Optimal Local Thresholds for Distributed Detection in Energy Harvesting Wireless Sensor Networks

Ghazaleh Ardeshiri, Hassan Yazdani, Azadeh Vosoughi

Department of Electrical Engineering and Computer Science
University of Central Florida

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Outline

- Introduction
- System Model and problem statement
- Optimizing local decision thresholds
- Simulation results
- Conclusions
The designs of wireless sensor networks to perform the task of distributed detection are often based on the conventional battery-powered sensors, leading into designs with a short lifetime, due to battery depletion.

Energy harvesting, which can collect energy from renewable resources of environment (e.g., solar, wind, and geothermal energy) promises a self-sustainable system with a lifetime.
System Model

$H_0: A$ is present

$H_1: A$ is absent

subject to additive $w_k$ and multiplicative $b_k$ observation noises

Figure 1: Our System model during one observation period.
System Model

Let $x_k$ denote the local observation at sensor $k$:

$$x_k = \begin{cases} g_k A + w_k & \mathcal{H}_1 \\ w_k & \mathcal{H}_0 \end{cases}$$

(1)

- $A$ is a known scalar signal
- $w_k \sim \mathcal{N}(0, \sigma^2_{w_k}) \rightarrow$ Additive noise
- $g_k \sim \mathcal{N}(0, \gamma_{g_k}) \rightarrow$ Multiplicative noise
- All observation noises are independent over time and among $K$ sensors.
System Model

During each observation period, sensor $k$ takes $N$ samples of $x_k$ to measure the received signal energy and applies an energy detector to make a binary decision, i.e., sensor $k$ decides whether or not signal $A$ is present.

$$\Lambda_k = \frac{1}{N} \sum_{n=1}^{N} |x_{k,n}|^2$$

where $\Lambda_k = 1$ for $d_k=1$ and $\Lambda_k = 0$ for $d_k=0$.

$$P_{f_k} = \Pr(\Lambda_k > \theta_k | \mathcal{H}_0) = \frac{\Gamma \left( \frac{N}{2}, \frac{N\theta_k}{\sigma_{w_k}^2} \right)}{\Gamma(N/2)}$$

$$P_{d_k} = \Pr(\Lambda_k > \theta_k | \mathcal{H}_1) = Q_{N/2} \left( \frac{\sqrt{\eta_k}}{\sigma_{w_k}}, \frac{\sqrt{N\theta_k}}{\sigma_{w_k}} \right)$$

Our goal is to optimize the local decision threshold $\theta_k$. 
Assumptions:

- Each sensor is able to harvest energy from the environment and stores it in a battery with the capacity $\mathcal{K}$ units of energy.
- The sensors communicate with the FC through orthogonal fading channels with channel gains $|h_k|$’s with parameters $\gamma h_k$.
- The sensors employ on-off keying signaling.
- We use the channel-inversion power, the number of energy units spent to convey a decision be inversely proportional to $|h_k|$.
- To avoid the battery depletion when $|h_k|$ is too small, we impose an extra constraint for channel quality.
Let $u_{k,t}$ represent the sensor output corresponding to the observation period $t$.

$$u_{k,t} = \begin{cases} \left\lceil \frac{\lambda}{|h_k|} \right\rceil & \Lambda_k > \theta_k, \ b_{k,t} > \left\lceil \frac{\lambda}{|h_k|} \right\rceil, \ |h_k|^2 > \zeta_k \\ 0 & \text{Otherwise} \end{cases}$$

(3)

- $b_{k,t}$ denote the battery state of sensor $k$
- $|h_k|$ is channel gain
- $\zeta_k$ is threshold of the channel quality
- $\lambda$ is a power regulation constant
We model $b_{k,t}$ in (3) as the following

$$b_{k,t} = \min\left\{ b_{k,t-1} - \left\lfloor \frac{\lambda}{|h_k|} \right\rfloor I_{u_{k,t-1}} + \Omega_{k,t}, K \right\}$$  \hspace{1cm} (4)$$

- $\Omega_{k,t} \in \{0, 1\}$ indicates units of harvesting energy a Bernoulli random variable, with $\Pr(\Omega_{k,t} = 1) = p_e$
- $I_{u_{k,t-1}} = \begin{cases} 1 & u_{k,t-1} > 0 \\ 0 & \text{Otherwise} \end{cases}$
Assuming $b_k$ in (4) is a stationary random process, one can compute the CDF and the pmf of $b_k$ in terms of $\mathcal{K}, p_e, \gamma_{h_k}$. Further, we use pmf of $b_k$ for our numerical results.

![CDF and PMF](image)

**Figure 2:** (a) CDF of $b_k$ for $\mathcal{K}=20$ and $p_e=0.5, 0.75, 0.82$, (b) pmf of $b_k$ for $\mathcal{K}=50$ and $p_e=0.8$. 

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We consider two detection performance metrics to find the optimal $\theta_k$'s:

- The detection probability at the FC, assuming that the FC utilizes the optimal fusion rule based on Neyman-Pearson optimality criterion.

