

# Analog Beamformer Design for Interference Exploitation based Hybrid Beamforming



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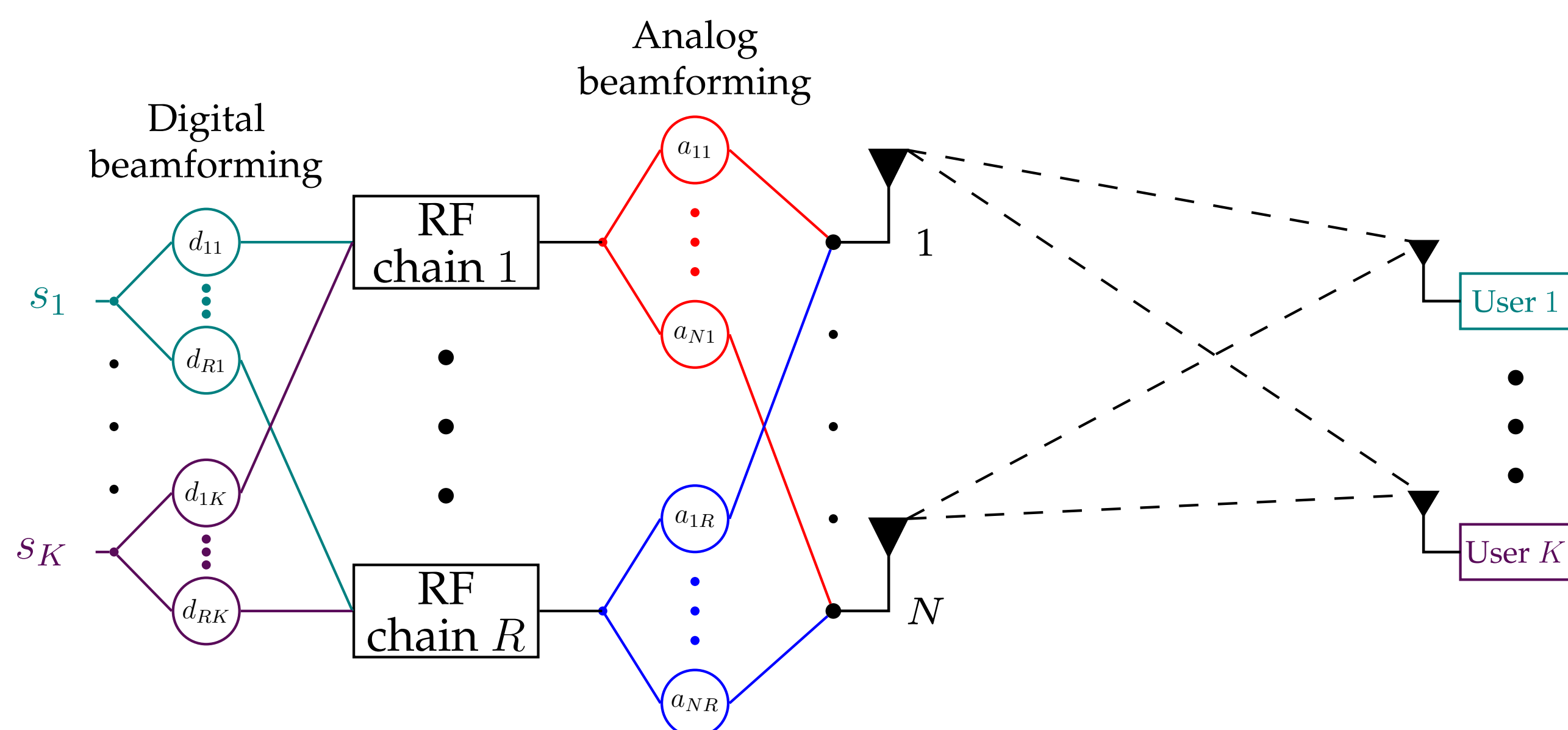
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## 1 Motivation

