Joint Subchannel and Power Allocation for Cognitive NOMA Systems with Imperfect CSI

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Outline

- Introduction
- System Model
- Problem Formulation
- Solution of the Optimization Problem
- Simulation Results
- Conclusions

Introduction

■ Background

- Because of the requirements of high spectral efficiency (SE) and system capacity, non-orthogonal multiple access (NOMA) technique has been considered as a promising candidate access technique for future communication systems.
- Cognitive radio networks (CRNs) with NOMA can further improve SE and support more secondary users (SUs).
- Resource allocation (RA) and energy efficiency (EE) are very important for the performance improvement of NOMA-based CRNs (N-CRNs).

Introduction

■ Motivation

- Current resource allocation algorithms (RAAs) mainly focus on accurate channel state information (CSI) and perfect successive interference cancellation (SIC) at the receivers, however, due to the inherent random nature of wireless channels, the effect of spectrum sensing errors and the limited interference cancellation at the receivers, RAAs under perfect CSI may be no longer feasible.

- To support practical applications of RAAs in N-CRNs, the designs of robust resource allocation are required to be reconsidered for improving the robustness of system and providing high reliability.

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System Model

- **A downlink underlay NOMA-based cognitive radio network (N-CRN) network**
  - One primary base station (PBS) servicing K PUs
  - One secondary base station (SBS) servicing M SUs
  - SUs access N licensed subchannels by the NOMA way
  - The bandwidth of each subchannel is B Hz
  - Each SU m only can occupy one subchannel
  - Each subchannel can be used by multiple users under the NOMA mode

System Model

- **The signal-tointerference-plus-noise ratio (SINR) of each SU can be formulated as**

$$r_{m,n} = p_{m,n}h_{m,n} \Big/ \left(\sum_{i=m+1}^{M} p_{i,n}h_{m,n} + N_{m,n}\right)$$

where $p_{m,n}$ is the allocated power from SBS to SU *m* on subchannel *n*. $h_{m,n}$ is the channel gain from SBS to SU *m* on subchannel *n*. $N_{m,n}$ is the interference power without SUs' links. Where $\sum_{i=m+1}^{M} p_{i,n}h_{m,n}$ denotes the inter-user interference after SIC.

- **The date rate of SU *m* on subchannel *n* is given as**

$$R_{m,n} = Ba_{m,n}\log_2\left(1 + r_{m,n}\right)$$

where $a_{m,n}$ is the subchannel allocation indicator.

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Problem Formulation

- Under **perfect CSI**, the **EE**-based maximization RA problem is formulated as

$$\max_{\{p_{m,n},a_{m,n}\}} \frac{\sum_n \sum_m R_{m,n}}{\sum_n \sum_m a_{m,n} p_{m,n} + P_c}$$

$$\begin{aligned} s.t.\ C_1:&\ \sum_n \sum_m a_{m,n} p_{m,n} g_{m,n,k} \le I^{th}, \\ C_2:&\ R_{m,n} \ge R_{m,n}^{min}, \\ C_3:&\ \sum_n \sum_m a_{m,n} p_{m,n} \le P^{max}, \\ C_4:&\ \sum_n a_{m,n} = 1, \\ C_5:&\ a_{m,n} \in \{0,1\}, \end{aligned}$$

where $g_{m,n,k}$ denotes the channel gain from SU *m* of subchannel *n* to PU *k*, and P_c is the circuit power consumption. I^{th} is the maximum interference power threshold of each PU, $R_{m,n}^{min}$ is the minimum data rate requirement and P^{max} is the maximum power of the SBS.

Problem Formulation

- **Considering a realistic estimation model with Gaussian error, channel gains become**

$$\mathcal{R}_h = \{h_{m,n} | \hat{h}_{m,n} + \Delta h_{m,n}, \Delta h_{m,n} \sim \mathcal{CN}(0, \sigma_{m,n}^2)\},$$

$$\mathcal{R}_g = \{g_{m,n,k} | \hat{g}_{m,n,k} + \Delta g_{m,n,k}, \Delta g_{m,n,k} \sim \mathcal{CN}(0, \sigma_{m,n,k}^2)\}$$

where $\hat{h}_{m,n}$ and $\hat{g}_{m,n,k}$ are the estimated channel gains. $\Delta h_{m,n}$ and $\Delta g_{m,n,k}$ are the estimation errors with variance $\sigma_{m,n}^2$ and $\sigma_{m,n,k}^2$ respectively.

