USING LINEAR PREDICTION TO MITIGATE END EFFECTS IN EMPIRICAL MODE DECOMPOSITION

Steven Sandoval, Matthew Bredin, Phillip L. De Leon

Klipsch School of Electrical and Computer Engineering

New Mexico State University, Las Cruces, NM 88003

Introduction

- Huang proposed the EMD algorithm which decomposes a signal into a set of Intrinsic Mode Functions (IMFs), $\{\varphi_k(t)\}$ via an iterative sifting algorithm
- Demodulation of IMFs leads to a time-frequency analysis of a signal
- Extensions/improvements to EMD include: Ensemble EMD (EEMD), Complete EEMD (CEEMD), and Improved CEEMD (ICEEMD)
- For this work, we utilize EMD with our proposed improvements [1]

The End Effect Problem in EMD

- A cubic spline interpolator is used to determine the envelopes of a given signal based on the extrema points
- Interpolation at signal boundaries becomes extrapolation, causing erratic behavior
- Rato proposed that *artificial extrema points be inserted* past the signal bounds, constraining the extrapolation at the boundaries of the signal [2]
- Our method is to use linear prediction (LP) to *artificially extend the signal*
- These two methods are complementary and can be used in conjunction

Methods for Mitigating End Effects

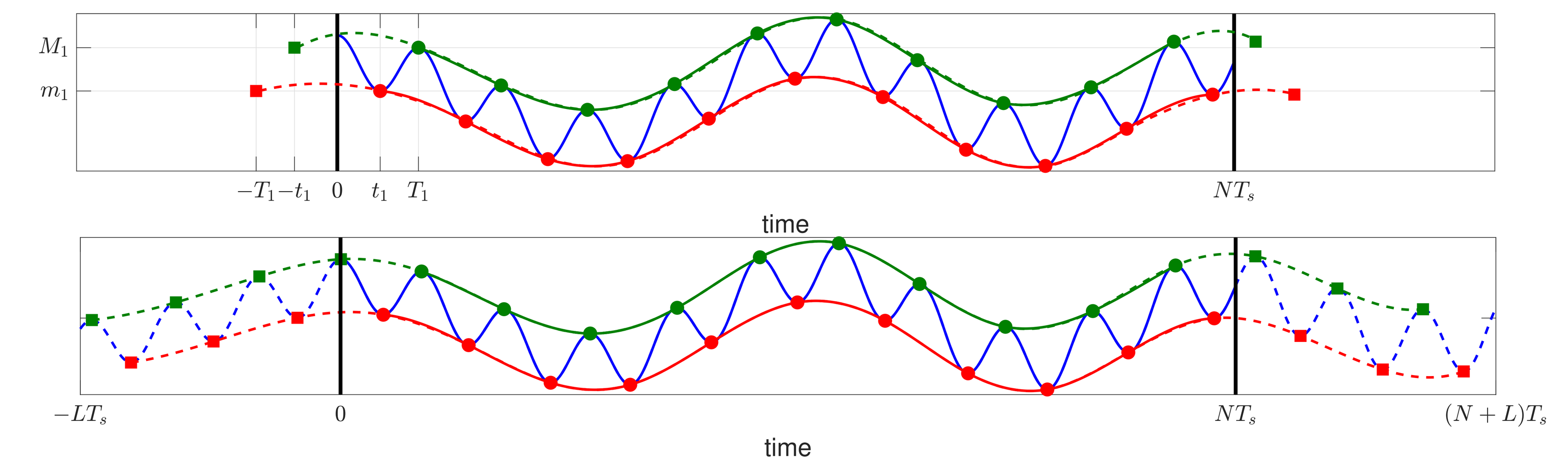

