Optimal Local Thresholds for Distributed Detection in Energy Harvesting Wireless Sensor Networks

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Outline

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The designs of wireless sensor networks for distributed detection are often based on battery-powered sensors, leading into designs with a short lifetime, due to battery depletion.

Energy harvesting, which collects energy from renewable resources promises a self-sustainable system with a lifetime that is not limited by battery lifetime.
We consider distributed detection of known signal $\mathcal{A}$ in a WSN, consisting of $K$ heterogeneous sensors.

Each sensor is able to harvest energy from the environment and stores it in a battery with the capacity $K$ units of energy.

The sensors communicate with the FC through orthogonal fading channels with channel gains $|h_k|$’s, $E\{|h_k|^2\} = \gamma h_k$.

The sensors employ on-off keying signaling.
Let $x_k$ denote the local observation at sensor $k$:

$$x_k = \begin{cases} g_k A + w_k & \mathcal{H}_1 \\ w_k & \mathcal{H}_0 \end{cases}$$

(1)

- $A$ is a known scalar signal
- $w_k \sim \mathcal{N}(0, \sigma^2_{w_k}) \rightarrow$ Additive noise
- $g_k \sim \mathcal{N}(0, \gamma_{g_k}) \rightarrow$ Multiplicative noise
- All observation noises are independent over time and among $K$ sensors.
System Model

During each observation period, sensor $k$ takes $N$ samples of $x_k$ to measure the received signal energy and applies an energy detector to decide whether or not signal $A$ is present.

$$\Lambda_k = \frac{1}{N} \sum_{n=1}^{N} |x_{k,n}|^2 \geq \begin{cases} d_k=1 & \theta_k \\ d_k=0 & \end{cases}$$

- $P_{f_k} = \Pr(\Lambda_k > \theta_k | \mathcal{H}_0)$
- $P_{d_k} = \Pr(\Lambda_k > \theta_k | \mathcal{H}_1)$

**Goal:** We optimize the local decision threshold $\theta_k$'s considering two detection performance metrics:

- The detection probability at the FC, assuming that the FC utilizes the optimal fusion rule based on Neyman-Pearson optimality criterion.

- Kullback-Leibler (KL) distance between the two distributions of the received signals at the FC conditioned on hypotheses $\mathcal{H}_0, \mathcal{H}_1$. 
System Model

Sensor $k$ uses the channel-inversion power control strategy, such that the number of energy units spent to convey a decision $d_k$ is inversely proportional to $|h_k|$. To avoid the battery depletion when $|h_k|$ is too small, we impose an extra constraint for channel quality.

Let $u_{k,t}$ be the sensor output corresponding to the observation period $t$.

$$u_{k,t} = \begin{cases} \lceil \frac{\lambda}{|h_k|} \rceil & \Lambda_k > \theta_k, \ b_{k,t} > \lceil \frac{\lambda}{|h_k|} \rceil, \ |h_k|^2 > \zeta_k \\ 0 & \text{Otherwise} \end{cases}$$

- $b_{k,t}$ denote the battery state of sensor $k$
- $|h_k|$ is channel gain
- $\zeta_k$ is threshold of the channel quality
- $\lambda$ is a power regulation constant
We model $b_{k,t}$ in (3) as the following

$$b_{k,t} = \min \left\{ b_{k,t-1} - \left\lfloor \frac{\lambda}{h_k} \right\rfloor u_{k,t} + \Omega_{k,t}, K \right\}$$

(4)

$\Omega_{k,t} \in \{0, 1\}$ indicates units of harvesting energy and it is a Bernoulli random variable, with $\Pr(\Omega_{k,t} = 1) = p_e$
Assuming $b_k$ in (4) is a stationary random process, one can compute the CDF and the pmf of $b_k$ in terms of $\mathcal{K}$, $p_e$, $\gamma h_k$ by some Monte Carlo simulations. Further, we use pmf of $b_k$ for our numerical results.

![Graph](image1.png)

**Figure 1:** (a) CDF of $b_k$ for $\mathcal{K}=20$ and $p_e=0.5, 0.75, 0.82$, (b) pmf of $b_k$ for $\mathcal{K}=50$ and $p_e=0.8$. 

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The received signal at the FC from sensor $k$ is $y_k = h_k u_k + n_k$, where $n_k \sim \mathcal{N} \left( 0, \sigma^2_{n_k} \right)$. The likelihood ratio at the FC is

$$\Delta_{\text{LRT}} = \sum_{k=1}^{K} \log \left( \frac{\sum_{u_k} f \left( y_k | u_k \right) \Pr \left( u_k | \mathcal{H}_1 \right)}{\sum_{u_k} f \left( y_k | u_k \right) \Pr \left( u_k | \mathcal{H}_0 \right)} \right)$$

(5)

Given $u_k$, $y_k$ is Gaussian:

$y_k | u_k=0 \sim \mathcal{N} \left( 0, \sigma^2_{n_k} \right)$ and $y_k | u_k=\left[ \frac{\lambda}{|h_k|} \right] \sim \mathcal{N} \left( \left[ \frac{\lambda}{|h_k|} \right] h_k, \sigma^2_{n_k} \right)$.