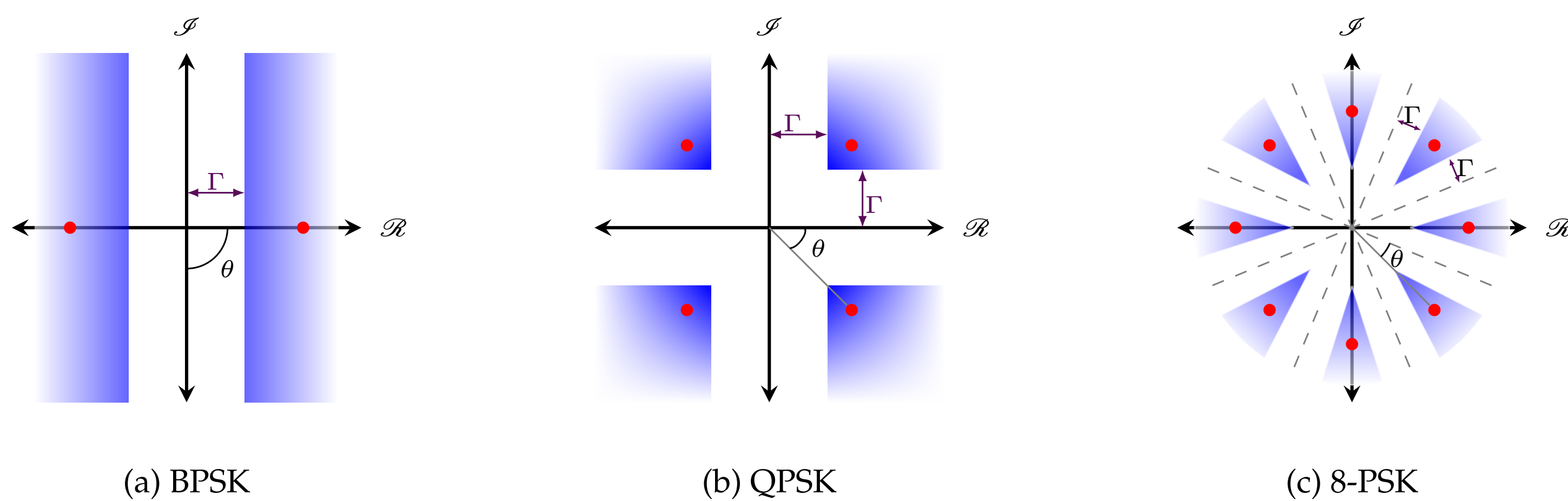
- Massive MIMO with the conventional fully-digital beamforming:
  - Extremely high hardware cost
  - Unwieldy operational power
- Hybrid beamforming with the conventional SINR maximization approach:
  - Reduced control over interference  $\Rightarrow$  increased transmit power
- Proposed solution:** Massive MIMO system comprising the following features:
  - Hybrid analog-digital beamforming
  - Symbol-level beamforming
  - Interference exploitation using constructive interference (CI) phenomena

## 2 System Model



- Co-channel multi-user downlink system with  $K$  single antenna users
- Each user has a maximum allowable symbol-error-rate (SER) requirement
- Base station (BS) equipped with  $N$  antennas and  $R$  RF chains, where  $R \leq N$
- Transmit symbols belong to  $M$ -PSK constellation, with  $s_k = e^{j\phi_k}$
- The digital beamformer (DBF)  $\mathbf{d}_k$  applied to transmit symbol  $s_k$
- Analog beamformers (ABF)  $\mathbf{a}_r$  are chosen from a predefined codebook  $\mathcal{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_L\}$ , where  $L \geq R$ . ABF matrix  $\mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_R]$
- The channel vector of the  $k$ th user  $\tilde{\mathbf{h}}_k$  is known at the BS
- The received signal  $y_k$  at the  $k$ th user is given by:  $y_k = \tilde{\mathbf{h}}_k^T \mathbf{A} \sum_{i=1}^K \mathbf{d}_i s_i + n_k$

## 3 CI-based Hybrid Beamforming



- Desired-region:** Complex space that is a threshold-margin  $\Gamma$  away from the corresponding decision boundaries
- CI-based beamforming enforces the received signals to the desired-regions of the corresponding transmit symbols
- Determine the threshold-margin  $\Gamma_k$  that ensure the SER requirement of the  $k$ th user
- CI-based joint analog-digital beamforming problem (**nonconvex**):

$$\text{minimize}_{\mathbf{A}, \{\mathbf{d}_k\}_{k \in \mathcal{K}}} \left\| \mathbf{A} \sum_{i=1}^K \mathbf{d}_i s_i \right\|^2 \quad (1a)$$