Figure: For a signal segment, the residue signal $r(t)$ (—), maxima $\{t_p, u_p\}$ (•) and minima $\{t_q, l_q\}$ (•), upper $u(t)$ (—) and lower $l(t)$ (—) envelopes, and signal boundaries (|) at $t=0$ and $t=NT_s$. Beyond the signal boundaries, we also illustrate (top) artificially-inserted maxima (•) and minima (•) obtained using Rato's mitigation method, as well as the subsequently estimated upper envelope $\breve{u}(t)$ (---) and lower envelope $\breve{l}(t)$ (---), and (bottom) the extension of the residue signal (---) using LP, as well as the maxima $\{\tilde{t}_p, \tilde{u}_p\}$ (•), minima $\{\tilde{t}_q, \tilde{l}_q\}$ (•), upper envelope $\tilde{u}(t)$ (---), and lower envelope $\tilde{l}(t)$ (---) that are obtained from the extended residue $\tilde{r}(t)$.

Sifting Algorithm with Proposed Mitigation

procedure $\varphi(t) = \mathrm{SIFT}(\ r(t),\ L,\ P\ )$

$$\tilde{r}(nT_s) = \begin{cases} \sum_{p=1}^{P} a^*_{P-p} r(nT_s + pT_s), -L \le n < 0 \\ r(nT_s), 0 \le n \le N \\ \sum_{p=1}^{P} a_p r(nT_s - pT_s), N < n \le N+L \end{cases}$$

  while $\frac{1}{NT_s}\int_0^{NT_s} |\tilde{e}(t)|^2 dt \ge \varepsilon$ **do**

    find all local maxima: $\tilde{u}_p = \tilde{r}(\tilde{t}_p),\ p = 1, 2, \ldots$

    find all local minima: $\tilde{l}_q = \tilde{r}(\tilde{t}_q),\ q = 1, 2, \ldots$

    insert artificial extrema (per Rato)

    interpolate: $\tilde{u}(t) =$ CublicSpline($\{\tilde{t}_p, \tilde{u}_p\}$)

    interpolate: $\tilde{l}(t) =$ CublicSpline($\{\tilde{t}_q, \tilde{l}_q\}$)

    $\tilde{e}(t) = [\tilde{u}(t) + \tilde{l}(t)]/2$

    $\tilde{r}(t) \leftarrow \tilde{r}(t) - \tilde{e}(t)$.

