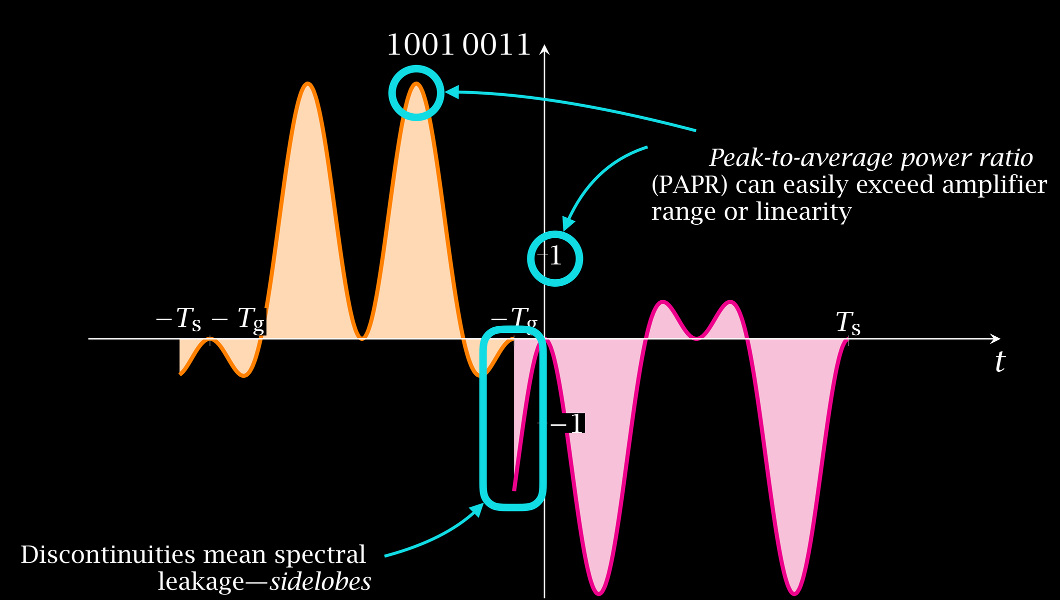
Orthogonal Precoding for Sidelobe Suppression in DFT-Based Systems Using Block Reflectors

Vaughan Clarkson

P.O. Box 920, Samford Village, Qld., 4520, AUSTRALIA, v.clarkson@ieee.org

Orthogonal frequency-division multiplexing (OFDM) has some important shortcomings.



Signal model

Over a single symbol, the complex-baseband transmitted signal $y(t)$ is

$$y(t) = \sum_{i=1}^{M} s_i \exp(j2\pi k_i f_s t)$$

where

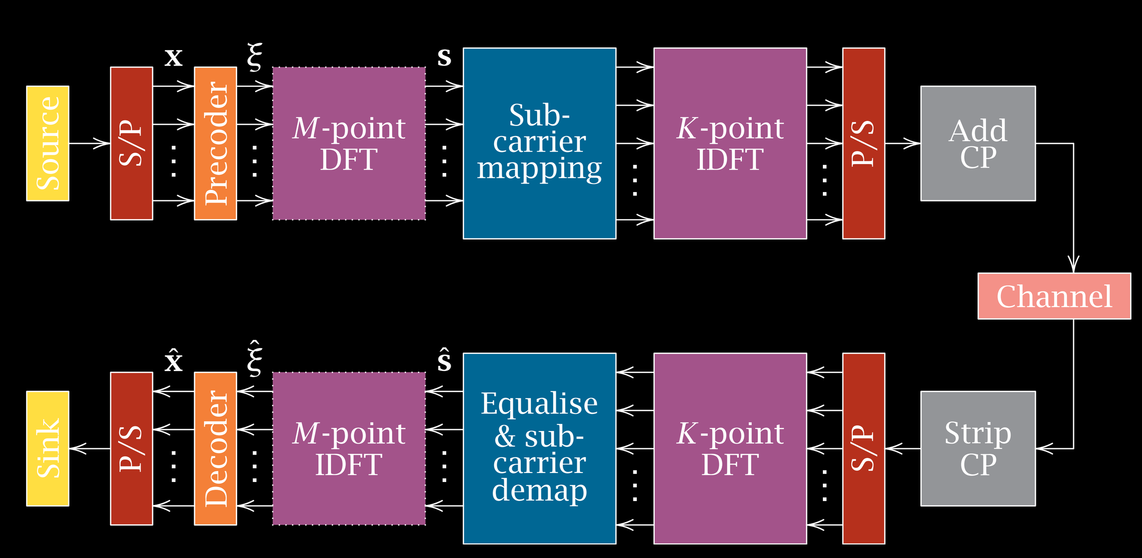
s_i is the amplitude on the ith subcarrier,

