**Objective**

Motivated with the concept of transform learning and the utility of rational wavelet transform in audio and speech processing, the objective of this work is to propose Rational Wavelet Transform Learning in the Statistical sense (RWLS) for natural images. The learned rational wavelet is used as the sparsifying transform for CS based reconstruction of natural images.

**Motivation**

- Wavelets are used extensively as sparsifying basis for images.
- Rational wavelets provide non-uniform frequency band representation.
- Lifting framework, extended to rational wavelets in [1] can be used to learn signal-matched rational wavelets.

**Lifting**

General dyadic wavelet

\[
\begin{array}{c}
\text{Odd/} \\
\text{Even Split} \\
\text{Interleave/combine} \\
\text{samples from the} \\
\text{two input streams} \\
\end{array}
\]

Lifting framework consists of three steps:

- **Split:** input into even and odd indexed samples.
- **Predict:** Predict one subband samples from the other
  Correspondingly, update analysis highpass and synthesis lowpass filters using:

\[
H_n^{\text{high}}(z) = H_n(z) - H_n(z)T(z)
\]

\[
F_n^{\text{low}}(z) = F_n(z) + F_n(z)T(z)
\]

- **Update:** Update the other subband samples with the predicted subband samples
  Correspondingly, update analysis lowpass and synthesis highpass filters using:

\[
H_n^{\text{low}}(z) = H_n(z) + H_n(z)S(z)
\]

\[
F_n^{\text{high}}(z) = F_n(z) - F_n(z)S(z)
\]

**Fractional Brownian Motion**

Fractional Brownian motion \( B_H(t) \) is a Gaussian, zero mean, self similar, non-stationary random process with stationary increments. The auto-covariance of the corresponding discrete time process \( B_H[n] \) is given by:

\[
G_{\text{cov}}[n,n] = \sigma_0^2 H(2n+1)\sin H\pi H
\]

**Compressive Sensing**

Mathematically:

\[
\text{Y}_{\text{est}} = A_x x_{\text{est}} + \text{N}_{\text{est}}, \quad m < n
\]

Can be solved using optimization framework:

\[
x = \arg \min || y - A x ||_2^2 \quad \text{subject to} \quad || W x ||_r \leq r.
\]

**REFERENCES**


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