  end while

  $\varphi(t) = \tilde{r}(t), 0 \le t \le NT_s$

end procedure

Convergence of Mitigation Methods

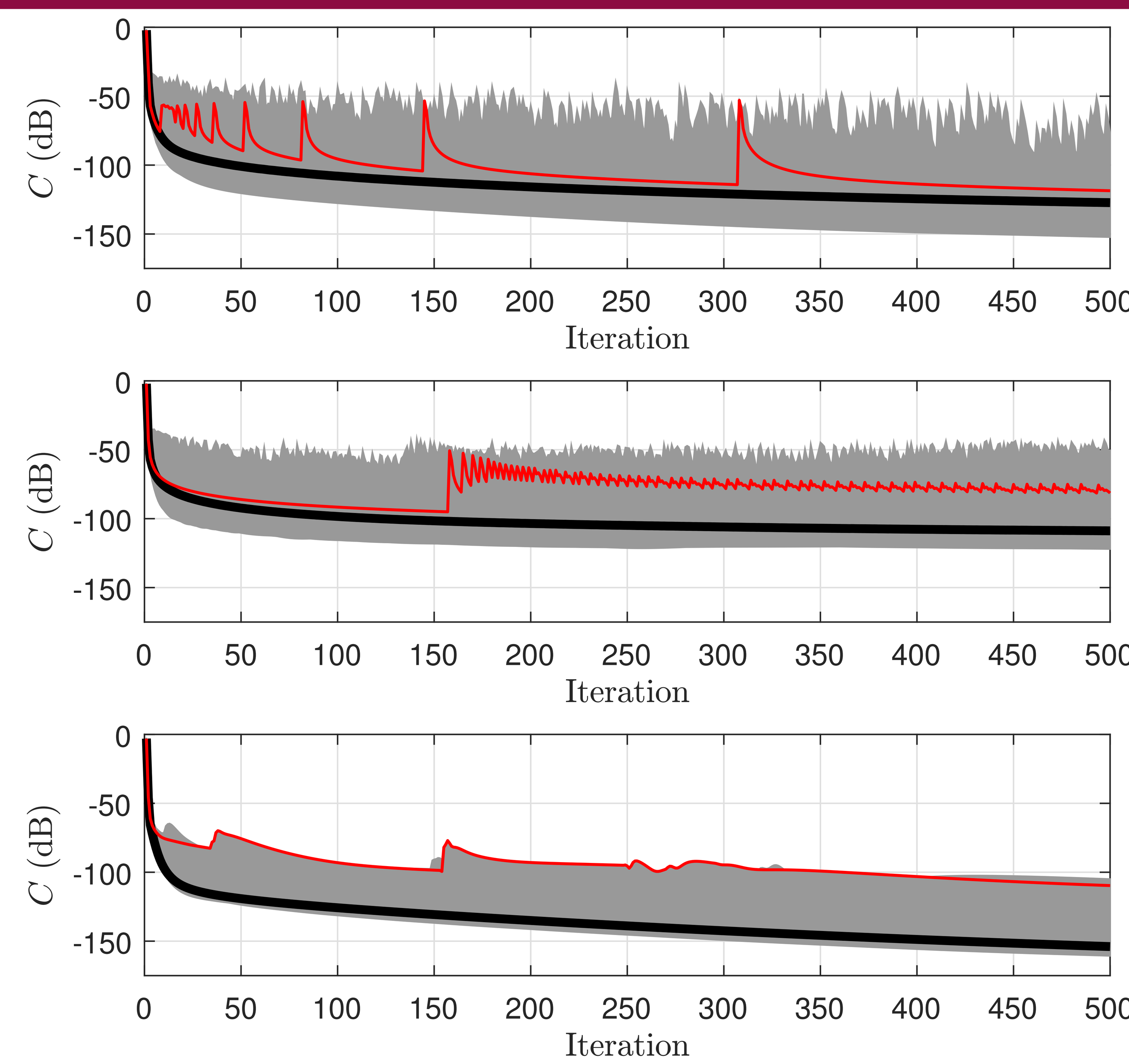

Figure: For 5000 trials of the convergence metric C in dB as a function of iteration, the mean value (▬) and the range (▬) for (top) no mitigation, (middle) Rato's mitigation only, (bottom) the proposed mitigation (LP+Rato's). The trial with the wost convergence (-) is also shown, note convergence instability.

