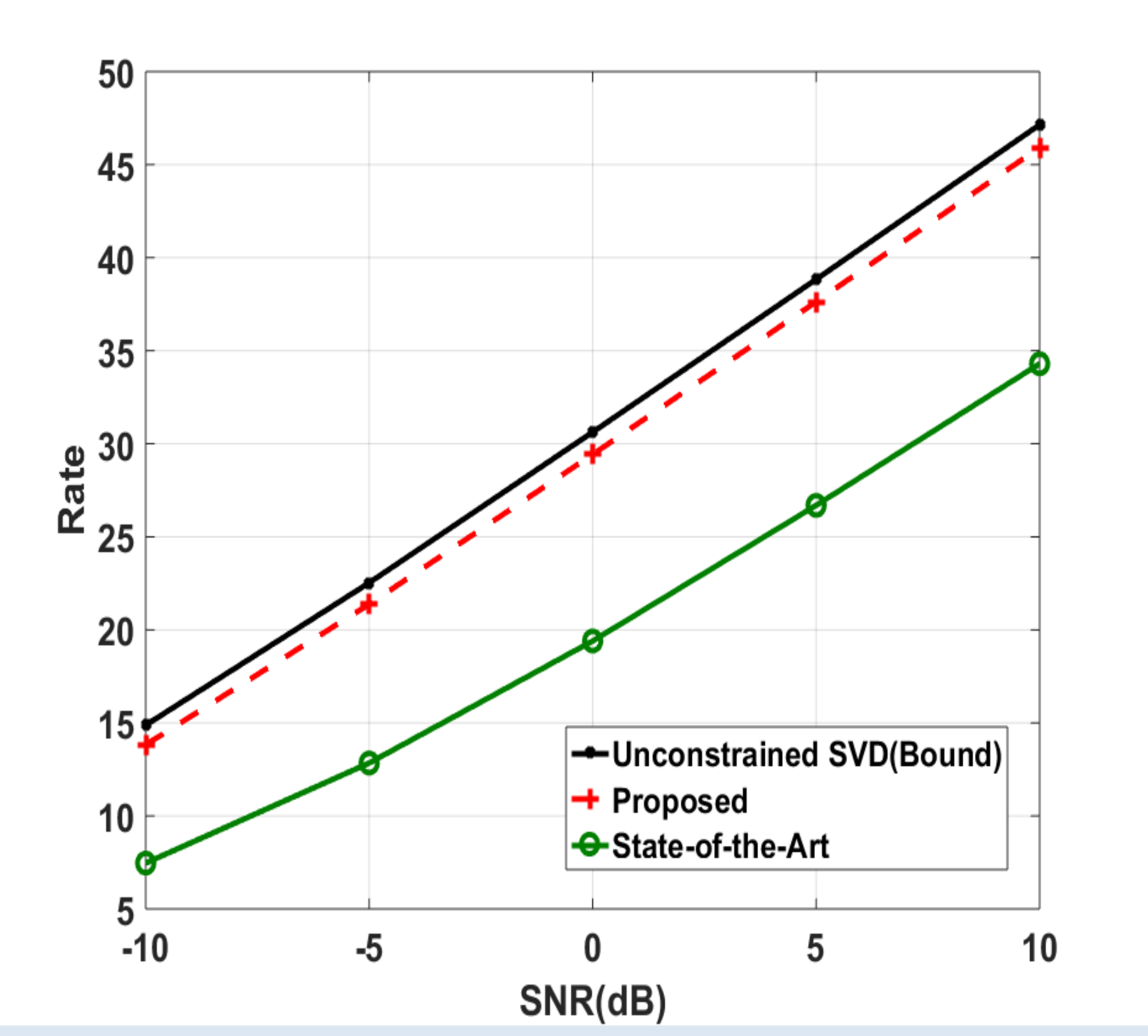

Fig. 3 Transmission Rate vs. SNR in a rich scattering channel

## Conclusion

The optimization of hybrid beamformer design for a multiantenna system was studied in this paper, and a low complexity hybrid beamforming algorithm is proposed. The coefficients of the analog beamformer were calculated by maximizing the data power and minimizing the interference power in the same time, after which the digital beamformer was derived based on a channel equivalent progress. The performance of the proposed algorithm was compared with state-of-the-art technique and shown advantage in terms of sphere of application (i.e., different antenna array sizes and channel sparsity).

## References

[1] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, "Spatially sparse precoding in millimeter wave mimo systems," *IEEE Transactions on Wireless Communications*, vol. 13, pp. 1499–1513, March 2014.

[2] S. Sohrabi and Y. Wei, "Hybrid digital and analog beamforming design for large-scale MIMO systems," *IEEE Int. Conf. Acoust. Speech Signal Process*, pp. 2929–2933, April. 2015.

[3] S. Sohrabi and Y. Wei, "Hybrid beamforming with finite-resolution phase shifters for large-scale MIMO systems," *IEEE Int. Workshop Signal Process. Wireless Commun*, pp. 136–140, Jun/Jul. 2015.

[4] A. M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Transactions on Signal Processing*, vol. 50, pp. 2563–2579, Oct 2002.

**Contact Information**

- [1]School of Engineering, University of Glasgow, Glasgow, G12 8QQ, UK — Email: b.yang.1@research.gla.ac.uk, Lei.Zhang@gla.ac.uk
- [2]Samsung Cambridge Solution Centre, Cambridge, CB4 0DS, UK — Email: mehdi.molu@gmail.com