

# INTERFERENCE ALIGNMENT ON MIMO X CHANNEL WITH SYNERGISTIC CSIT

Tong Zhang and P. C. Ching

Department of Electronic Engineering, The Chinese University of Hong Kong, Hong Kong

## ABSTRACT

The achievable degree of freedom (DoF) boosting has been demonstrated on a single-input single-output (SISO) X channel by using outdated and instantaneous channel state information at transmitter (CSIT) synergistically, in contrast to that of using completely outdated CSIT. However, the means by which the DoF gain can be obtained in a multiple-input multiple-output (MIMO) system remains unclear. This paper proposes an interference alignment scheme with synergistic CSIT for MIMO X channel. We show that the achievable DoF is greater than the optimal DoF obtained with outdated CSIT, and that achievable DoF equals to the optimal value with outdated CSIT and transmitter cooperation.

**Index Terms**— MIMO X channel, degree of freedom, interference alignment, synergistic CSIT

## 1. INTRODUCTION

The degree of freedom (DoF) refers to the number of independent channels available for communication when the transmit power approaches infinity. The DoF can denote the approximation of sum channel capacity in a high signal-to-noise ratio (SNR) regime [1]. The mathematical definition of DoF can be found in [2].

X channel is a canonical model for studying the fundamental limits of wireless communication [2, 3], which consists of two transmitters and two receivers and each transmitter has messages for each receiver. On the X channel, the DoF outer bound can be achieved by interference alignment, which aligns the interference and liberates as many interference-free channels as possible for communication [4]. This scheme is based on instantaneous channel state information at transmitter (CSIT), which may be unavailable when the channel is fast-varying. A DoF-optimal scheme with outdated CSIT is first introduced in [5] for the  $K$ -user multiple-input single-output (MISO) broadcasting channel (BC), e.g.  $4/3$  DoF for  $K = 2$ . The BC and the X channel differs in that the information desired by each receiver are delivered by co-located and separated transmitters in the BC and the X channel respectively, which is equivalent to whether transmitter cooperation is provided to the X channel [6–8].

It is first revealed by [9] that interference alignment on the single-input single-output (SISO) X channel with outdated CSIT can achieve  $8/7$  DoF. The DoF is improved to  $6/5$  in [10], and this value is the upper bound for linear interference alignment [3]. The DoF in the SISO X channel with outdated CSIT has been observed to be less than that in the two-user MISO BC (equivalent to SISO X channel with transmitter cooperation), viz.  $6/5$  vs  $4/3$  [11].

When the feedback delay is less than the channel coherence time, both the outdated and instantaneous CSIT can be synergistically utilized to align interference [12, 13]. In the SISO X channel, the achievable DoF ( $4/3$ ) with synergistic CSIT is greater than that obtained by the scheme using only outdated CSIT ( $6/5$ ), and is the same as the optimal DoF of two-user MISO BC with outdated CSIT. This finding shows that using synergistic CSIT in the SISO X channel is better than using completely outdated CSIT.

Unlike the SISO system, the MIMO X channel can have more than one spatial dimension of signal space [4]. However, how to align interference by which DoF gain can be attained in the MIMO X channel through synergistic CSIT remains unclear. In this paper, we propose an interference alignment scheme with synergistic CSIT for the MIMO X channel. An optimization problem is established for determining optimal length of transmission time slots at different phases in order to maximize the DoF. The results show that a greater DoF can be obtained by using this approach than by using instantaneous CSIT only, or any of the outdated CSIT [6–8]. In fact, the attained DoF is the same as the optimal DoF in the MIMO X channel with outdated CSIT and transmitter cooperation. Under symmetric antenna setting, the ratio of instantaneous CSIT time slots to total time slots being used is not more than  $1/3$ .