- the KL distance between the two distributions of the received signals at the FC conditioned on hypothesis $\mathcal{H}_0, \mathcal{H}_1$
The received signal at the FC from sensor $k$ is $y_k = h_k u_k + n_k$, where the additive communication channel noise $n_k \sim \mathcal{N}(0, \sigma^2_{n_k})$. The likelihood ratio at the FC is

$$\Delta_{\text{LRT}} = \sum_{k=1}^{K} \log \left( \frac{\sum_{u_k} f(y_k|u_k, \mathcal{H}_1) \Pr(u_k|\mathcal{H}_1)}{\sum_{u_k} f(y_k|u_k, \mathcal{H}_0) \Pr(u_k|\mathcal{H}_0)} \right)$$

(5)

Given $u_k$, $y_k$ is Gaussian, i.e., $y_k|u_k=0 \sim \mathcal{N}(0, \sigma^2_{n_k})$ and

$$y_k|u_k=\left\lfloor \frac{\lambda}{|h_k|} \right\rfloor \sim \mathcal{N}\left(\left\lfloor \frac{\lambda}{|h_k|} \right\rfloor h_k, \sigma^2_{n_k}\right).$$
The probabilities $\Pr(u_k|H_1)$, $\Pr(u_k|H_0)$ in (5) are

- $\Pr(u_k = \lceil \frac{\lambda}{|h_k|} \rceil | H_1) = P_{d_k} \rho_k q_k = \alpha_k$
- $\Pr(u_k = \lceil \frac{\lambda}{|h_k|} \rceil | H_0) = P_{f_k} \rho_k q_k = \beta_k$

where $\rho_k = \Pr(b_k > \lceil \frac{\lambda}{|h_k|} \rceil)$ and $q_k = \Pr(|h_k|^2 > \zeta_k)$.

Given a threshold $\tau$, the optimal likelihood ratio test (LRT) is $\Delta_{LRT} \geq \frac{H_1}{H_0} \tau$. The $P_F, P_D$ at the FC

$$P_F = \Pr(\Delta_{LRT} > \tau | H_0) = Q\left(\frac{\tau - \mu |\Delta|_{H_0}}{\sigma |\Delta|_{H_0}}\right)$$

$$P_D = \Pr(\Delta_{LRT} > \tau | H_1) = Q\left(Q^{-1}(a)\sigma |\Delta|_{H_0} + \mu |\Delta|_{H_0} - \mu |\Delta|_{H_1}\right)$$

$$= \frac{Q^{-1}(a)\sigma |\Delta|_{H_0} + \mu |\Delta|_{H_0} - \mu |\Delta|_{H_1}}{\sigma |\Delta|_{H_1}}$$
Kullback-Leibler distance (KL) between the two distributions of the received signals at the FC

\[ KL_k = \int_{y_k} f(y_k|\mathcal{H}_1) \log \left( \frac{f(y_k|\mathcal{H}_1)}{f(y_k|\mathcal{H}_0)} \right) dy_k \tag{8} \]

One can approximate \( KL_k \) in (8) by the KL distance of two Gaussian distributions

\[ KL_k \approx \frac{1}{2} \log \left( \frac{\sigma_{y_k|\mathcal{H}_0}^2}{\sigma_{y_k|\mathcal{H}_1}^2} \right) + \frac{\sigma_{y_k|\mathcal{H}_1}^2 - \sigma_{y_k|\mathcal{H}_0}^2}{2\sigma_{y_k|\mathcal{H}_0}^2} + \left( \mu_{y_k|\mathcal{H}_1} - \mu_{y_k|\mathcal{H}_0} \right)^2 \tag{9} \]
In this section, we consider:

- **Case I:** Numerically find $\theta_k$’s which maximize $P_D$ in (7) $\to$ $K$-dimensional search is required $\to$ computational complexity!

- **Case II:** Finding $\theta_k$’s which maximize $KL_{tot} = \sum_{k=1}^{K} KL_k$, using the $KL_k$ approximations in (9) $\to$ Only one dimensional search $\to$ computationally efficient.

- **Special cases:** Assume all sensors employ the same local threshold $\theta_k = \theta$.

We then compare $P_D$ evaluated at the $\theta_k$’s obtained from mentioned cases.
Simulation results

Figure 3: (a) $P_D$ vs. $P_F$  
(b) $P_D$ vs. $P_{av}$
Conclusion

- We studied a distributed detection problem in a wireless network with $K$ heterogeneous energy harvesting sensors and investigated the optimal local decision thresholds for given transmission and battery state models.

- Our numerical results indicate that the thresholds obtained from maximizing the KL distance are near-optimal and computationally very efficient, as it requires only $K$ one-dimensional searches, as opposed to a $K$-dimensional search required to find the thresholds that maximize the detection probability.

- The performance gap between each scheme and its corresponding special case indicates that when sensors are heterogeneous, it is advantageous to use different local thresholds according to sensors’ statistics.
Questions?