Tensor-Factorization-Based 3D Single Image Super-Resolution with Semi-Blind Point Spread Function Estimation

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Objective

- Proposing a fast volumetric single image super-resolution technique with a semi-blind estimation of the Point Spread Function (PSF).

Tensor factorization

Any tensor can be written as the sum of F simple tensors (the outer product of 1D arrays)

\[ X = \sum_{f=1}^{F} U^1(\cdot; f) \odot U^2(\cdot; f) \odot U^3(\cdot; f) = [U^1, U^2, U^3] \]  

where \( U^1, U^2, U^3 \) are sets of the 1D arrays. When \( F \) is minimal, the decomposition (CPD) is unique under mild conditions. [1]

The SISR problem

Image degradation: the LR image (\( Y \)) is a down-sampled (\( D \), by factor \( r \)) and blurred (\( H \)) HR image (\( X \)) with some additive white noise \( N \). Using CPD for \( X \) it is

\[ Y = [D_1 H_1 U^1, D_2 H_2 U^2, D_3 H_3 U^3] + N \]  

The non-convex minimization problem is then

\[ \min_{U,\bar{\sigma}} \| Y - \sum_{i=1}^{L} D_i H_i (\sigma_i U^1, D_2 H_2 (\sigma_2 U^2, D_3 H_3 (\sigma_3 U^3)) \|_F^2 \]  

The algorithm

- **Input:** \( Y, r, F, \sigma, \epsilon, N, M \)
- 1) for \( i = 0 \: N \) do
- 2) \( H_1, H_2, H_3 \leftarrow \bar{\sigma} \)
- 3) for \( j = 0 \: M \) do
- 4) update \( U_1, U_2, U_3 \) sequentially using (4)
- 5) \( \bar{U} \leftarrow \bar{U} \)
- 6) while residual > \( \epsilon \) do
- 7) update \( \sigma \) using (5)
- 8) **Output:** \( X \), the estimated HR image

Image estimation

\[ \begin{align*}
\min_{U_1} \frac{1}{2} \| Y^{(1)} - D_1 H_1 U^1 (D_2 H_2 U^2 \odot D_3 H_3 U^3) \|^2_F \\
\min_{U_2} \frac{1}{2} \| Y^{(2)} - D_2 H_2 U^2 (D_1 H_1 U^1 \odot D_3 H_3 U^3) \|^2_F \\
\min_{U_3} \frac{1}{2} \| Y^{(3)} - D_3 H_3 U^3 (D_1 H_1 U^1 \odot D_2 H_2 U^2) \|^2_F
\end{align*} \]  

(4)

\[ \text{blurring can be written as } F^{-1}(F \bar{h}(\bar{\sigma}) \cdot FX) \]  

(5)

Conclusion

- **Processing the 3D volumes in less than 5 min**
- **Image quality is at least as good as the state of the art**

References