*Notation:* The channel matrix from transmitter  $T_i, i = 1, 2$  to receiver  $R_j, j = 1, 2$  is written as  $\mathbf{H}_i^{[j]}$ . The entries of channel matrices are independent and identically distributed (i.i.d.) across time and space and drawn from a continuous distribution. One symbol desired by  $R_j, j = 1, 2$  and transmitted by  $T_i, i = 1, 2$  are written as  $\mathbf{x}_i^{[j]}$ . The number of transmit and receive antennas are denoted by  $M_i, i = 1, 2$  and  $N_j, j = 1, 2$ , respectively. The complex scalar, vector and matrix are denoted by  $\mathbf{h}, \mathbf{h}$  and  $\mathbf{H}$ , respectively.  $\mathbf{R}$  denote Gaussian random matrices generated offline.

This work is partially supported by the HKSAR Research Grant Council under GRF Grant Project # CUHK 14201714

## 2. INTERFERENCE ALIGNMENT SCHEME

We examine the proposed algorithm under three scenarios and design the transmission and decoding scheme accordingly.

### 2.1. Transmission and Decoding

We denote the receiver and transmitter with more antennas as  $R_1$  and  $T_1$ . Thus, we have  $N_2 \leq N_1$  and  $M_2 \leq M_1$ .

**Case A:**  $M_1 + M_2 \leq N_1$

$M_1 + M_2$  DoF can be obtained. In this process, transmitter  $T_1$  sends  $M_1$  symbols for receiver  $R_1$  and transmitter  $T_2$  sends  $M_2$  symbols for receiver  $R_1$  in one time slot. Given that  $M_1 + M_2$  linearly independent equations have been obtained at receiver  $R_1$ , all  $M_1 + M_2$  symbols can be decoded without interference.

**Case B:**  $N_1 < M_1 + M_2 \leq N_1 + N_2$

The design principle of the scheme is given as follows. In the first  $t_1$  time slots, both transmitters send symbols intended for receiver  $R_1$ . In the next  $t_2$  time slots, both transmitters send symbols intended for receiver  $R_2$ . In the last  $t_3$  time slots, both transmitters send the information that helps the receivers to decode previous symbols. These parameters  $t_1$ ,  $t_2$  and  $t_3$  will be determined later on.

*The first  $t_1$  time slots (Phase-I):* This phase occurs before the feedback. The transmitters  $T_1, T_2$  send  $(M_1 + M_2)t_1$  symbols, i.e.,  $\mathbf{x}_{1,1}^{[1]}, \dots, \mathbf{x}_{1,t_1}^{[1]}$  and  $\mathbf{x}_{2,1}^{[1]}, \dots, \mathbf{x}_{2,t_1}^{[1]}$  vectors, respectively, where  $\mathbf{x}_{i,1}^{[1]}, i = 1, 2$  is a vector consists of  $M_i, i = 1, 2$  symbols sent in the time slot 1. Then, we have the received signals as follows:

$$R_1 : \quad \mathbf{y}_1(p) = \mathbf{H}_{1,1}(p)\mathbf{x}_{1,p}^{[1]} + \mathbf{H}_{2,1}(p)\mathbf{x}_{2,p}^{[1]} \quad (1)$$

and

$$R_2 : \quad \mathbf{y}_2(p) = \underbrace{\mathbf{H}_{1,2}(p)\mathbf{x}_{1,p}^{[1]}}_{\underline{\mathcal{L}}_p^{[1]}} + \underbrace{\mathbf{H}_{2,2}(p)\mathbf{x}_{2,p}^{[1]}}_{\underline{\mathcal{J}}_p^{[1]}} \quad (2)$$

where  $p = 1, \dots, t_1$  and  $\underline{\mathcal{L}}_p^{[1]}$  and  $\underline{\mathcal{J}}_p^{[1]}$  are vectors with  $N_2$  dimensions related to  $T_1$  and  $T_2$ , respectively. The receiver  $R_1$  has seen total  $N_1 t_1$  linearly independent equations in  $t_1$  time slots. There are  $(M_1 + M_2)t_1$  symbols need to be decoded. Since we require  $M_1 + M_2 > N_1$ , i.e.,  $(M_1 + M_2 - N_1)t_1 > 0$ , the desired symbols cannot be decoded without interference.