k_i is the index of the ith subcarrier and

f_s is the subcarrier spacing.

The symbol at $t = 0$ extends from $-T_g$ (or $-T_{cp}$) to $T_s = 1/f_s$.

System model



Orthogonal precoding

Orthogonal precoding is implemented by rotating the individual constellations in n dimensions.

- Let $\bar{\mathbf{x}}$ be an M-dimensional vector containing the QAM data.
- Zero pad $\bar{\mathbf{x}}$ with R zeros to make an N-dimensional vector

$$\mathbf{x} = \begin{pmatrix} \mathbf{0}_R \\ \bar{\mathbf{x}} \end{pmatrix}.$$

- Set $\mathbf{s} = \mathbf{Q}\mathbf{x}$ for some unitary matrix $\mathbf{Q}$—this is the *orthogonal precoding*.
- Equivalently, $\mathbf{s} = \bar{\mathbf{Q}}\bar{\mathbf{x}}$ where $\bar{\mathbf{Q}}$ has M orthogonal columns.
- The elements of $\mathbf{s}$ now represent the subcarrier amplitudes that are transmitted like standard OFDM.

Power spectral density

The spectrum of an individual OP-OFDM symbol is

$$Y(f) = \sum_{i=1}^{M} a_i^*(f) s_i$$

where

$$a_i(f) = T \exp\left[-j\pi(T_s - T_{cp})(f - k_i f_s)\right] \operatorname{sinc}\left[\pi(T_s + T_{cp})(f - k_i f_s)\right].$$

- With the functions $a_i(f)$, $i = 1, \ldots, M$, grouped as a column vector $\mathbf{a}(f)$, we have $Y(f) = \mathbf{a}^H(f)\mathbf{s}$.
- With independent uniform QAM data, the *power spectral density* of OP-OFDM is

We can use $\bar{\mathbf{Q}}$ to shape the PSD!

$$G_Y(f) = \frac{\eta}{T}\|\bar{\mathbf{Q}}^H\mathbf{a}(f)\|^2$$

where η is the power of each symbol in the source stream.

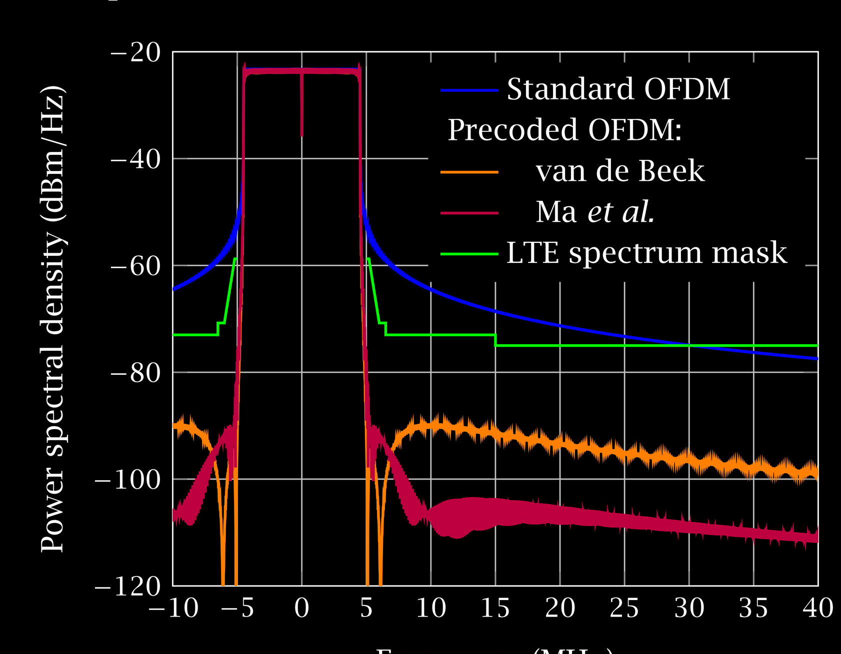
Choosing Q

We'll review two methods of choosing $\mathbf{Q}$ for sidelobe suppression.