$$\text{subject to } \left| \text{Im} \left( s_k^* \tilde{\mathbf{h}}_k^T \mathbf{A} \sum_{i=1}^K \mathbf{d}_i s_i \right) \right| \leq \left( \text{Re} \left( s_k^* \tilde{\mathbf{h}}_k^T \mathbf{A} \sum_{i=1}^K \mathbf{d}_i s_i \right) - \gamma_k \right) \tan \theta, \quad \forall k \in \mathcal{K} \quad (1b)$$

$$\mathbf{a}_r \in \mathcal{C}, \quad \forall r \in \mathcal{R} \quad (1c)$$

where  $\gamma_k = \Gamma_k / \sin(\theta)$   $\theta = \pi/M$

- Equivalent single-group multicast problem (**nonconvex**):

$$\text{minimize}_{\mathbf{A}, \mathbf{b}} \|\mathbf{A}\mathbf{b}\|^2 \quad (2a)$$

$$\text{subject to } \left| \text{Im}(\mathbf{h}_k^T \mathbf{A}\mathbf{b}) \right| - \left( \text{Re}(\mathbf{h}_k^T \mathbf{A}\mathbf{b}) - \gamma_k \right) \tan \theta \leq 0, \quad \forall k \in \mathcal{K} \quad (2b)$$

$$\mathbf{a}_r \in \mathcal{C}, \quad \forall r \in \mathcal{R} \quad (2c)$$

where  $\mathbf{h}_k = \tilde{\mathbf{h}}_k s_k^*$   $\mathbf{b} = \frac{1}{K} \sum_{k=1}^K \mathbf{d}_k s_k^*$

- The problem is combinatorial and needs exhaustive search over the codebook  $\mathcal{C}$  to obtain the optimal solution

## 4 Proposed Suboptimal Solution

- Decompose the problem into two parts: ABF design and DBF design
  - ABF design:** Select ABFs from the predefined codebook, employing the proposed methods
  - DBF design:** For the fixed ABF matrix  $\mathbf{A}$ , design the optimal DBFs by solving problem (2) (**convex quadratic problem**)

## 5 Analog Beamformers Design Techniques

### Method 1: Margin widening and selection operator (MWASO)

- Define ABF codebook matrix  $\mathbf{C} = [\mathbf{c}_1, \dots, \mathbf{c}_L] \in \mathbb{C}^{N \times L}$
- Solve the convex problem

$$\text{minimize}_{\mathbf{b} \in \mathbb{C}^L, \delta \in \mathbb{R}} \|\mathbf{b}\|_1 + \delta \quad (3a)$$

$$\text{subject to } \left| \text{Im}(\mathbf{h}_k^T \mathbf{C}\mathbf{b}) \right| - \left( \text{Re}(\mathbf{h}_k^T \mathbf{C}\mathbf{b}) - (\gamma_k - \delta) \right) \tan \theta \leq 0, \quad \forall k \in \mathcal{K} \quad (3b)$$

