Optimal Local Thresholds for Distributed Detection in Energy Harvesting Wireless Sensor Networks

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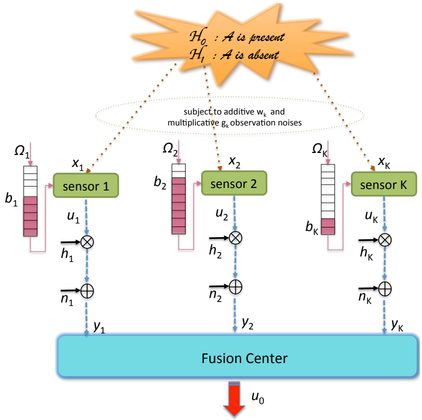
Outline

- Introduction
- System Model and problem statement
- Optimizing local decision thresholds
- Simulation results
- Conclusions

Introduction

- The designs of wireless sensor networks for distributed detection are often based on battery-powered sensors, leading into designs with a short lifetime, due to battery depletion.
- Energy harvesting, which collects energy from renewable resources promises a self-sustainable system with a lifetime that is not limited by battery lifetime.

System Model



- We consider distributed detection of known signal $\mathcal{A}$ in a WSN, consisting of K heterogeneous sensors.
- Each sensor is able to harvest energy from the environment and stores it in a battery with the capacity $\mathcal{K}$ units of energy.
- The sensors communicate with the FC through orthogonal fading channels with channel gains $|h_k|$'s, $\mathrm{E}\{|h_k|^2\} = \gamma_{h_k}$.
- The sensors employ on-off keying signaling.

System Model

Let x_k denote the local observation at sensor k:

$$x_k = \begin{cases} g_k \mathcal{A} + w_k & \mathcal{H}_1 \\ w_k & \mathcal{H}_0 \end{cases} \tag{1}$$

- $\mathcal{A}$ is a known scalar signal
- $w_k \sim \mathcal{N}(0, \sigma^2_{w_k}) \rightarrow$ Additive noise
- $g_k \sim \mathcal{N}(0, \gamma_{g_k}) \rightarrow$ Multiplicative noise
- All observation noises are independent over time and among K sensors.

System Model

During each observation period, sensor k takes N samples of x_k to measure the received signal energy and applies an energy detector to decide whether or not signal $\mathcal{A}$ is present.

$$\Lambda_k = \frac{1}{N}\sum_{n=1}^{N} |x_{k,n}|^2 \gtrless_{d_k=0}^{d_k=1} \theta_k \tag{2}$$

- $P_{f_k} = \Pr(\Lambda_k > \theta_k | \mathcal{H}_0)$
- $P_{d_k} = \Pr(\Lambda_k > \theta_k | \mathcal{H}_1)$
- **Goal:** We optimize the local decision threshold θ_k's considering two detection performance metrics:
  - The detection probability at the FC, assuming that the FC utilizes the optimal fusion rule based on Neyman-Pearson optimality criterion.
  - Kullback-Leibler (KL) distance between the two distributions of the received signals at the FC conditioned on hypotheses $\mathcal{H}_0, \mathcal{H}_1$.

System Model

- Sensor k uses the channel-inversion power control strategy, such that the number of energy units spent to convey a decision d_k is inversely proportional to $|h_k|$. To avoid the battery depletion when $|h_k|$ is too small, we impose an extra constraint for channel quality.
- Let $u_{k,t}$ be the sensor output corresponding to the observation period t.

$$u_{k,t} = \begin{cases} \lceil \frac{\lambda}{|h_k|} \rceil & \Lambda_k > \theta_k, \ b_{k,t} > \lceil \frac{\lambda}{|h_k|} \rceil, \ |h_k|^2 > \zeta_k \\ 0 & \text{Otherwise} \end{cases} \tag{3}$$

  - $b_{k,t}$ denote the battery state of sensor k
  - $|h_k|$ is channel gain
  - ζ_k is threshold of the channel quality
  - λ is a power regulation constant

