**Key Points**

- We tackle the hyperspectral super-resolution problem using matrix factorization and first-order optimization.
- We devise a novel inexact block coordinate descent method which employs hybrid proximal gradient and Frank-Wolfe updates.

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**Hyperspectral Super-Resolution (HSR)**

- **Spectral sensors:** capture scenes in multiple spectral bands
  - Hyperspectral (HS) sensors
    - Specifications:
      - WORIS
      - HYSpRIS
      - P-1 LA
      - Sensor number: 294
      - Wavelength range: 0.4 - 2.5 μm
      - Spatial resolution: 20 m
      - HSI image has low-spatial and high-spectral resolution.
  - Multispectral (MS) sensors
    - Specifications:
      - Sensor number: 96
      - Wavelength range: 0.4 - 1.5 μm
      - Spatial resolution: 1.2 m
      - MS image has high-spectral and low-spatial resolution.

- **Super-resolution (SR), high spatial-spectral resolution, sensors? Not exist.**

- **HSR:** recover an SR image from an HS-MS image pair.

- **Applications:** high-spectral-resolution mapping of, e.g., minerals, urban surface materials, plant species, etc.

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**Problem Statement**

- **Signal model:**
  - MS image model: \( Y_S = F X + V_S \)
  - HS image model: \( Y_H = X G + V_H \)
  - \( X \in \mathbb{R}^{N \times M} \) is the spectral-spatial matrix of the MS image;
  - \( Y_S \in \mathbb{R}^{N \times S} \) is the spectral-spatial matrix of the MS image;
  - \( Y_H \in \mathbb{R}^{N \times H} \) is the spectral degradation matrix;
  - \( G \in \mathbb{R}^{H \times M} \) is the spectral degradation matrix;
  - \( V_S \) and \( V_H \) are noise.

- **Assumption:** The SR image has low rank, i.e., \( X \approx AS \)
  - \( N < \min(M, L) \)
  - \( A \) and \( S \) follow the linear mixture model
    - \( A \in \mathbb{R}^{M \times L} \)
    - \( S \in \mathbb{R}^{N \times K} \) where \( S = [s_1, \ldots, s_K] \text{ and } K = 1, \ldots, N \).

- **Structured matrix factorization (SMF) formulation:**
  - \( A \) and \( S \) follow the linear mixture model.
  - \( A \in \mathbb{R}^{M \times L} \) and \( S \in \mathbb{R}^{N \times K} \).
  - \( A \) is from an abundance map of the AVIRIS Cuprite dataset;
  - \( S \) is from an abundance map of the FUMI-2016 dataset.

- **Exact Block Coordinate Descent (EBCD):**
  - Exact BCD works by recursively solving
    - \( S^{t+1} = \arg \min_{S} f(A, S) \)
    - \( A^{t+1} = \arg \min_{A} f(A, S^{t+1}) \)
  - It guarantees convergence to a stationary point of the SMF for HSR.

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**Simulations**

- **Algorithms under comparison:**
  - PGiBCD: PG update for both \( A \) and \( S \).
  - FWiBCD: FW update for both \( A \) and \( S \).
  - FUMI: The state-of-the-art exact BCD algorithm.

- **Synthetic data experiment:**
  - Settings:
    - \( N = 9 \), \( L = 10^{19} \), \( M = 224 \), \( L_H = 259 \), \( M_H = 6 \).
    - \( A \) is from the USGS digital spectral library.
    - \( S \) is from an abundance map of the AVIRIS Cuprite dataset.
  - **Results:**
    - Average performance over 100 independent trials

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**Conclusion**

- An hybrid inexact BCD scheme was proposed for HSR.
- Computational and convergence issues were dealt.
- Numerical results showed promising runtime performance.

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**Reference**

