RECOVERING SIGNALS FROM THEIR FROG TRACE

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BACKGROUND

Goal: Characterization of ultra-short ($\sim 10^{-15}$ seconds) optical pulses

Problem: Electronic sensors cannot measure the phase information of the pulse, only its Fourier magnitude. The problem of recovering a signal from its Fourier magnitude, known as the **phase retrieval problem**, is ill-posed.

Solution: Correlating the signal with its shifted versions, before measuring the Fourier magnitude, for several different shifts. The acquired data is a **quartic phaseless** function of the signal. This technique, proposed in 1993, is called **Frequency Resolved Optical Gating (FROG).**

THE FROG PROBLEM

The FROG trace (measurements) is given by

$$|\hat{y}_{m,k}| = \left| \sum_{n=0}^{N-1} x_n x_{n+mL} e^{-2\pi i k n/N} \right|,$$

$$k = 1, \dots, N, \qquad m = 1, \dots, \lceil N/L \rceil.$$

The parameter L determines the redundancy of the data.

Under what conditions the measurements $|\hat{y}_{m,k}|$ determine the signal x uniquely?

MAIN RESULT

Let $x \in \mathbb{C}^N$ be a B-bandlimited signal for some $B \leq N/2$. Then, generic signals are determined uniquely, up to symmetries, from only 3B measurements if $N/L \geq 4$.

If the power spectrum of x is also available (as often the case in practice), then 2B measurements are enough provided that $N/L \geq 3$.

CONTRIBUTION

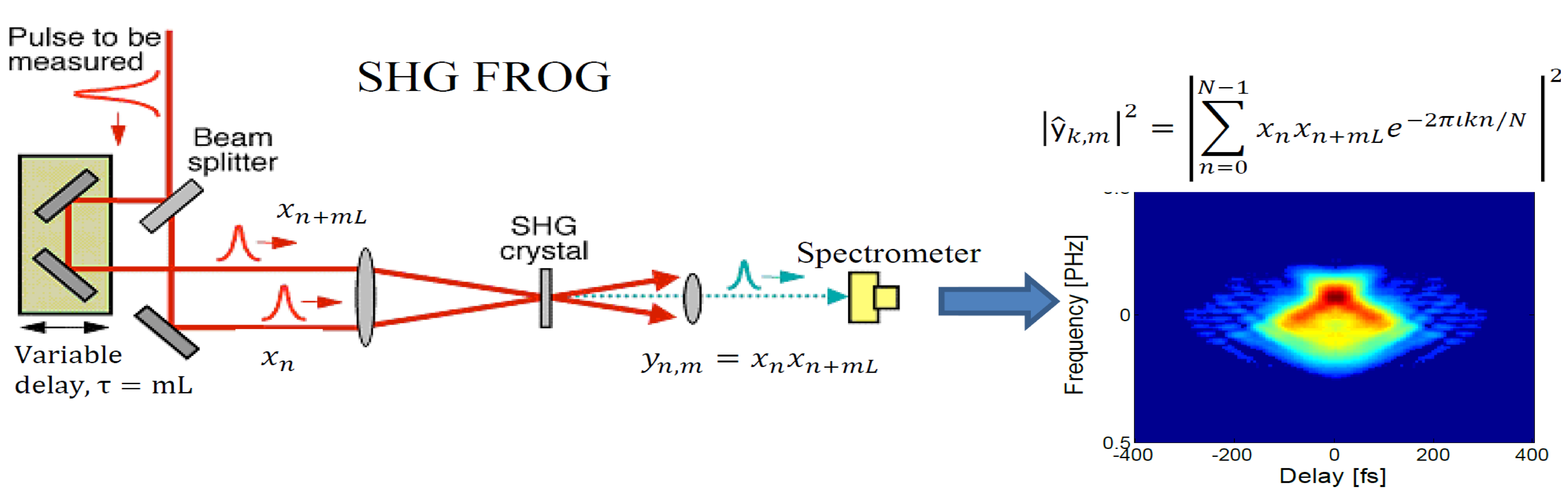
FROG is the most popular method for full characterization of ultra-short optical pulses in the last two decades.

We present the **first theoretical result** on FROG, deriving the **fundamental conditions for exact recovery**.

FUTURE WORK

1. Analysis of the **phaseless blind STFT** model, arising in ptychography and blind FROG, in which two different signals correlate each other.
2. The computational and algorithmic aspects of FROG.

SCHEMATIC ILLUSTRATION OF THE FROG TECHNIQUE



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