- **Considering the impact of estimation errors, problem can be reformulated as the RRA problem**

$$\max_{\{p_{m,n}, a_{m,n}\}} \frac{\sum_n \sum_m R_{m,n}}{\sum_n \sum_m a_{m,n} p_{m,n} + P_c}$$

$$s.t.\ \bar{C}_1 : \Pr\left[I_k \geq I^{th} | \Delta g_{m,n,k} \in \mathcal{R}_g\right] \leq \sigma_k,$$

$$\bar{C}_2 : \Pr\left[R_{m,n} \leq R_{m,n}^{min} | \Delta h_{m,n} \in \mathcal{R}_h\right] \leq \epsilon_{m,n},$$

$$C_3 - C_5,$$

where σ_k and I_k are the outage probability threshold and the received interference power of PU k, respectively. $\epsilon_{m,n}$ is the outage probability threshold of each SU.

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Transformation of Optimization Problem

- **Based on the time-sharing approach, the integer indicator $a_{m,n}$ can be slacked as the continuous interval [0,1], i.e.,**

$$\tilde{p}_{m,n} = a_{m,n} p_{m,n}$$

- **The robust counterpart constraint $\bar{C}_1$ can be rewritten as:**

$$\Pr_{\Delta g_{m,n,k} \in \mathcal{R}_g} \left(\sum_n \sum_m \tilde{p}_{m,n} \Delta g_{m,n,k} \geq \bar{I}^{th} \right) \leq \sigma_k,$$

where $\bar{I}^{th} = I^{th} - \sum_n \sum_m \tilde{p}_{m,n} \hat{g}_{m,n,k}$ is the interference power gap. Thus, we have

$$\sum_n \sum_m \tilde{p}_{m,n} \tilde{g}_{m,n,k} \leq I^{th},$$

where $\tilde{g}_{m,n,k} = \hat{g}_{m,n,k} + \sigma_{m,n,k} Q^{-1}(\sigma_k)$, and $Q^{-1}(\cdot)$ is the inverse Gaussian Q-function.

Transformation of Optimization Problem

- Similarly, the rate outage probability of the SU $\bar{C}_2$ becomes

$$\tilde{R}_{m,n} \geq R_{m,n}^{min},$$

where $\tilde{R}_{m,n} = Ba_{m,n}\log_2(1+\tilde{p}_{m,n}\tilde{h}_{m,n}/H_{m,n})$. $H_{m,n} = \sum_{i=m+1}^{M} \tilde{p}_{i,n}\tilde{h}_{m,n} + a_{m,n}N_{m,n}$.
And $\tilde{h}_{m,n} = \hat{h}_{m,n} + \sigma_{m,n}Q^{-1}(1-\epsilon_{m,n})$.

- Thus, we have the **deterministic** optimization problem with the convex constraints, i.e.,

$$\max_{\{\tilde{p}_{m,n},a_{m,n}\}} \frac{\sum_n \sum_m R_{m,n}}{\sum_n \sum_m \tilde{p}_{m,n} + P_c}$$

$$s.t.\ \tilde{C}_1: \sum_n \sum_m \tilde{p}_{m,n}\tilde{g}_{m,n,k} \leq I^{th},$$

$$\tilde{C}_2: \tilde{R}_{m,n} \geq R_{m,n}^{min},$$

$$\tilde{C}_3: \sum_n \sum_m \tilde{p}_{m,n} \leq P^{max},$$

$$\tilde{C}_4: \sum_n a_{m,n} \leq 1.$$

Transformation of Optimization Problem

- **By using the Dinkelbach method, the objective function becomes**

$$\max_{\tilde{p}_{m,n},a_{m,n}} \sum_n \sum_m R_{m,n} - \theta(\sum_n \sum_m \tilde{p}_{m,n} + P_c).$$

where θ is a nonnegative parameter.