1. The method of van de Beek:
   (a) Choose R out-of-band frequencies, f_r, $r = 1, \ldots, R$, at which $G_Y(f_r) = 0$.
      - Then each $\mathbf{a}(f_r)$ should be in the null space of $\bar{\mathbf{Q}}$.
   (b) Construct a matrix $\mathbf{C}_{\mathrm{vdB}} = (\mathbf{a}(f_1), \ldots, \mathbf{a}(f_R))$.
   (c) Perform a singular value decomposition (SVD) $\mathbf{C}_{\mathrm{vdB}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$.
      - Set $\mathbf{Q} = \mathbf{U}$.
2. The method of Ma *et al.*:
   (a) Choose a set ϕ of out-of-band frequencies on which to minimise $\sum_{f \in \phi} G_Y(f)$.
   (b) Construct a matrix $\mathbf{C}_{\mathrm{Ma}}$ whose columns are $\mathbf{a}(f)$, $f \in \phi$.
      $\Rightarrow$ Choose $\bar{\mathbf{Q}}$ to minimise $\|\bar{\mathbf{Q}}^H\mathbf{C}_{\mathrm{Ma}}\|_F^2$.
   (c) Perform an SVD $\mathbf{C}_{\mathrm{Ma}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$.
      - Set $\mathbf{Q} = \mathbf{U}$.

Suppression performance

E-UTRA/LTE parameters, 600 subcarriers, 8 sacrificed.



Householder reflection

Unitary transformation needn't be computationally expensive when you don't care what happens outside a low-dimensional subspace.

- If $\mathbf{y} \neq \mathbf{z}$ are unit basis vectors for two subspaces then the *Householder matrix* is

$$\mathbf{H} = \mathbf{I} - \mathbf{g}\mathbf{g}^H \quad \text{where} \quad \mathbf{g} = \sqrt{2}\,\frac{\mathbf{y} - \mathbf{z}}{\|\mathbf{y} - \mathbf{z}\|}$$

- Observe that $\mathbf{H}$ is unitary, $\mathbf{H}\mathbf{y} = \mathbf{z}$ and $\mathbf{H}\mathbf{z} = \mathbf{y}$.
- Computing $\mathbf{H}\mathbf{x}$ is cheap since $\mathbf{H}\mathbf{x} = \mathbf{x} - \mathbf{g}(\mathbf{g}^H\mathbf{x}) \Leftarrow$ all vector-vector operations and so $O(N)$ rather than $O(N^2)$.
- Since $\mathbf{H}^2 = \mathbf{I}$, this unitary transformation is the *Householder reflection*.

Block reflector

Householder reflection can be generalised to higher-dimensional subspaces in a number of different ways.

- Examples: the WY representation, the *basis-kernel* representation and successive Householder reflection.
- Suppose $\mathbf{Y}$ and $\mathbf{Z}$ are orthonormal basis matrices for two R-dimensional subspaces.
- Take the SVD of $\mathbf{Y}^H\mathbf{Z} = \boldsymbol{\Theta}\mathbf{D}\boldsymbol{\Phi}^H$.
- The *block reflector* is

$$\mathbf{H} = \mathbf{I} - \mathbf{G}\mathbf{G}^H \quad \text{where} \quad \mathbf{G} = (\mathbf{Y}\boldsymbol{\Theta} - \mathbf{Z}\boldsymbol{\Phi})(\mathbf{I} - \mathbf{D})^{-1/2}.$$

- Again, $\mathbf{H}$ is unitary, $\mathbf{H}\mathbf{Y}\boldsymbol{\Theta} = \mathbf{Z}\boldsymbol{\Phi}$ and $\mathbf{H}\mathbf{Z}\boldsymbol{\Phi} = \mathbf{Y}\boldsymbol{\Theta}$.
- Again, $\mathbf{H}\mathbf{x}$ is cheap to compute since $\mathbf{H}\mathbf{x} = \mathbf{x} - \mathbf{G}(\mathbf{G}^H\mathbf{x})$ and $\mathbf{G}$ is an $N \times R$ matrix $\Rightarrow$ $O(NR)$ rather than $O(N^2)$.

