Cramér–Rao Bound for Line Constrained Trajectory Tracking  
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Prior Work
- Background subtraction (Ebadi & Ones ICIP 2015)
  \[ \minimize_{L,S} \|L\|_1 + \lambda_1 \|S\|_1 \quad \text{subject to} \quad X = L + S \]
  \[ \minimize_{L,S} \|L\|_1 + \lambda_1 \|S\|_{2,1} \quad \text{subject to} \quad X = L + S \]
- Add trajectory constraint (Elnakeeb & Mitra ISIT 2017)
  Observation: with proper rotation, sparse contribution is rank 1

Solution via ALM
Lin, Chen & Ma, UIUC Technical Report, 2009
- Update step for L:
  \[ L_{k+1} = \arg\min_L \mathcal{L}(L, S, R, S_R, M_{S_k}, N_{S_k}, V_{S_k}, R_k) \]
- Similar updates for S, R, and S_R

Cramér–Rao Bound
Adapt RPCA results to include linear constraint (Tang & Nehorai TSP 2011)
\[ \text{MSE}_{L,S} \geq \text{new CRB} \leq \text{old CRB} \]
\[ f(m, n, N, \min(s, m + n - 1)) \]
\[ f(m, n, N, s) \]
\[ \min(s, m + n - 1) \] rather than \( s \)

Incorporating linear constraint → tighter CRB
\[ f(a, b, c, d) = d - c + \frac{abc}{ab - d} \]
\[ N = (m + n)r - r^2; \ m, \ n \ text{are the dimensions,} \ r \ text{is the rank} \]

Our Contributions
- Derive CRB for linear trajectory estimation
- 4 dB improvement over background subtraction and 1.2 dB away from CRB
- Real video: accurate tracking (trajectory not fully linear)
- Performance improvement with addition of “linear” constraint

Optimization
\[ \minimize_{L,S,R} \|L\|_1 + \lambda_1 \|S\|_{2,1} + \lambda_3 \|RS\|_1 \]
subject to \( X = L + S \)

Simulations
- LE-ALM: our new method
- EBS-MD: Efficient Background Subtraction via Matrix Decomposition
- BS-MD: Background Subtraction via Matrix Decomposition

Real Data
- Noisy frame (SNR=30 dB)
- Estimation

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