- $\Pr \left( u_k = \left[ \frac{\lambda}{|h_k|} \right] | \mathcal{H}_1 \right) = P_{d_k} \Pr \left( b_k > \left[ \frac{\lambda}{|h_k|} \right] \right) \Pr \left( |h_k|^2 > \zeta_k \right) = \alpha_k$
- $\Pr \left( u_k = \left[ \frac{\lambda}{|h_k|} \right] | \mathcal{H}_0 \right) = P_{f_k} \Pr \left( b_k > \left[ \frac{\lambda}{|h_k|} \right] \right) \Pr \left( |h_k|^2 > \zeta_k \right) = \beta_k$
Given a threshold $\tau$, the optimal likelihood ratio test (LRT) is

$$\Delta_{\text{LRT}} \geq \mathcal{H}_1 \tau.$$ The $P_F, P_D$ at the FC

$$P_F = \Pr (\Delta_{\text{LRT}} > \tau | \mathcal{H}_0) = Q \left( \frac{\tau - \mu_{\Delta|\mathcal{H}_0}}{\sigma_{\Delta|\mathcal{H}_0}} \right) \quad (6)$$

$$P_D = \Pr (\Delta_{\text{LRT}} > \tau | \mathcal{H}_1)$$

$$= Q \left( \frac{Q^{-1}(a) \sigma_{\Delta|\mathcal{H}_0} + \mu_{\Delta|\mathcal{H}_0} - \mu_{\Delta|\mathcal{H}_1}}{\sigma_{\Delta|\mathcal{H}_1}} \right) \quad (7)$$

where $\mu_{\Delta|\mathcal{H}_i}$ and $\sigma^2_{\Delta|\mathcal{H}_i}$ are functions of statistics of conditional random variables $y_k | u_k = \lceil \frac{\lambda}{|h_k|} \rceil$ and $y_k | u_k = 0$.

We note that $P_D$ expression depends on all our optimization variables $\theta_k$’s through $\alpha_k, \beta_k$’s in $\mu_{\Delta|\mathcal{H}_i}$ and $\sigma^2_{\Delta|\mathcal{H}_i}$.
KL distance between the two distributions of the received signals at FC is

$$KL_k = \int_{y_k} f(y_k|\mathcal{H}_1) \log \left( \frac{f(y_k|\mathcal{H}_1)}{f(y_k|\mathcal{H}_0)} \right) dy_k \quad (8)$$

We approximate $KL_k$ in (8) by the KL distance of two Gaussian distributions with the means $\mu_{y_k|\mathcal{H}_0}$, $\mu_{y_k|\mathcal{H}_1}$, and the variances $\sigma^2_{y_k|\mathcal{H}_0}$ and $\sigma^2_{y_k|\mathcal{H}_1}$, respectively.

$$KL_k \approx \frac{1}{2} \log \left( \frac{\sigma^2_{y_k|\mathcal{H}_1}}{\sigma^2_{y_k|\mathcal{H}_0}} \right) + \frac{\sigma^2_{y_k|\mathcal{H}_1} - \sigma^2_{y_k|\mathcal{H}_0} + (\mu_{y_k|\mathcal{H}_1} - \mu_{y_k|\mathcal{H}_0})^2}{2\sigma^2_{y_k|\mathcal{H}_0}} \quad (9)$$
Simulation Results

We consider:

- **Scheme I**: Numerically find $\theta_k$’s which maximize $P_D$ in (7) → $K$-dimensional search is required → computational complexity!

- **Scheme II**: Finding $\theta_k$’s which maximize $KL_{tot} = \sum_{k=1}^{K} KL_k$, using the $KL_k$ approximation in (9) → $K$ one-dimensional search is required → computationally efficient.

- Special case: Assume all sensors employ the same local threshold $\theta_k = \theta$ and compare Scheme I and Scheme II.

We then compare $P_D$ evaluated at the $\theta_k$’s obtained from these schemes.
Simulation results

Figure 2: (a) $P_D$ vs. $P_F$  
(b) $P_D$ vs. $\kappa$
We studied a distributed detection problem in a WSN with $K$ heterogeneous energy harvesting sensors and investigated the optimal local decision thresholds for given transmission and battery state models.

Our numerical results indicate that the thresholds obtained from maximizing the total KL distance are near-optimal and computationally very efficient, as it requires only $K$ one-dimensional searches, as opposed to a $K$-dimensional search required to find the thresholds that maximize the detection probability at FC.

The performance gap between each scheme and its corresponding special case indicates that when sensors are heterogeneous, it is advantageous to use different local thresholds according to sensors’ statistics.
Questions?