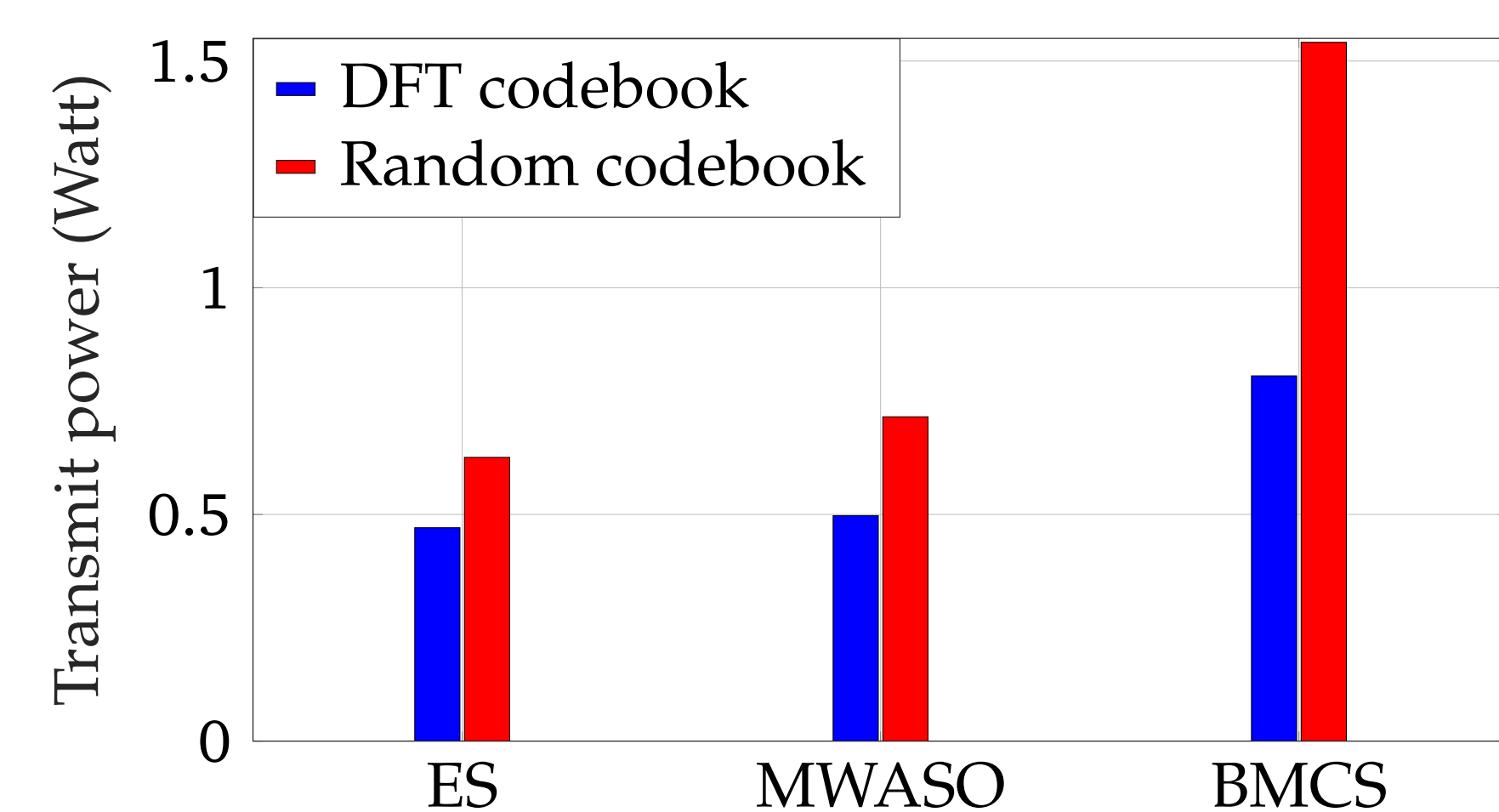
- The  $\ell_1$ -norm term  $\|\mathbf{b}\|_1$  promotes sparsity over vector  $\mathbf{b}$
- $\delta$  controls the margin between received signals and corresponding decision boundaries
- The optimization variable  $\delta$  also facilitates achieving any desired sparsity on vector  $\mathbf{b}$  by relaxing threshold-margin constraint
- Columns of matrix  $\mathbf{C}$  corresponding to the non-zero elements of  $\mathbf{b}$  form the ABF matrix
- ABFs dependent on transmit symbols  $\Rightarrow$  need to be reselected at every time-slot