*The next  $t_2$  time slot (Phase-II):* This phase occurs before the feedback. Transmitters  $T_1$  and  $T_2$  send  $(M_1 + M_2)t_1$  symbols, i.e.,  $\mathbf{x}_{1,t_1+1}^{[2]}, \dots, \mathbf{x}_{1,t_1+t_2}^{[2]}$ , and  $\mathbf{x}_{2,t_1+1}^{[2]}, \dots, \mathbf{x}_{2,t_1+t_2}^{[2]}$  vectors, respectively, where  $\mathbf{x}_{i,t_1+1}^{[2]}, i = 1, 2$  is a vector consists of  $M_i, i = 1, 2$  symbols sent in the time slot  $t_1 + 1$ . We have the received signals as follows:

$$R_1 : \quad \mathbf{y}_1(q) = \underbrace{\mathbf{H}_{1,1}(q)\mathbf{x}_{1,q}^{[2]}}_{\underline{\mathcal{L}}_q^{[2]}} + \underbrace{\mathbf{H}_{2,1}(q)\mathbf{x}_{2,q}^{[2]}}_{\underline{\mathcal{J}}_q^{[2]}} \quad (3)$$

and

$$R_2 : \quad \mathbf{y}_2(q) = \mathbf{H}_{1,2}(q)\mathbf{x}_{1,q}^{[2]} + \mathbf{H}_{2,2}(q)\mathbf{x}_{2,q}^{[2]} \quad (4)$$

where  $q = t_1 + 1, \dots, t_1 + t_2$  and  $\underline{\mathcal{L}}_q^{[2]}$  and  $\underline{\mathcal{J}}_q^{[2]}$  are vectors with  $N_1$  dimensions related to  $T_1$  and  $T_2$ , respectively. The receiver  $R_2$  has seen total  $N_2 t_1$  linearly independent equations in  $t_2$  time slots. There are  $(M_1 + M_2)t_2$  symbols need to be decoded. Since we require  $M_1 + M_2 > N_1 \geq N_2$ , i.e.,  $(M_1 + M_2 - N_2)t_2 > 0$ , the desired symbols cannot be decoded without interference.

To facilitate the decoding, we need to provide  $(M_1 + M_2 - N_1)t_1$  and  $(M_1 + M_2 - N_1)t_2$  extra linearly independent equations to the receivers  $R_1$  and  $R_2$ , respectively. These additional equations can exist in receivers  $R_2$  and  $R_1$ , as long as  $(M_1 + M_2 - N_1)t_1 \leq N_2 t_1$  and  $(M_1 + M_2 - N_2)t_2 \leq N_1 t_2$ , but not in the desired receivers  $R_1$  and  $R_2$ . This condition is equivalent to  $M_1 + M_2 \leq N_1 + N_2$ . In the phase-III, via synergistic CSIT, we can design a transmission scheme that enables the receivers to swap these equations.

*The last  $t_3$  time slots (Phase-III):* This phase comes after the feedback. The outdated CSIT for the first  $t_1 + t_2$  time slots and instantaneous CSIT for the last  $t_3$  time slots are obtained. The channel matrices for this phase are constant, because of within channel coherence time [13]. Transmitter  $T_1$  obtains  $\underline{\mathcal{L}}_1^{[1]}, \dots, \underline{\mathcal{L}}_{t_1}^{[1]}, \underline{\mathcal{L}}_{t_1+1}^{[2]}, \dots, \underline{\mathcal{L}}_{t_1+t_2}^{[2]}$  and transmitter  $T_2$  obtains  $\underline{\mathcal{J}}_1^{[1]}, \dots, \underline{\mathcal{J}}_{t_1}^{[1]}, \underline{\mathcal{J}}_{t_1+1}^{[2]}, \dots, \underline{\mathcal{J}}_{t_1+t_2}^{[2]}$ . Our aim is to deliver useful information that can reconstruct previous interference at receivers while providing additional equations for decoding. Thus, at the time slot  $t_1 + t_2 + 1$ , transmitter  $T_1$  sends

$$\mathbf{V}_1^{[1]}(t_1 + t_2 + 1)\underline{\mathcal{L}}_1^{[1]} + \mathbf{V}_1^{[2]}(t_1 + t_2 + 1)\overline{\mathcal{L}}_{t_1+1}^{[2]} \quad (5)$$

