A Performance Analysis on the Optimal Number of Measurements for Coded Compressive Imaging

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Motivation

• Practical coded compressive imaging settings
  – Focal Plane Array (FPA) imaging
    • Gathers noisy undersampled measurements of spatially modulated light intensity from a scene
    • Spatial modulation can be performed at sub-pixel level using a DMD
    • Reconstruction using sparse recovery algorithm
  – Magnetic Particle Imaging (MPI)
    • Allows fast imaging of magnetic nanoparticle (MNP) samples in a FOV
    • System matrix (SM) calibration is done using coded scenes with MNP samples at multiple positions
    • SM reconstruction using compressive sensing

• Investigation of the trade-off between input pSNR, number of measurements, and image quality
Previous Work

- Practical signal transmission in radar/sonar with a fixed power budget (Yang et al., 2017)
  - Measurement matrix with Gaussian iid entries
  - Gaussian, Bernoulli-Gaussian, and least favourite distributions for signal models
  - Sparsity level should be known
  - Based on state evolution technique proposed for approximate message passing algorithm
Assumptions

• FPA Imaging
  – Constant integration time for measurements
    • Per-frame integration time is divided among different spatial modulations
    • $k$ different modulations -> Input SNR scales by $1/k$
  – Signal energy per pixel increases with pixel size
• MPI
  – Signal energy decays linearly with number of coded scenes
Real Domain : FPA Imaging

- $k$ different DMD encodings
- $x \in \mathbb{R}^N$, $y \in \mathbb{R}^{nk}$ (FPA with $n$ pixels) ($N > n$)
- Super-resolution factor $d = \frac{N}{n}$ & Compression ratio $m = \frac{k}{d}$
- Forward model: $y = \tau Ax + n$ where $n \in \mathbb{R}^{nk}$ is AWGN and $A = D\Lambda$
- $\tau = \frac{d}{k}$ reflects the effects of increased pixel size and decreased integration time per DMD mask, given constant noise level
\[ y = \tau A x + n \]

\[
\min_x \alpha_1 \|Fx\|_1 + \alpha_2 TV(x) \quad \text{subject to} \quad \left\|Ax - \frac{y}{\tau}\right\|_2 \leq \frac{\epsilon}{\tau}, x[i] \geq 0 \ \forall i
\]

- \( F \): Sparsifying transform such as the Fourier
- \( TV(.\)\) : Total variation operator
- \( \epsilon \): Bound on the noise
- Weighted sum is due to superior performance
- ADMM based reconstruction algorithm (Kar et al., 2018)
Complex Domain: Magnetic Particle Imaging
System Calibration

\[ y = Xp + n \]

- \( y \in \mathbb{C}^M \): measurements, \( p \in \mathbb{R}^N \): Calibration Scene
- \( X \in \mathbb{C}^{M \times N} \): System matrix (SM), \( n \in \mathbb{C}^N \): Complex AWGN

- Taking multiple measurements using different \( p \), a single row of \( X \), i.e. \( x^{(i)} \):
  \[ y^{(i)} = P^T x^{(i)} + n \]
  - \( P \): Binary coding scene, \( y^{(i)} \): \( i \)-th row of SM sensed with \( P \)
Magnetic Particle Imaging – Reconstruction

\[ y^{(i)} = P^T x^{(i)} + n \]

\[ \min_x \|Dx^{(i)}\|_1 \quad \text{subject to} \quad \|P^T x^{(i)} - y^{(i)}\|_2 \leq \varepsilon_i \]

- \( D \): Sparsifying transform such as the DCT
- Entries of \( P \) are drawn from a symmetric Bernoulli distribution
- ADMM based reconstruction algorithm (Ilbey et al., 2018)
- Can be considered as a special case of the FPA-imaging problem
Compressive Sensing Perspective

