



# CHANNEL DEPENDENT MUTUAL INFORMATION IN INDEX MODULATIONS

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## Outline

- Introduction
- Problem statement
- Proposed solution
- Results
- Conclusions

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## Introduction

- Link Adaptation requires Effective SNR metric

$$\bar{\gamma} = \varphi^{-1} \left( \frac{1}{N} \sum_n^N \varphi(\gamma_n) \right)$$

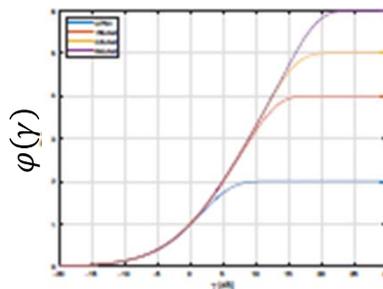
- This metric “averages” the SNR ( $\gamma_n$ ) of each symbol (of length N) within a codeblock.
- In this way it can be mapped to the corresponding modulation & coding scheme (ESM) in order to carry out Adaptive Code Modulation and better control the error rate.
- In the literature, this mapping is based on the Mutual Information (MI-ESM).

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## Introduction

- However, MI-ESM involves the expectation of a Random Variable (RV) (without a closed-form solution).
- In the literature, the computation of MI-ESM is performed off-line via Monte Carlo long simulations and the results are stored into a look-up table.

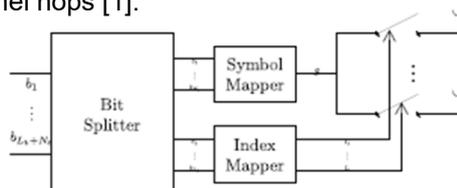




## Introduction

### HOWEVER

- Index Modulations, such as Spatial Modulation (SM) or Polarized Modulation (PM), convey the information in the radiated symbol and also in the channel hops [1].



- Thus, the constellation of symbols depends on the channel realization at each time instant.
- The computation of MI-ESM cannot be performed off-line due to this channel dependency.

[1] P. Henarejos, A. Pérez-Neira, Dual Polarized Modulation and Reception for Next Generation Satellite Communications, IEEE Transactions on Communications, Vol. 63, No. 10, pp. 3803-3812, October 2015.

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## Introduction

- The problem statement is:

How can MI-ESM be computed in Index Modulation?

- We propose a closed-form expression based on approximations of the integral-based expressions.
- Thanks to these approximations, we are able to obtain very low computational complex expressions with a valid accuracy and

enable link adaptation for Index Modulations

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## Problem Statement

- System model of Index Modulations (SM, PM, etc.)

$$\begin{aligned} \mathbf{y} &= \sqrt{\gamma} \mathbf{H} \mathbf{x} + \mathbf{w} \\ &= \sqrt{\gamma} \mathbf{h}_l s + \mathbf{w} \end{aligned}$$

where  $\mathbf{x} = \mathbf{1}_s$  is the all-zero vector except at position  $l$ , that is 1,  $\gamma$  is the average received SNR,  $s$  is the radiated symbol and  $l$  is the hopping index.

- The statistics of channel  $\mathbf{H}$  depend on the physical domain (spatial, polarization, etc.)

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## Problem Statement

- Mutual Information

$$\begin{aligned} I(\mathbf{y}; s, l) &= I(\mathbf{y}; s; l) + I(\mathbf{y}; l) \\ &= H(s) + H(l) - h(s, l | \mathbf{y}) \end{aligned}$$

where  $s$  and  $l$  are independent RV,  $H(x)$  is the entropy of  $X$  and  $h(x)$  is the differential entropy of  $X$ .  $s$  belongs to a known constellation, such as QAM.

Note that:

$$\begin{aligned} H(s) &= \log_2 S \\ H(l) &= \log_2 t \end{aligned}$$

where  $S$  is the size of constellation of  $s$  and  $t$  is the number of channel elements (number of antennas, polarizations, etc.)

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## Problem Statement

- Assuming white Gaussian noise, and after some mathematical manipulations,  $h(s, l|y)$  can be expressed as

$$\begin{aligned}
 h(S, L|Y) &= \frac{1}{tS} \sum_{s \in \mathcal{S}} \sum_{l=1}^t \mathbb{E} \left\{ \log_2 \left( \frac{\sum_{s' \in \mathcal{S}} \sum_{l'=1}^t f_{Y|S,L}(\mathbf{y}, s', l')}{f_{Y|S,L}(\mathbf{y}, s, l)} \right) \right\} \\
 &= \frac{1}{tS} \sum_{s \in \mathcal{S}} \sum_{l=1}^t \mathbb{E}_{\mathbf{W}'} \left\{ \log_2 \left( \sum_{s' \in \mathcal{S}} \sum_{l'=1}^t e^{-\gamma (\|\mathbf{h}_l s - \mathbf{h}_l s' + \mathbf{w}'\|^2 - \|\mathbf{w}'\|^2)} \right) \right\}
 \end{aligned}$$

where  $f_{Y|S,L}(\mathbf{y}, s, l)$  is the joint probability density function of  $y$ ,  $s$  and  $l$ , and  $\mathbf{W}' \sim \mathcal{CN}(\mathbf{0}, \gamma^{-1} \mathbf{I})$ .