Accuracy of Mitigation Methods

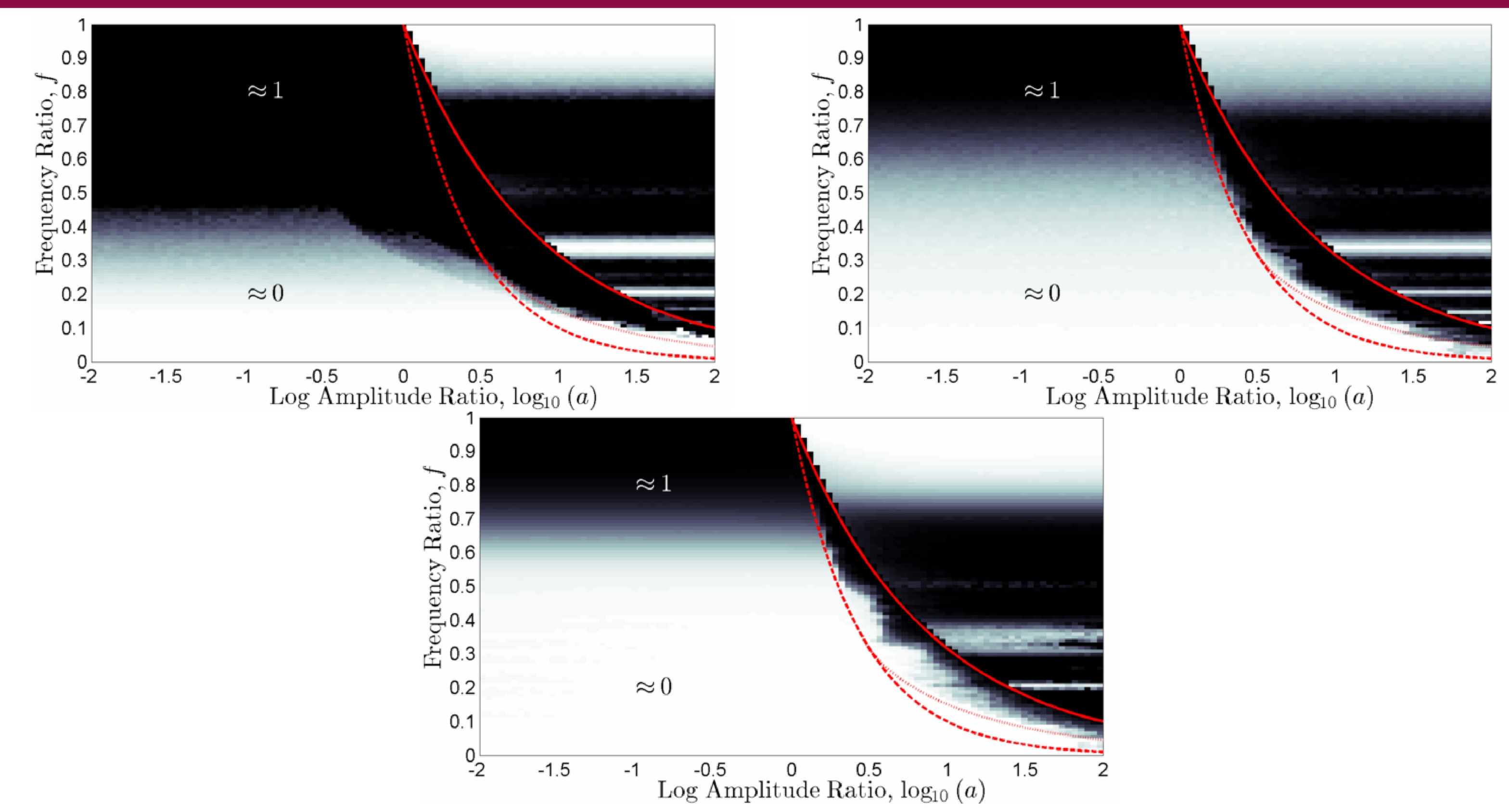

Figure: The expected error surface $\mathbb{E}[J(a,f)]$ when using (top left) no mitigation, (top right) Rato's mitigation, and (bottom center) the proposed mitigation (LP+Rato's). Additionally, the theoretical bifurcation curves $af^2 = 1$ (—), $af = 1$ (---), and $af\sin(3\pi f/2) = 1$ (—) derived in [3] are overlaid.

Conclusions

- The use of linear prediction with Rato's mitigation gives promising results.
- *Accuracy:* The expected mean error $\mathbb{E}[\bar{J}]$ is reduced.
- *Convergence:* The convergence metric C for the trial mean and worst case trial have smaller error.
- *Convergence:* The expected mean error surface $\mathbb{E}[J(a,f)]$ is smoother (reduced variance).

The proposed method is expected to have the most impact in cases where the area of interest within the signal extends up to the signal boundaries, such as in online or block EMD.

References

[1] S. Sandoval and P. L. De Leon "Advances in empirical mode decomposition for computing instantaneous amplitudes and instantaneous frequencies," *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, Mar. 2017.

[2] R. Rato, M. Ortigueira, and A. Batista, "On the HHT, its problems and some solutions," *Mechanical Syst. Signal Process.,vol. 22, no. 6, pp. 1374–1394*, 2008.

[3] G. Rilling and P. Flandrin,"One or two frequencies? The empirical mode decomposition answers," *IEEE Trans. Signal Process., vol. 56, no. 1, pp. 85–95*, 2008.

[4] S. Sandoval, M. Bredin, and P. L. De Leon "Using linear prediction to mitigate end effects in empirical mode decomposition," *Proc. IEEE Global Conf. Signal Info. Process.*, Nov. 2018.