• Theorem 1.2 (Candes, 2008)

\[ \| \hat{x} - x \|_2 \leq C_0 s^{-0.5} \| x - x_s \|_1 + C_1 \epsilon \]  

\[ C_0 = 2 \frac{1-(1-\sqrt{2})\delta_{2s}}{1-(1+\sqrt{2})\delta_{2s}} , \quad C_1 = 4 \frac{\sqrt{1+\delta_{2s}}}{1-(1+\sqrt{2})\delta_{2s}} \]

- \( \hat{x} \): Estimate of \( x \)
- \( x_s \): \( s \) — sparse version of \( x \)
- \( \epsilon = m\sigma^2 \): Bound on the noise
- \( \delta_{2s} \): Restricted isometry constant

• Increasing \( m \) increases the second term in (1)
• \( \delta_{2s} \) is monotonically decreasing function of \( m \)
• \( C_0 \) and \( C_1 \) calculations are NP-hard, thus the bound is an NP-hard problem
• There exists an optimal number of measurements for a given problem, but its solution is impractical
ADMM Based Reconstruction

• ADMM
• Problem formulation:

\[
\begin{align*}
\text{minimize} & \quad f_1(x) + f_2(z) \\
\text{subject to} & \quad Gx + Qz - r = 0
\end{align*}
\]

• \(f_1(.)\) and \(f_2(.)\) separable convex functions
• Two small problems instead of one large problem
• Updates \(x\) and \(z\) alternatingly
Solved Problem

\[
\begin{align*}
\min_{x} & \quad \alpha_1 \|Fx\|_1 + \alpha_2 TV(x) \\
\text{subject to} & \quad \|Bx - y\|_2 \leq \epsilon, \ x[i] \geq 0 \ \forall i
\end{align*}
\]

Solved Problem in ADMM Form

\[
\begin{align*}
\min_{x,z} & \quad f_1(x) + f_2(z) \\
\text{subject to} & \quad Gx + Qz - r = 0
\end{align*}
\]

Where

\[
f_1(x) = \mathcal{I}_{(x \geq 0)}(x), \quad f_2(z) = \alpha_1 \|Fz^{(1)}\|_1 + \alpha_2 TV(z^{(2)}) + \mathcal{I}_{(\|Bz^{(3)} - y\|_2 \leq \epsilon)}(z^{(3)})
\]

\[
G = [I \ I \ B]^T, \quad Q = -I, \quad r = 0, \quad z = [z^{(1)} \ z^{(2)} \ z^{(3)}]^T
\]

Efficient solutions of ADMM steps (Kar et al., 2018)
Results: FPA imaging

- Image size: $360 \times 360$
- FPA size: $30 \times 30$, $60 \times 60$, $90 \times 90$
  - Super resolution ratios (d): 144, 36, 16 respectively
- Input pSNR levels (for full integration time): $40 \, \text{dB}$, $50 \, \text{dB}$, $60 \, \text{dB}$
- Compression ratios (m): $0.05$, $0.10$, $0.15$, ..., $0.80$
- Each experiment is repeated 10 times with different noise & mask realizations
Results: FPA imaging

30 × 30 FPA
Results: FPA imaging

30 × 30 FPA

60 × 60 FPA
Results: FPA imaging

- Reconstruction improves up to some measurement level, and decrease afterwards
- As the noise level decreases, optimal number of measurements favors more measurements
- All three images result in similar performance and optimal number of measurements
- Reconstruction performance decreases with lower FPA resolution
Results : MPI

- Image size : $40 \times 20$
- Input pSNR levels : 0 dB, 10 dB,..., 40 dB
- Compression ratios ($m$) : 0.05, 0.10, 0.15, ..., 0.80
Conclusions & Future Work

• Practical analysis of two coded compressive imaging techniques
  – FPA imaging and MPI
  – Under different noise, super resolution, compression ratio settings

• Optimal number of measurements favor higher number of measurements as the input pSNR increases, and vice versa

• Finding it analytically requires knowledge of sparsity level which is impractical

• Shortcomings
  – Linear scaling in signals
  – Additional non-idealities such as photon noise
THANK YOU

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