- **We use the lower bound $\tilde{R}_{m,n}$ to substitute $R_{m,n}$. Thus the deterministic optimization problem becomes**

$$\max_{\tilde{p}_{m,n},a_{m,n}} \sum_n \sum_m \tilde{R}_{m,n} - \theta(\sum_n \sum_m \tilde{p}_{m,n} + P_c)$$

$$\begin{aligned} s.t.\ \tilde{C}_1 :&\ \sum_n \sum_m \tilde{p}_{m,n}\tilde{g}_{m,n,k} \leq I^{th}, \\ \tilde{C}_2 :&\ \tilde{R}_{m,n} \geq R_{m,n}^{min}, \\ \tilde{C}_3 :&\ \sum_n \sum_m \tilde{p}_{m,n} \leq P^{max}, \\ \tilde{C}_4 :&\ \sum_n a_{m,n} \leq 1. \end{aligned}$$

the above problem is convex because the Hessian matrix of $\tilde{R}_{m,n}$ is positive.

Robust Resource Allocation Algorithm

- The **Lagrange function** is given by

$$L(\{\tilde{p}_{m,n}\},\{a_{m,n}\},\lambda,\beta,\{\lambda_m\},\{\beta_{m,n}\}) = \sum_n\sum_m \tilde{R}_{m,n} - \theta(\sum_n\sum_m \tilde{p}_{m,n} + P_c) + \beta(P^{max} - \sum_n\sum_m \tilde{p}_{m,n}) + \lambda(I^{th} - \sum_n\sum_m \tilde{p}_{m,n}\tilde{g}_{m,n,k}) + \sum_m \lambda_m(1-\sum_n a_{m,n}) + \sum_n\sum_m \beta_{m,n}(\tilde{R}_{m,n} - R^{min}_{m,n})$$

where $\lambda, \beta, \{\lambda_m\}$ and $\{\beta_{m,n}\}$ are non-negative Lagrange multipliers. Thus, the corresponding dual problem is

$$\min_{\{\lambda,\beta,\lambda_m,\beta_{m,n}\}} \max_{\tilde{p}_{m,n},a_{m,n}} L(\cdot)$$
$$s.t.\ \lambda \geq 0, \lambda_m \geq 0, \beta \geq 0, \beta_{m,n} \geq 0.$$

- According to the Karush-Kuhn-Tucker conditions, the optimal power allocation can be obtained by

$$p^*_{m,n} = [\frac{B(1+\beta_{m,n})\hat{H}_{m,n}}{\ln 2(\theta+\beta+\lambda\tilde{g}_{m,n,k})\tilde{h}_{m,n}} - \frac{\hat{H}_{m,n}}{\tilde{h}_{m,n}}]^+,$$

where $[x]^+ = \max\{0,x\}$. $\hat{H}_{m,n} = N_{m,n} + \tilde{h}_{m,n}\sum_{i=m+1}^{M} p^*_{i,n}$.

Robust Resource Allocation Algorithm

- And the optimal subchannel allocation is

$$a_{m^*,n} = 1 \,\middle|\, m^* = \max_{m} \psi_{m,n}, \forall n \ .$$

where the auxiliary variable $\psi_{m,n}$ is

$$\psi_{m,n} = B(1+\beta_{m,n}) \log_2(1 + \frac{p^*_{m,n}\bar{h}_{m,n}}{\tilde{h}_{m,n}\sum\limits_{i=m+1}^{M} p^*_{i,n} + N_{m,n}}) - p^*_{m,n}(\theta + \lambda \tilde{g}_{m,n,k} + \beta).$$

- Based on subgradient methods , the dual variables are updated as

$$\lambda(t+1) = [\lambda(t) - t_1 \times (I^{th} - \sum_{n}\sum_{m} \tilde{p}_{m,n}\tilde{g}_{m,n,k})]^+,$$

$$\beta(t+1) = [\beta(t) - t_2 \times (P^{max} - \sum_{n}\sum_{m} \tilde{p}_{m,n})]^+,$$

$$\beta_{m,n}(t+1) = [\beta_{m,n}(t) - t_3 \times (\tilde{R}_{m,n} - R^{min}_{m,n})]^+,$$

where $t \geq 0$ denotes the iteration index and $t_i,\ i \in \{1,2,3\}$ are the positive step sizes. The algorithm can obtain good convergence when the step sizes are appropriately chosen.

