Cramér–Rao Bound for Line Constrained Trajectory Tracking

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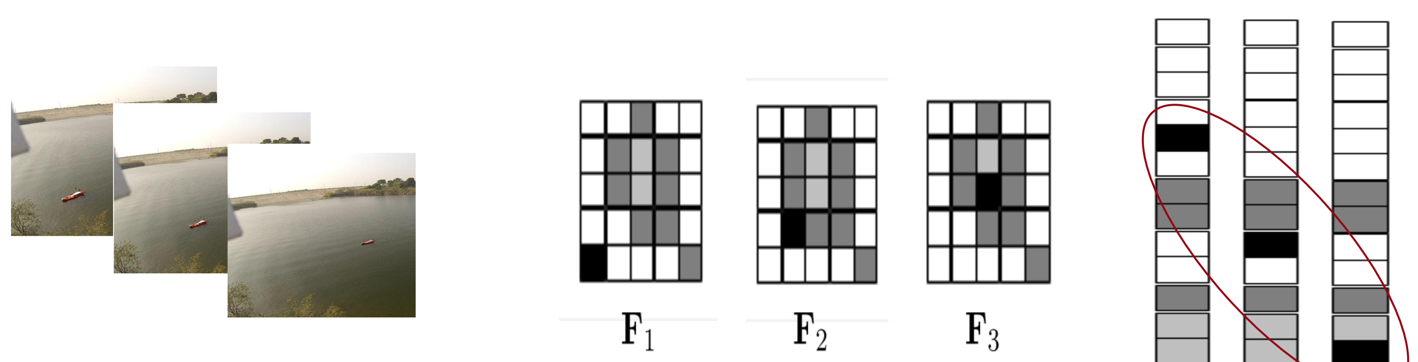
Tracking Targets from Video

$X = [vec(\mathbf{F1}), vec(\mathbf{F2}), ..]$



Signal model: $X = L + S$

L = slowly changing background, low rank

S = object of interest, sparse and "linear"

Prior Work

- Background subtraction (Ebadi & Ones ICIP 2015)

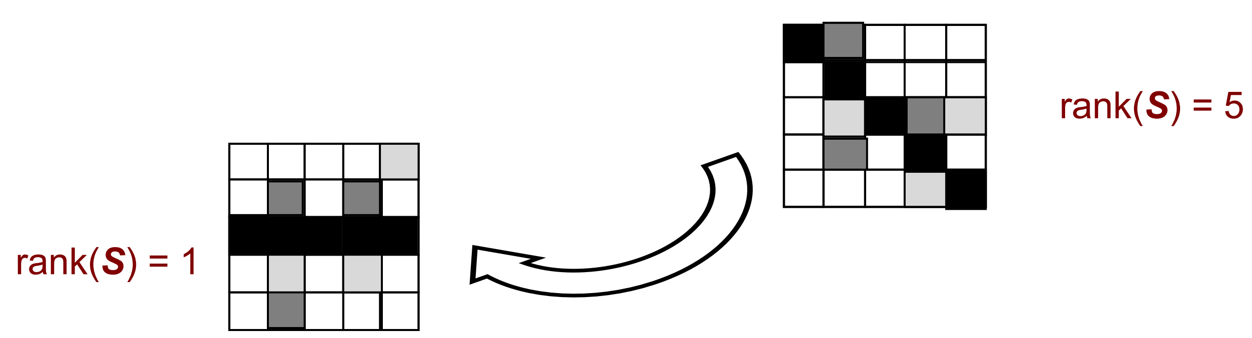
$$\underset{L,S}{\text{minimize}} \ \|L\|_* + \lambda_1 \|S\|_1 \quad \text{subject to} \quad X = L + S$$

- Group + element sparsity (Liu, Zhao, Yao & Qi, IEEE Trans Image Proc 2015)

$$\underset{L,S}{\text{minimize}} \ \|L\|_* + \lambda_1 \|S\|_{2,1} \quad \text{subject to} \quad X = L + S$$

- Add trajectory constraint (Elnakeeb & Mitra ISIT 2017)

Observation: with proper rotation, sparse contribution is rank 1



Our Contributions

- Derive CRB for linear trajectory estimation
- 4 dB improvement over background subtraction and 1.2 dB away from CRB
- Real video: accurate tracking (trajectory not fully linear)
- Performance improvement with addition of "linear" constraint

Optimization

$$\underset{L,S,R}{\text{minimize}} \ \|L\|_* + \lambda_1 \|S\|_{2,1} + \lambda_2 \|RS\|_*$$
$$\text{subject to} \quad X = L + S, \quad RR^T = I_m$$

rotation constraint

Solution via ALM

Lin, Chen &Ma, UIUC Technical Report, 2009

- Update step for L:

$$L_{k+1} = \underset{L}{\text{argmin}} \ \mathcal{L}(L, S_k, R_k, S_{r_k}, M_k, N_k, V_k, \beta_k)$$