Fast orthogonal precoding

The procedure for transmission and reception involves first computing the $\mathbf{G}$ matrix of the block reflector.

- Using either van de Beek's or Ma *et al.*'s method, obtain the $\mathbf{U}$ matrix from the SVD.
- Take the first R columns of $\mathbf{U}$ and assign it to $\mathbf{Y}$.
- Choose which R subcarriers to sacrifice and assign those columns of the identity matrix to $\mathbf{Z}$.
- Follow the recipe to create $\mathbf{G}$ from $\mathbf{Y}$ and $\mathbf{Z} \Rightarrow \mathbf{Q} = \mathbf{H}$.

To transmit:

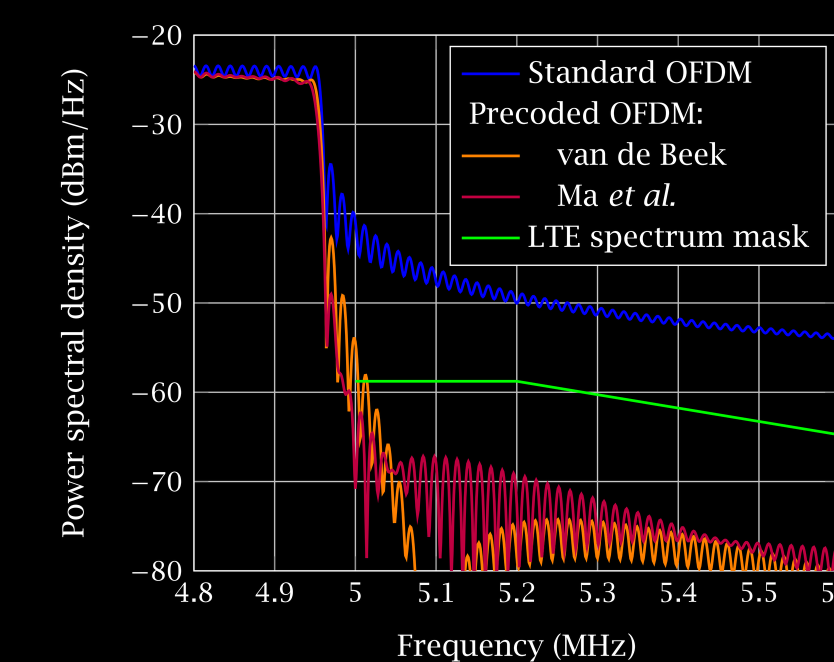
- Set $\mathbf{s} = \mathbf{H}\mathbf{x} = \mathbf{x} - \mathbf{G}(\mathbf{G}^H\mathbf{x})$.

To receive:

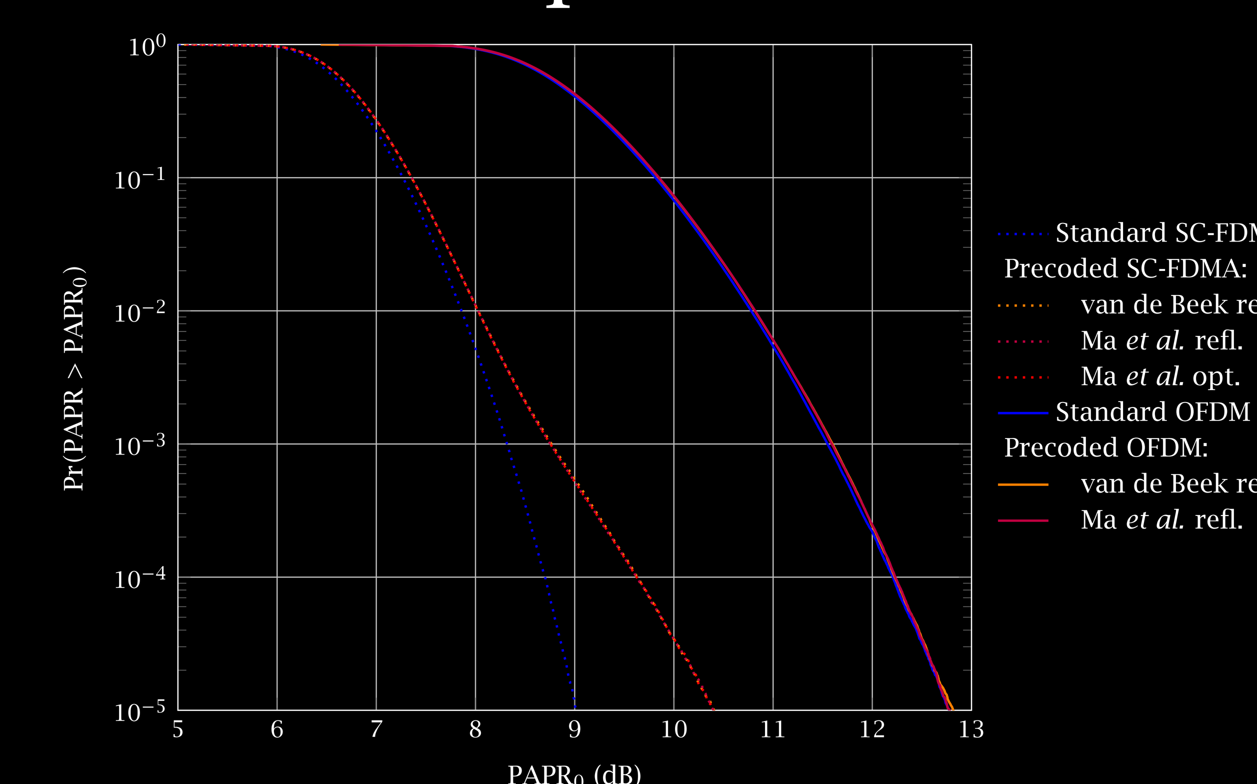
- Set $\hat{\mathbf{x}} = \mathbf{H}\hat{\mathbf{s}} = \hat{\mathbf{s}} - \mathbf{G}(\mathbf{G}^H\hat{\mathbf{s}})$.

Suppression performance

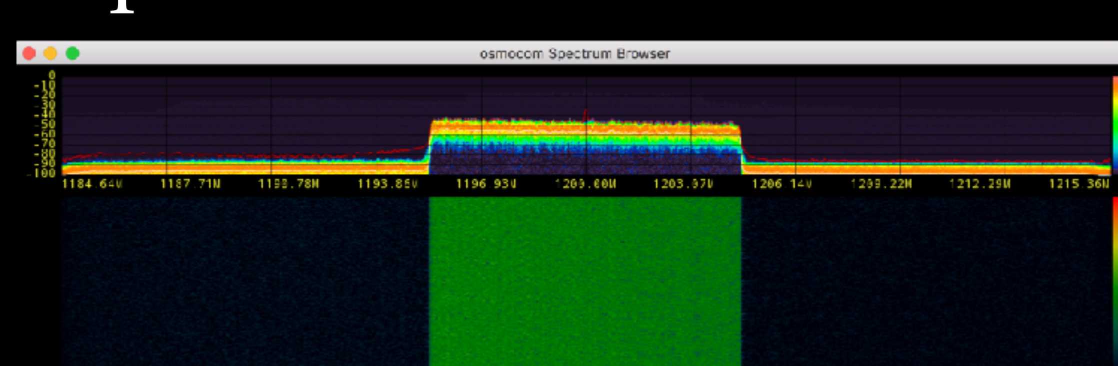
E-UTRA/LTE parameters, 660 subcarriers (55 PRB), 8 sacrificed.



PAPR performance



Experimental results: SDR



Credit: Konstanty Bialkowski, UQ

Why ❤️ OP-OFDM?

- Steep sidelobe suppression.
- Acceptable computational cost: $O(NR)$.
- Negligible sacrifice of subcarriers: $R = 8$.
- No ISI, no BER sacrifice.
  - Great for 256-QAM operation.
  - Great for TDD operation.
- Compatible with MIMO, SC-FDMA (DFT-s-OFDM), CP, etc.