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Simulation Results

SIMULATION PARAMETERS

Parameters	Values	Parameters	Values
M	2	N	8
K	2	σ^2	0.001 W
$I_{m,n}$	0.01 W	P_c	0.25 W
P^{max}	0.25 W	B	1 Hz
I^{th}	0.015 W	$R_{m,n}^{min}$	1 bps/Hz
$\hat{h}_{m,n}$	[0,1]	$\hat{g}_{m,n}^{k}$	[0,0.1]
$\sigma_{m,n}$	[0,1]	$\sigma_{m,n,k}$	[0,0.1]
σ_k	0.1	$\epsilon_{m,n}$	0.1

Simulation Results

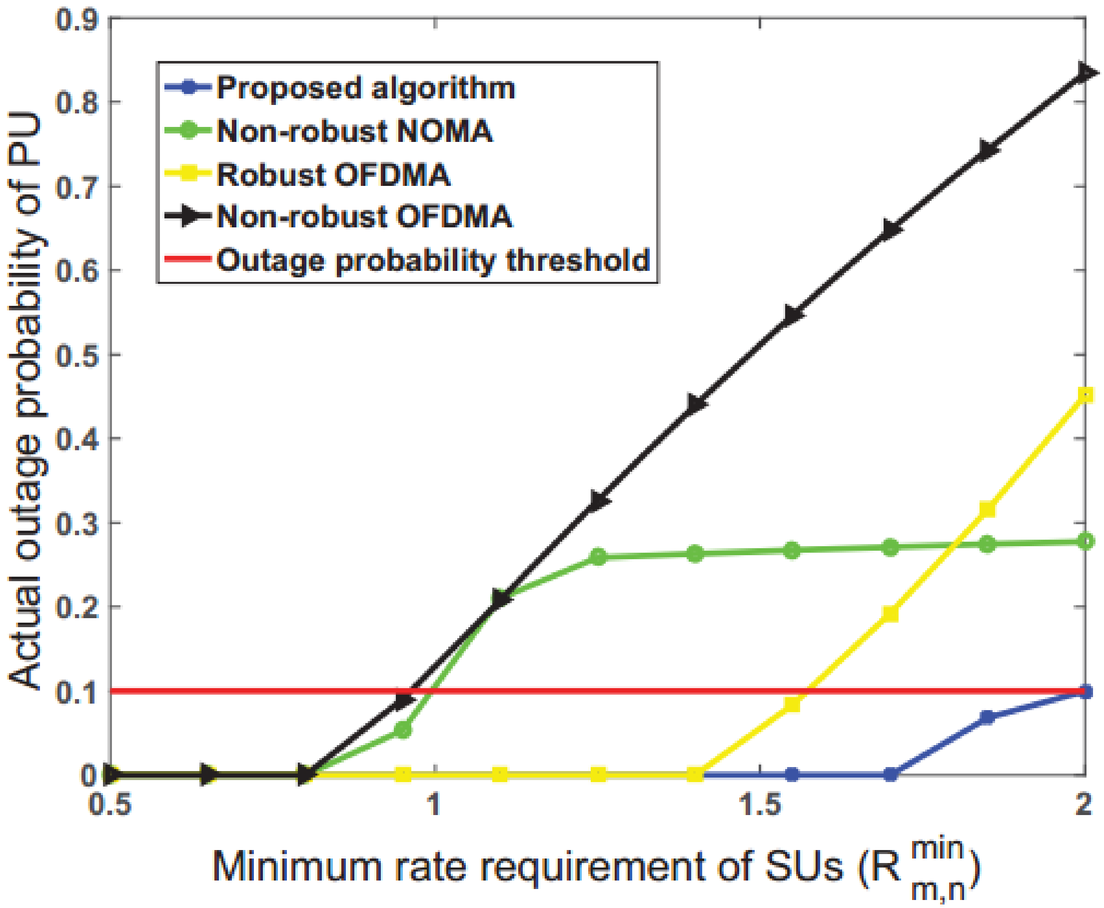

Fig. 1. Outage probability of PU versus the minimum rate of SU.