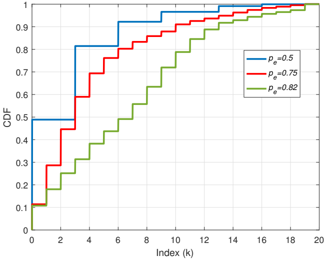
Battery state Model

We model $b_{k,t}$ in (3) as the following

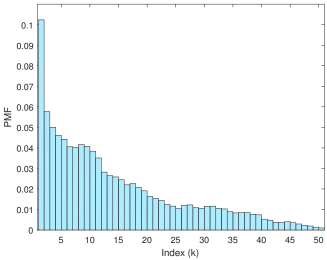
$$b_{k,t} = \min\Big\{ b_{k,t-1} - \lceil \frac{\lambda}{|h_k|} \rceil I_{u_{k,t}} + \Omega_{k,t} \ , \ \mathcal{K} \Big\} \tag{4}$$

$\Omega_{k,t} \in \{0, 1\}$ indicates units of harvesting energy and it is a Bernoulli random variable, with $\Pr(\Omega_{k,t}=1)=p_e$

- Assuming b_k in (4) is a stationary random process, one can compute the CDF and the pmf of b_k in terms of $\mathcal{K}, p_e, \gamma_{h_k}$ by some Monte Carlo simulations. Further, we use pmf of b_k for our numerical results.



(a)



(b)

Figure 1: (a) CDF of b_k for $\mathcal{K}=20$ and $p_e=0.5, 0.75, 0.82$, (b) pmf of b_k for $\mathcal{K}=50$ and $p_e=0.8$.

Optimal LRT Fusion Rule and P_D, P_F Expressions

The received signal at the FC from sensor k is $y_k = h_k u_k + n_k$, where $n_k \sim \mathcal{N}\left(0, \sigma_{n_k}^2\right)$. The likelihood ratio at the FC is

$$\Delta_{\text{LRT}} = \sum_{k=1}^{K} \log\left(\frac{\sum_{u_k} f\left(y_k|u_k\right) \Pr\left(u_k|\mathcal{H}_1\right)}{\sum_{u_k} f\left(y_k|u_k\right) \Pr\left(u_k|\mathcal{H}_0\right)}\right) \tag{5}$$

Given u_k, y_k is Gaussian:

$y_k|_{u_k=0} \sim \mathcal{N}\left(0, \sigma_{n_k}^2\right)$ and $y_k|_{u_k=\lceil\frac{\lambda}{|h_k|}\rceil} \sim \mathcal{N}\left(\lceil\frac{\lambda}{|h_k|}\rceil h_k, \sigma_{n_k}^2\right)$.

- $\Pr\left(u_k = \lceil\frac{\lambda}{|h_k|}\rceil|\mathcal{H}_1\right) = P_{d_k} \Pr\left(b_k > \lceil\frac{\lambda}{|h_k|}\rceil\right) \Pr\left(|h_k|^2 > \zeta_k\right) = \alpha_k$
- $\Pr\left(u_k = \lceil\frac{\lambda}{|h_k|}\rceil|\mathcal{H}_0\right) = P_{f_k} \Pr\left(b_k > \lceil\frac{\lambda}{|h_k|}\rceil\right) \Pr\left(|h_k|^2 > \zeta_k\right) = \beta_k$

Optimal LRT Fusion Rule and P_D, P_F Expressions

- Given a threshold τ, the optimal likelihood ratio test (LRT) is $\Delta_{\mathrm{LRT}} \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \tau$. The P_F, P_D at the FC

$$P_F = \Pr(\Delta_{\mathrm{LRT}} > \tau|\mathcal{H}_0) = Q\big(\frac{\tau - \mu_{\Delta|\mathcal{H}_0}}{\sigma_{\Delta|\mathcal{H}_0}}\big) \quad (6)$$

$$P_D = \Pr(\Delta_{\mathrm{LRT}} > \tau|\mathcal{H}_1) = Q\left(\frac{Q^{-1}(a)\sigma_{\Delta|\mathcal{H}_0} + \mu_{\Delta|\mathcal{H}_0} - \mu_{\Delta|\mathcal{H}_1}}{\sigma_{\Delta|\mathcal{H}_1}}\right) \quad (7)$$

where $\mu_{\Delta|\mathcal{H}_i}$ and $\sigma^2_{\Delta|\mathcal{H}_i}$ are functions of statistics of conditional random variables $y_k\big|_{u_k=\lceil \frac{\lambda}{|h_k|}\rceil}$ and $y_k\big|_{u_k=0}$.