### Method 2: Best matching code selection (BMCS) method

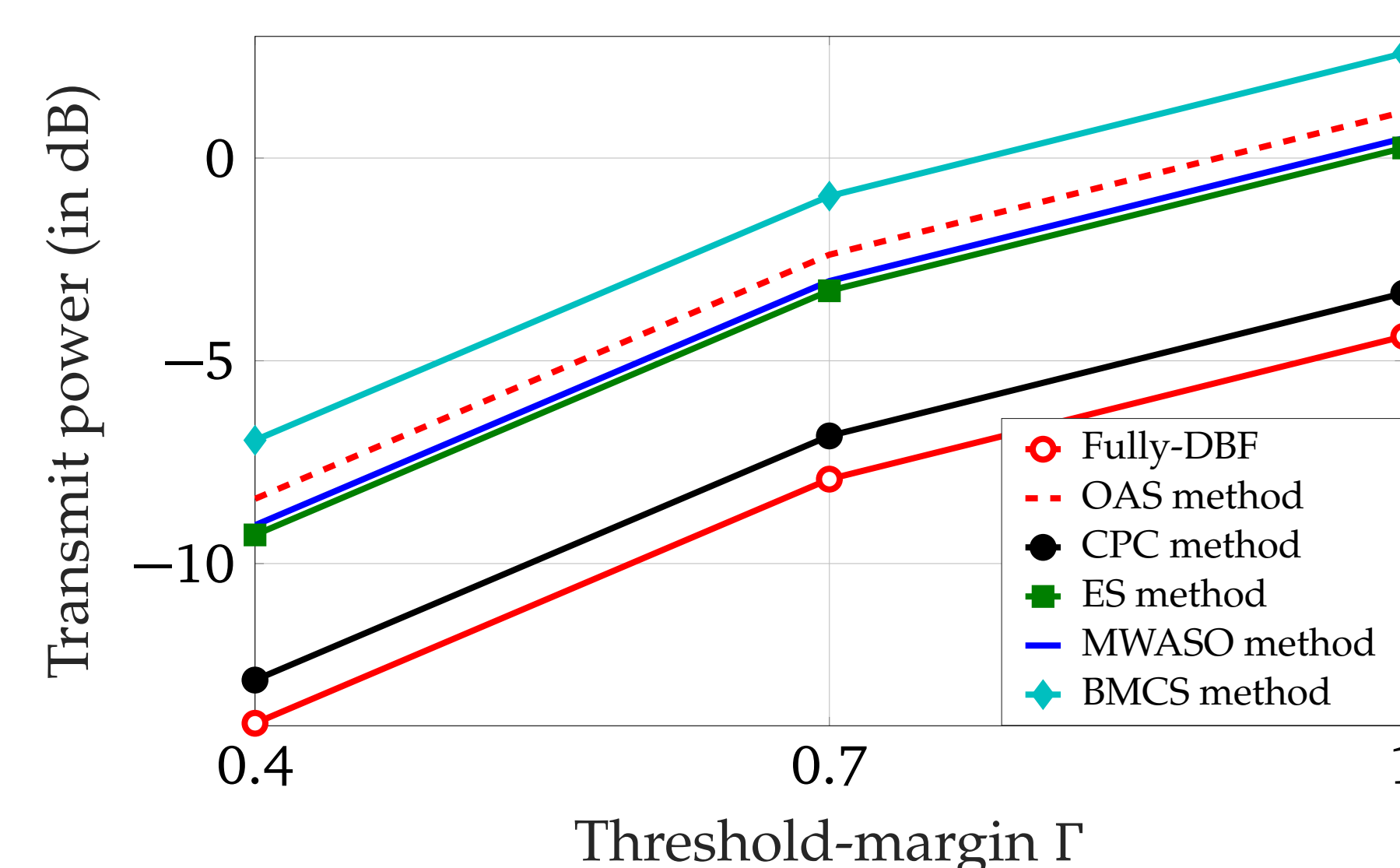
- For each user, select an ABF from the codebook  $\mathcal{C}$  that maximizes the absolute value of inner product with its channel vector (array gain)
  - Initialization:  $\tilde{\mathcal{C}} = \mathcal{C}$
  - for  $k = 1 : K$  do
  - $\mathbf{a}_k = \text{argmax}_{\mathbf{c} \in \tilde{\mathcal{C}}} |\mathbf{c}^T \tilde{\mathbf{h}}_k|$
  - $\tilde{\mathcal{C}} \leftarrow \tilde{\mathcal{C}} \setminus \mathbf{a}_k$
  - end for
- ABFs independent of transmit symbols  $\Rightarrow$  can be fixed for multiple time-slots
- Significantly reduced computational complexity when compared to MWASO

## 6 Numerical Results

- Simulation settings:  $N = L = 16, R = K = 2, M = 4, \Gamma = 0.7$
- Comparison of 1) DFT codebook, 2) random phase constant magnitude codebook



- Comparison of different ABF design techniques



- Fully-DBF: CI-based fully-digital beamforming, with  $R = N$
- OAS method: Optimal  $R$  antenna selection (exhaustive search) with CI-based fully-digital beamforming for  $N = R$
- CPC method: Conjugate phase of channel method [Liang *et al*-14] with expensive high-resolution continuous-valued phase shifters
- ES method: Optimal ABFs selection employing exhaustive search over codebook  $\mathcal{C}$  and subsequent optimal DBF design