where the beamforming matrices  $\mathbf{V}_1^{[1]}, \mathbf{V}_1^{[2]}$  are with dimensions  $M_1 \times N_1$  and  $M_1 \times N_2$  respectively.  $\underline{\mathcal{L}}_1^{[1]}$  can be derived by  $[\underline{\mathcal{L}}_1^{[1]}; \dots; \underline{\mathcal{L}}_{\alpha t_1}^{[1]}] = \mathbf{R}_{N_1 \alpha t_1 \times N_2 t_1} [\underline{\mathcal{L}}_1^{[1]}; \dots; \underline{\mathcal{L}}_{t_1}^{[1]}]$  whilst  $\overline{\mathcal{L}}_{t_1+1}^{[2]}$  can be obtained from  $[\overline{\mathcal{L}}_{t_1+1}^{[1]}; \dots; \overline{\mathcal{L}}_{t_1+\beta t_2}^{[1]}] = \mathbf{R}_{N_2 \beta t_2 \times N_1 t_2} [\underline{\mathcal{L}}_{t_1+1}^{[1]}; \dots; \underline{\mathcal{L}}_{t_1+t_2}^{[1]}]$ , where we define

$$(M_1 + M_2 - N_1)/N_1, \quad (M_1 + M_2 - N_2)/N_2 \quad (6)$$

as  $\alpha$  and  $\beta$ , respectively. Transmitter  $T_2$  sends

$$\mathbf{V}_2^{[1]}(t_1 + t_2 + 1)\underline{\mathcal{J}}_1^{[1]} + \mathbf{V}_2^{[2]}(t_1 + t_2 + 1)\overline{\mathcal{J}}_{t_1+1}^{[2]} \quad (7)$$

where  $\mathbf{V}_2^{[1]}$  is with dimension  $M_2 \times N_1$ , and  $\mathbf{V}_2^{[2]}$  is with dimension  $M_2 \times N_2$ . Similarly, we can compute  $\underline{\mathcal{J}}_1^{[1]}$  from  $[\underline{\mathcal{J}}_1^{[1]}; \dots; \underline{\mathcal{J}}_{\alpha t_1}^{[1]}] = \mathbf{R}_{N_1 \alpha t_1 \times N_2 t_1} [\underline{\mathcal{J}}_1^{[1]}; \dots; \underline{\mathcal{J}}_{t_1}^{[1]}]$  and  $\overline{\mathcal{J}}_{t_1+1}^{[2]}$  from  $[\overline{\mathcal{J}}_{t_1+1}^{[1]}; \dots; \overline{\mathcal{J}}_{t_1+\beta t_2}^{[1]}] = \mathbf{R}_{N_2 \beta t_2 \times N_1 t_2} [\underline{\mathcal{J}}_{t_1+1}^{[1]}; \dots; \underline{\mathcal{J}}_{t_1+t_2}^{[1]}]$ . Next, we omit  $t_1 + t_2 + 1$  in brackets for

simplicity. For received signals,

$$R_1 : \quad \underline{\mathbf{y}}_1 = \underbrace{\mathbf{H}_{1,1}\mathbf{V}_1^{[1]}\underline{\mathcal{L}}_{\underline{1}}^{[1]} + \mathbf{H}_{2,1}\mathbf{V}_2^{[1]}\underline{\mathcal{J}}_{\underline{1}}^{[1]}}_{\text{desired signal}} + \underbrace{\mathbf{H}_{1,1}\mathbf{V}_1^{[2]}\underline{\mathcal{L}}_{t_1+1}^{[2]} + \mathbf{H}_{2,1}\mathbf{V}_2^{[2]}\underline{\mathcal{J}}_{t_1+1}^{[2]}}_{\text{interference}} \quad (8)$$

and

$$R_2 : \quad \underline{\mathbf{y}}_2 = \underbrace{\mathbf{H}_{1,2}\mathbf{V}_1^{[2]}\underline{\mathcal{L}}_{t_1+1}^{[2]} + \mathbf{H}_{2,2}\mathbf{V}_2^{[2]}\underline{\mathcal{J}}_{t_1+1}^{[2]}}_{\text{desired signal}} + \underbrace{\mathbf{H}_{1,2}\mathbf{V}_1^{[1]}\underline{\mathcal{L}}_{\underline{1}}^{[1]} + \mathbf{H}_{2,2}\mathbf{V}_2^{[1]}\underline{\mathcal{J}}_{\underline{1}}^{[1]}}_{\text{interference}} \quad (9)$$