- We note that P_D expression depends on all our optimization variables θ_k's through α_k, β_k's in $\mu_{\Delta|\mathcal{H}_i}$ and $\sigma^2_{\Delta|\mathcal{H}_i}$.

KL Expression

- KL distance between the two distributions of the received signals at FC is

$$KL_k = \int_{y_k} f(y_k|\mathcal{H}_1) \log \left(\frac{f(y_k|\mathcal{H}_1)}{f(y_k|\mathcal{H}_0)} \right) dy_k \tag{8}$$

- We approximate KL_k in (8) by the KL distance of two Gaussian distributions with the means $\mu_{y_k|\mathcal{H}_0}$, $\mu_{y_k|\mathcal{H}_1}$, and the variances $\sigma^2_{y_k|\mathcal{H}_0}$ and $\sigma^2_{y_k|\mathcal{H}_1}$, respectively.

$$KL_k \approx \frac{1}{2} \log\left(\frac{\sigma^2_{y_k|\mathcal{H}_0}}{\sigma^2_{y_k|\mathcal{H}_1}}\right) + \frac{\sigma^2_{y_k|\mathcal{H}_1} - \sigma^2_{y_k|\mathcal{H}_0} + (\mu_{y_k|\mathcal{H}_1} - \mu_{y_k|\mathcal{H}_0})^2}{2\sigma^2_{y_k|\mathcal{H}_0}} \tag{9}$$

Simulation Results

We consider:

- Scheme I: Numerically find θ_k's which maximize P_D in (7) $\rightarrow$ K-dimensional search is required $\rightarrow$ computational complexity!
- Scheme II: Finding θ_k's which maximize $KL_{tot} = \sum_{k=1}^{K} KL_k$, using the KL_k approximation in (9) $\rightarrow$ K one-dimensional search is required $\rightarrow$ computationally efficient.
- Special case: Assume all sensors employ the same local threshold $\theta_k = \theta$ and compare Scheme I and Scheme II.

We then compare P_D evaluated at the θ_k's obtained from these schemes.

Simulation results

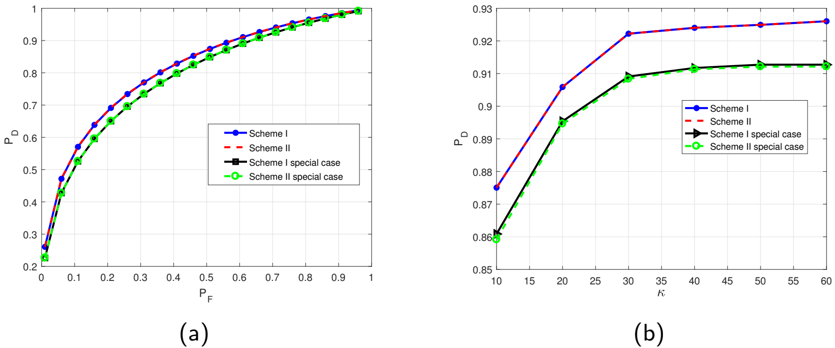


Figure 2: (a) P_D vs. P_F
(b) P_D vs. $\mathcal{K}$

Conclusions

- We studied a distributed detection problem in a WSN with K heterogeneous energy harvesting sensors and investigated the optimal local decision thresholds for given transmission and battery state models.
- Our numerical results indicate that the thresholds obtained from maximizing the total KL distance are near-optimal and computationally very efficient, as it requires only K one-dimensional searches, as opposed to a K-dimensional search required to find the thresholds that maximize the detection probability at FC.
- The performance gap between each scheme and its corresponding special case indicates that when sensors are heterogeneous, it is advantageous to use different local thresholds according to sensors' statistics.

Thank You

Questions?