Lagrange Multipliers

$S_r = RS$

- Similar updates for $S, R,$ and S_r

Cramér–Rao Bound

Adapt RPCA results to include linear constraint (Tang & Nehorai TSP 2011)

$$\text{MSE}_{L,S} \geq \text{new CRB} \leq \text{old CRB}$$

$f(m, n, N, \min(s, m+n-1))$ $\qquad$ $f(m, n, N, s)$

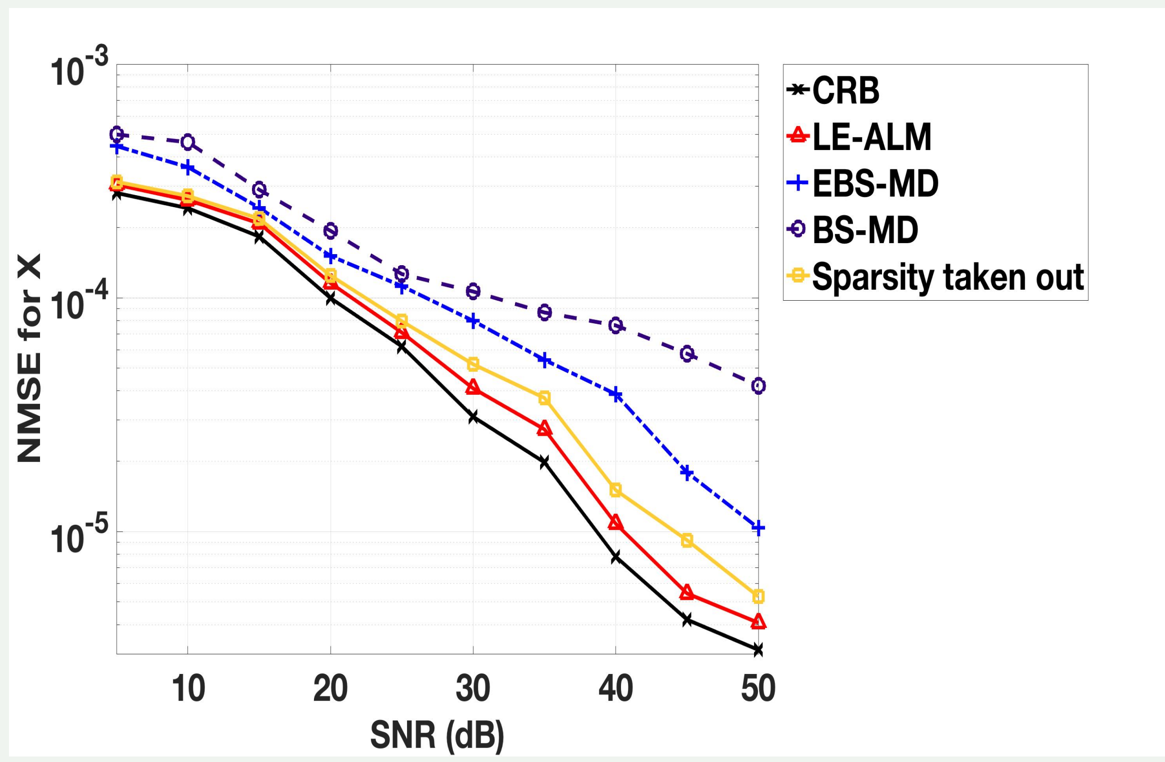
min $(s, m+n-1)$ rather than s

Incorporating linear constraint → tighter CRB

$$f(a,b,c,d) = d - c + \frac{abc}{ab - d}$$

$N = (m+n)r - r^2$; m, n are the dimensions, r is the rank

Simulations



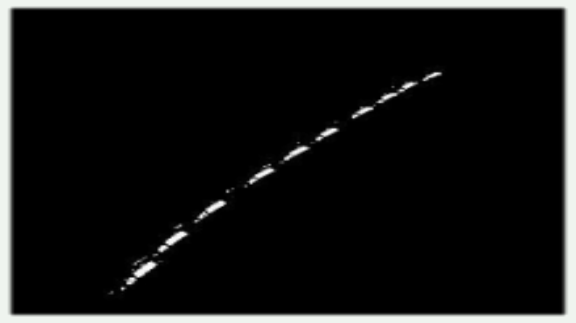
LE-ALM: our new method
EBS-MD: Efficient Background Subtraction via Matrix Decomposition
BS-MD: Background Subtraction via Matrix Decomposition

- LE-ALM has superior performance, nearly achieves CRB
- Linear constraint improves performance
- Sparsity constraint $\lambda_1 \|S\|_{2,1}$ is also needed

Real Data

$$\text{NMSE} = \frac{\|\theta_0 - \hat{\theta}\|_F^2}{\|\theta_0\|_F^2}$$

Noisy frame (SNR=30 dB)



Estimation

LE-ALM has excellent MSE even in low SNR

LE-ALM works well despite not fully linear trajectory

NMSE for L and S for various SNR values

SNR (in dB)	e_L	e_S
10	5.97×10^{-5}	4.83×10^{-4}
20	3.92×10^{-5}	2.21×10^{-4}
30	1.69×10^{-5}	1.24×10^{-4}
40	0.55×10^{-5}	0.62×10^{-4}
50	0.013×10^{-5}	0.18×10^{-4}

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