To construct the interference received in the last  $t_3$  time slots based on the signals received in the  $t_2$  and  $t_1$  time slots, we require  $\mathbf{H}_{1,1}\mathbf{V}_1^{[2]} = \mathbf{H}_{2,1}\mathbf{V}_2^{[2]} = \mathbf{F}_1$  and  $\mathbf{H}_{1,2}\mathbf{V}_1^{[1]} = \mathbf{H}_{2,2}\mathbf{V}_2^{[1]} = \mathbf{F}_2$ . Since  $[\mathbf{H}_{1,1}, -\mathbf{H}_{2,1}]\begin{bmatrix} \mathbf{V}_1^{[2]} \\ \mathbf{V}_2^{[2]} \end{bmatrix} = \mathbf{0}$  and  $[\mathbf{H}_{1,2}, -\mathbf{H}_{2,2}]\begin{bmatrix} \mathbf{V}_1^{[1]} \\ \mathbf{V}_2^{[1]} \end{bmatrix} = \mathbf{0}$ , we can derive the beamforming matrices  $\mathbf{V}_1^{[2]}$  and  $\mathbf{V}_2^{[2]}$  by the nullspace of  $[\mathbf{H}_{1,1}, -\mathbf{H}_{2,1}]$  and the beamforming matrices  $\mathbf{V}_1^{[1]}$  and  $\mathbf{V}_2^{[1]}$  by the nullspace of  $[\mathbf{H}_{1,2}, -\mathbf{H}_{2,2}]$ . The subtraction  $\underline{\mathbf{y}}_1^{[1]} - \mathbf{F}_1(\underline{\mathcal{L}}_{t_1+1}^{[2]} + \underline{\mathcal{J}}_{t_1+1}^{[2]})$  and  $\underline{\mathbf{y}}_2^{[2]} - \mathbf{F}_2(\underline{\mathcal{L}}_{\underline{1}}^{[1]} + \underline{\mathcal{J}}_{\underline{1}}^{[1]})$  are made in receivers  $R_1$  and  $R_2$ . After that, we obtain  $N_1$  new linearly independent equations at receiver  $R_1$  except the  $M_2 \leq N_2 \leq M_1$ ,  $M_1 + 2M_2 < N_1 + N_2$  or  $M_1 \leq N_2$ ,  $M_1 + M_2 < N_2 + N_1/2$  antenna settings and  $N_2$  new linearly independent equations at receiver  $R_2$  except the  $M_2 \leq N_1 \leq M_1$ ,  $M_1 + 2M_2 < N_1 + N_2$  or  $M_1 \leq N_1$ ,  $M_1 + M_2 < N_1 + N_2/2$  antenna settings. Finally, the  $(M_1 + M_2 - N_1)t_1$  and  $(M_1 + M_2 - N_2)t_2$  new linearly independent equations can be obtained at receivers  $R_1$  and  $R_2$  respectively, as long as  $N_2\alpha t_1 \leq N_1t_2$  and  $N_1\beta t_2 \leq N_2t_1$ .