Simulation Results

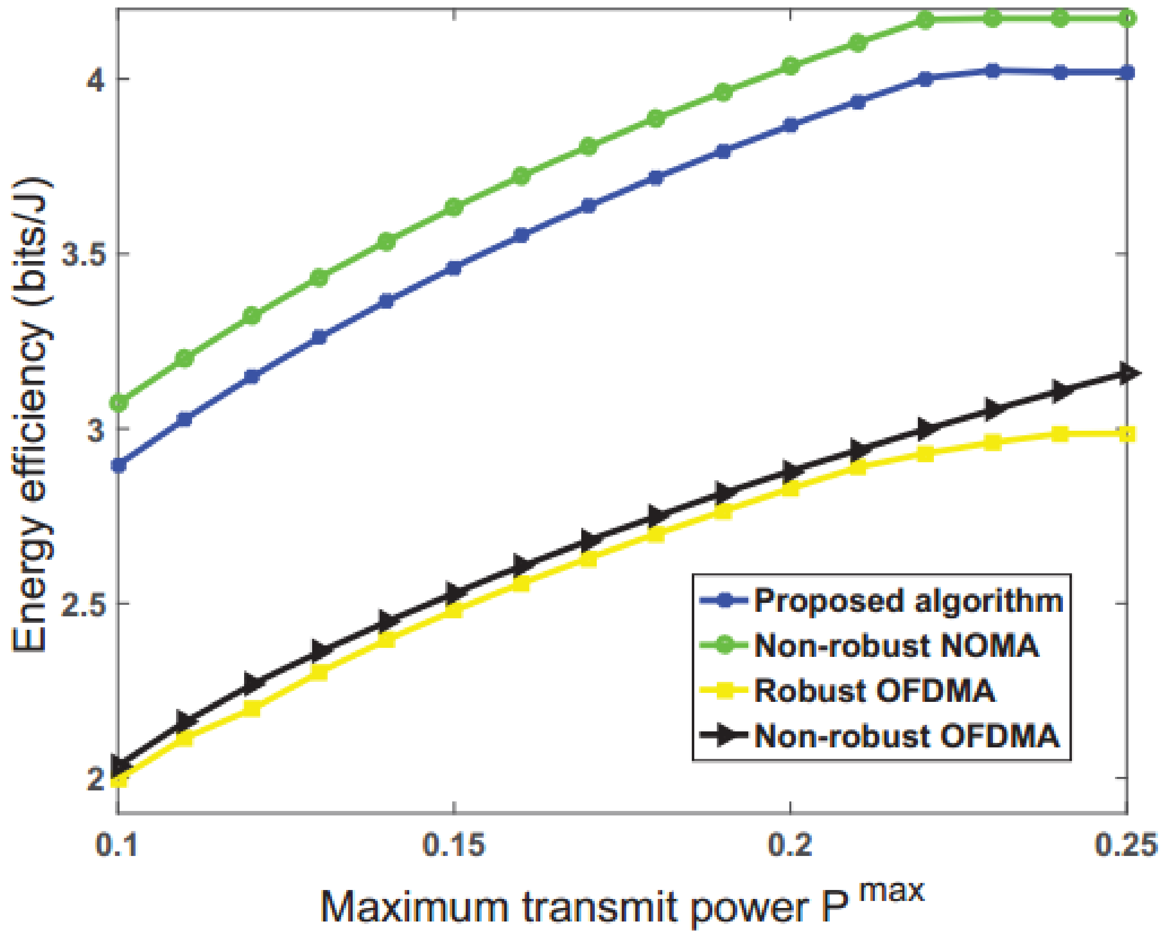

Fig. 2. The total EE versus the maximum transmit power of the SBS.

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Conclusions

- A robust power allocation and subchannel assignment algorithm was proposed to maximize the total EE of SUs for cognitive NOMA systems under taking channel uncertainties and outage probabilities into account.
- Based on Gaussian CSI error models, we transformed the robust rate constraint and the robust interference power constraint into the convex constraints.
- By slacking the integer subchannel allocation factor into a continuous variable, the original problem was converted into a convex problem by using the subtractive-form auxiliary variable
- Based on the Lagrangian dual approach and the subgradient updating methods, the closed-form solutions were obtained.
- The effectiveness of the proposed algorithm was verified by comparing with the existing algorithms.

The end , thanks !