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## Problem Statement

- Due to the channel dependency, the expectation cannot be computed off-line since it depends on the channel realization.
- The expectation has to be computed at each time instant where the link adaptation has to take place.
- It requires a very high complex computation to be carried out on-line.

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## Proposed Solution

- Following the approach of [1], we compute the Taylor Series Expansion of the expectation function, which yields into its moments.
- The first order approximation is:

$$I_{(1)}(\mathbf{y}; s, l) \simeq \log_2 \left( \frac{tS}{\mathfrak{G}(\mathcal{D}_{sl})} \right)$$

where  $\mathcal{D}_{sl} \doteq \sum_{s' \in \mathcal{S}} \sum_{l'=1}^t e^{-\gamma(\|\mathbf{x}_{sl} - \mathbf{x}_{s'l'}\|^2)}$ ,  $\mathbf{x}_{sl} \doteq \mathbf{h}_l s$  and  $\mathfrak{G}(\mathbf{x})$  is the geometric mean of  $\mathbf{x}$ .

- The second order approximation is more involved and it can be found in the paper

[1] P. Henarejos and A. I. Perez-Neira, "Capacity analysis of index modulations over spatial, polarization, and frequency dimensions," *IEEE Trans. Commun.*, vol. 65, no. 12, pp. 5280–5292, Dec. 2017.

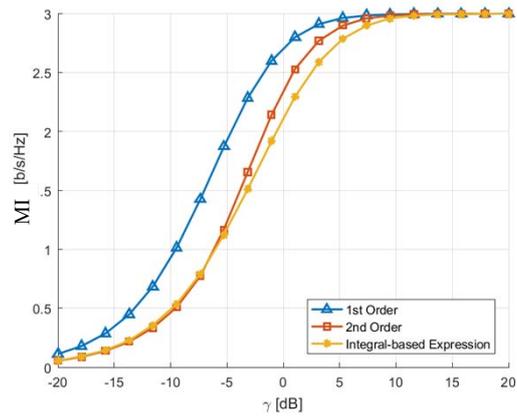


## Results

- We compare the first and second order approximations with the integral-based expression.
- We generate a very large number of Rayleigh channel realizations and average the MI results to obtain a smooth curve.
- As expected,
  - as we increase the order of the approximation, the curve becomes tighter and closer to the integral-based expression;
  - The approximation is an upper bound  $f(\mathbb{E}_{\mathbf{X}}\{x\}) \leq \mathbb{E}_{\mathbf{X}}\{f(x)\}$



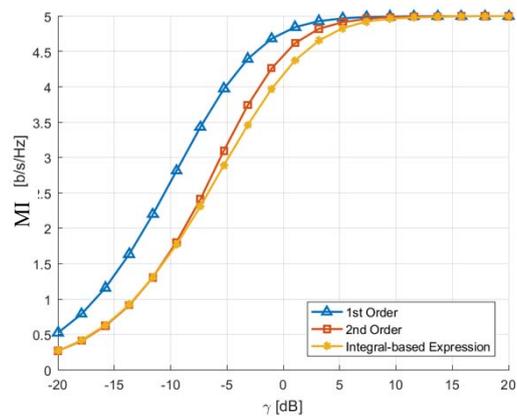
- We simulate a  $2 \times 2$  MIMO channel (1 bit) conveying a QPSK symbol (2 bits).



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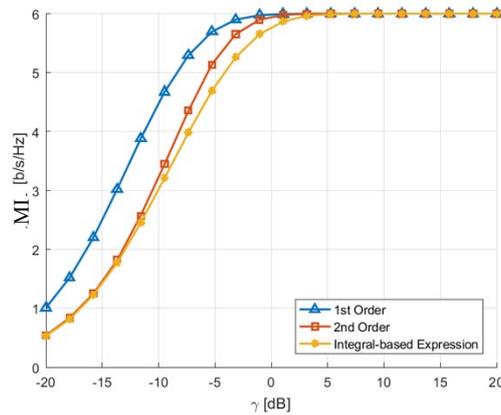
- Other schemes:  $2 \times 2$  MIMO channel (1 bit) conveying a 16-QAM symbol (4 bits).



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## CTTC<sup>®</sup> Results

- Other schemes:  $4 \times 4$  MIMO channel (2 bit) conveying a 16-QAM symbol (4 bits).



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## CTTC<sup>®</sup> Conclusions

- We introduced the problem of implementing link adaptation in Index Modulations, such as Spatial Modulation or Polarized Modulation.
- If the channel is time varying, it is **unaffordable** to compute the Mutual Information at each time instant.
- With our approach it is **possible** to obtain a smooth curve by using closed-form expressions, **decreasing** the computational complexity and allowing to perform the link adaptation.
- Finally, we depicted the first and second order approximations compared with integral-based expression for several configurations and constellation size.

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Thank you!

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