Here, we determine parameters  $t_1$ ,  $t_2$  and  $t_3$ .  $(M_1 + M_2)t_1$  symbols desired by receiver  $R_1$  are sent in phase-I and  $(M_1 + M_2)t_2$  symbols desired by receiver  $R_2$  are sent in phase-II. For each receiver obtaining sufficient additional equations, the length of  $t_3$  time slots equals to  $\max\{\alpha t_1, \beta t_2\}$ . To maximize the achievable DoF, we need to find the scheme with least time and the most decodable symbols. For this purpose, we establish a maximization problem under the feasibility of subtraction constraints.

$$\begin{aligned} \max_{\{t_1, t_2\}} & \frac{(M_1 + M_2)t_1}{t_1 + t_2 + \alpha t_1} + \frac{(M_1 + M_2)t_2}{t_1 + t_2 + \beta t_2} \\ \text{s.t.} & \quad 0 < N_2\alpha t_1 \leq N_1t_2, \quad 0 < N_1\beta t_2 \leq N_2t_1 \end{aligned} \quad (10)$$

Next, the problem is reformulated w.r.t.  $x = t_2/t_1$ .

$$\begin{aligned} \max_x & \frac{N_1}{\frac{N_1}{M_1 + M_2}x + 1} + \frac{N_2}{\frac{N_2}{(M_1 + M_2)x} + 1} \\ \text{s.t.} & \quad (N_2\alpha)/N_1 \leq x \leq N_2/(N_1\beta) \end{aligned} \quad (11)$$

The derivative of the objective function is

$$\frac{(M_1 + M_2)N_2^2}{((M_1 + M_2)x + N_2)^2} - \frac{(M_1 + M_2)N_1^2}{(N_1x + M_1 + M_2)^2} \quad (12)$$

The maximal point can be obtained from the above equation and is given by

$$x^* = \frac{N_2(M_1 + M_2 - N_1)}{N_1(M_1 + M_2 - N_2)} \quad (13)$$

This solution can satisfy the constraints. Thus, we obtain the optimal solution to the problem (11). Also, the gap between the objective function that is DoF upper bound and the real DoF function is zero at this point. This approach shows that we can select  $t_1 = N_1(M_1 + M_2 - N_2)$ ,  $t_2 = N_2(M_1 + M_2 - N_1)$  and  $t_3 = (M_1 + M_2 - N_1)(M_1 + M_2 - N_2)$ . Consequently, the DoF is

$$\frac{N_1(M_1 + M_2 - N_2) + N_2(M_1 + M_2 - N_1)}{M_1 + M_2 - \frac{N_1N_2}{M_1 + M_2}} \quad (14)$$

*Remark:* An intuitive explanation for the optimal solution is given hereunder. In the last  $t_3$  time slots, to efficiently utilize the simultaneous transmission of  $T_1, T_2$  that is desired by both receivers, both receivers need equal time to acquire their desired equations. Thus, we have

$$t_3 = \alpha t_1 = \beta t_2 \quad (15)$$

This is equivalent to

$$\frac{N_2(M_1 + M_2 - N_1)}{N_1(M_1 + M_2 - N_2)} = \frac{t_2}{t_1} \quad (16)$$

which is exactly the optimal solution we obtained.

**Case C:**  $N_1 + N_2 < M_1 + M_2$

In the case B, there exists sufficient additional equations in receivers  $R_1$  and  $R_2$ , i.e.,  $(M_1 + M_2 - N_2)t_2 \leq N_1t_2$  and  $(M_1 + M_2 - N_1)t_1 \leq N_2t_1$ . However, this condition cannot be satisfied in the case C, provided that the  $M_1$  and  $M_2$  antennas are used for transmission in phase-I and -II. To exhaust the transmission ability in phase-I and -II, and retrieve enough additional equations in phase-III, we should take  $N_1$  and  $N_2$  antennas for phase-I and -II transmission. This is a significant difference between cases B and A. The design principle of cases B and C are the same.

Again, we try to determine the parameters  $t_1, t_2$  and  $t_3$  that act in getting the highest DoF. Considering that the transmission is similar, we can establish the following equation for deriving the optimal solution,

$$t_3 = \frac{N_2}{N_1}t_1 = \frac{N_1}{N_2}t_2 \quad (17)$$

Thus, we have  $t_2/t_1 = N_2^2/N_1^2$ , i.e.,  $t_1 = N_1^2$ ,  $t_2 = N_2^2$  and  $t_3 = N_1N_2$ . Finally, we have the following DoF:

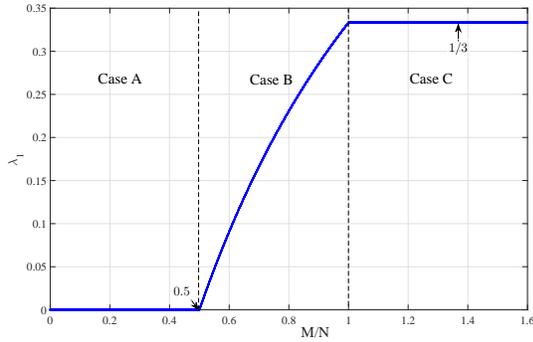
$$\frac{N_1^2(N_1 + N_2) + N_2^2(N_1 + N_2)}{N_1^2 + N_2^2 + N_1N_2} \quad (18)$$

## 2.2. Outdated and Instantaneous CSIT Usage Analysis

We define  $\lambda_1$  and  $\lambda_2$  as the ratios of instantaneous CSIT time to total time and outdated CSIT time to total time, respectively. Via direct computation, we obtain the ratios as follows:

$$\begin{aligned} \text{Case A : } \lambda_1 &= 0, \lambda_2 = 0 \\ \text{Case B : } \lambda_1 &= \frac{(M_1 + M_2 - N_2)}{N_1 \frac{M_1 + M_2 - N_2}{M_1 + M_2 - N_1} + (M_1 + M_2)} \\ \text{Case C : } \lambda_1 &= \frac{N_1 N_2}{N_1^2 + N_2^2 + N_1 N_2} \end{aligned} \quad (19)$$

In cases B and C,  $\lambda_2 = 1 - \lambda_1$ . When the antenna setting is symmetric,  $\lambda_1$  degrades to  $1 - \frac{2}{2(M/N)+1}$  and  $\frac{1}{3}$  for cases B and C, respectively.



**Fig. 1.** Behavior of  $\lambda_1$  as a function of  $M/N$ .

Figure 1 shows that in case C under symmetric antenna setting, the proportion of instantaneous CSIT is highest and equals to  $1/3$ . In that case, we have  $N < M$ , which indicates that the number of used transmit antennas is equal to the number of receive antennas in our proposed scheme.

## 3. PERFORMANCE COMPARISON

In this section, we compare our proposed scheme with three DoF results on MIMO X channel, namely, optimal DoF with outdated CSIT and transmitter cooperation, achievable DoF with instantaneous and no CSIT, and optimal DoF with outdated CSIT. The achievable DoF of our proposed scheme using synergistic CSIT is at least as good as, if not better than, the optimal DoF obtained with outdated CSI [7, 8].

### 3.1. Optimal DoF with Outdated CSIT and Transmitter Cooperation

The two-user MIMO BC has one transmitter of  $M$  antennas sending independent messages for two receivers, where each receiver is equipped with  $N_1$  and  $N_2$  antennas. Unlike the two-user MIMO BC that has one transmitter, the MIMO X channel has two separate transmitters. By substituting  $M =$

$M_1 + M_2$  into the optimal DoF of two-user MIMO BC [14], we can obtain the equivalent optimal DoF on the MIMO X channel under outdated CSIT and transmitter cooperation.

$$\begin{aligned} \text{Case A : DoF} &= M_1 + M_2, \\ \text{Case B : DoF} &= \frac{N_1(M_1 + M_2 - N_2) + N_2(M_1 + M_2 - N_1)}{M_1 + M_2 - \frac{N_1 N_2}{M_1 + M_2}} \\ \text{Case C : DoF} &= \frac{N_1^2(N_1 + N_2) + N_2^2(N_1 + N_2)}{N_1^2 + N_2^2 + N_1 N_2} \end{aligned} \quad (20)$$

Specifically, when  $M_1 = M_2 = M$  &  $N_1 = N_2 = N$ , the eqn. (20) can be simplified to

$$\begin{aligned} \text{Case A } (2M \leq N) : \text{DoF} &= 2M, \\ \text{Case B } (N < 2M \leq 2N) : \text{DoF} &= \frac{4MN}{2M + N} \\ \text{Case C } (2N < 2M) : \text{DoF} &= \frac{4N}{3} \end{aligned} \quad (21)$$

The proposed scheme with synergistic CSIT can achieve that optimal DoF, i.e., eqn.(20). Note that X channel with transmitter cooperation can obtain a higher DoF than the one without cooperation in the presence of outdated CSIT [11].

### 3.2. DoF with Instantaneous and No CSIT

We compare the proposed scheme with the approach using instantaneous CSIT in the last  $t_3$  time slots [4] only, which is an alternative CSIT usage mode to ours. In the  $t_1 + t_2$  time slots, the time division multiple access (TDMA) scheme is used, because of the absence of CSIT.

With a symmetric antenna setting, the alignment that uses instantaneous CSIT [4] can achieve the DoF upper bound on the X channel.

$$\text{DoF}_{\text{Instant.}} = \min\{2 \min(M, N), \frac{4}{3} \max(M, N)\} \quad (22)$$

In cases B and C, the approach using instantaneous CSIT only has the following DoF:

$$\begin{aligned} \text{Case B : } \frac{2N^2 + \min\{2M, \frac{4}{3}N\}(2M - N)}{2M + N} &\leq \frac{4MN}{2M + N} \\ \text{Case C : } \frac{2N + \min\{2N, \frac{4}{3}M\}}{3} &\leq \frac{4N}{3} \end{aligned} \quad (23)$$

Thus, our proposed method outperforms the one using solely instantaneous CSIT with symmetric antenna configuration.

## 4. CONCLUSION

This paper analyzes the achievable DoF on MIMO X channel with synergistic CSIT and proposes an efficient interference alignment scheme to address the inherent problem. The performance of proposed scheme is compared with the DoF results on MIMO X channel under different conditions, and the advantages of proposed scheme are validated via simulation.

## 5. REFERENCES

- [1] S. A. Jafar, *Interference alignment: A new look at signal dimensions in a communication network*, Now Publishers Inc, 2011.
- [2] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication over mimo x channels: Interference alignment, decomposition, and performance analysis," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3457–3470, Aug 2008.
- [3] S. Lashgari, A. S. Avestimehr, and C. Suh, "Linear degrees of freedom of the x-channel with delayed csit," *IEEE Transactions on Information Theory*, vol. 60, no. 4, pp. 2180–2189, April 2014.
- [4] S. A. Jafar and S. Shamai, "Degrees of freedom region of the mimo x channel," *IEEE Transactions on Information Theory*, vol. 54, no. 1, pp. 151–170, Jan 2008.
- [5] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4418–4431, July 2012.
- [6] A. Ghasemi, M. J. Abdoli, and A. K. Khandani, "On the degrees of freedom of mimo x channel with delayed csit," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2012, pp. 1892–1896.
- [7] D. T. H. Kao and A. S. Avestimehr, "Linear degrees of freedom of the mimo x-channel with delayed csit," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2014, pp. 366–370.
- [8] D. T. H. Kao and A. S. Avestimehr, "Linear degrees of freedom of the mimo x-channel with delayed csit," *IEEE Transactions on Information Theory*, vol. PP, no. 99, pp. 1–1, 2016.
- [9] H. Maleki, S. A. Jafar, and S. Shamai, "Retrospective interference alignment over interference networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 228–240, June 2012.
- [10] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "On the degrees of freedom of x channel with delayed csit," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2011, pp. 767–770.
- [11] J. Abdoli, A. Ghasemi, and A. K. Khandani, "Interference and x networks with noisy cooperation and feedback," *IEEE Transactions on Information Theory*, vol. 61, no. 8, pp. 4367–4389, Aug 2015.
- [12] A. Wagdy, A. El-Keyi, T. Khattab, and M. Nafie, "A degrees of freedom-optimal scheme for siso x channel with synergistic alternating csit," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2014, pp. 376–380.
- [13] N. Lee, R. Tandon, and R. W. Heath, "Distributed space-time interference alignment with moderately delayed csit," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 1048–1059, Feb 2015.
- [14] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region of the two-user mimo broadcast channel with delayed csit," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, 2011, pp. 199–203.