MASSIVE MIMO CHANNEL ESTIMATION FOR MILLIMETER WAVE SYSTEMS VIA MATRIX COMPLETION

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At a glance

- We focus on the **estimation of narrowband millimeter wave channel** for massive multiple input multiple output systems with hybrid analog beamforming architecture.
- We introduce a **joint optimization formulation** for mmWave massive MIMO channel estimation incorporating both the sparsity and low rank properties [1].
- We develop a machine learning algorithm based on the **Alternating Direction Method of Multipliers (ADMM)** for efficient recovery of massive MIMO channel matrices.

I. The Problem

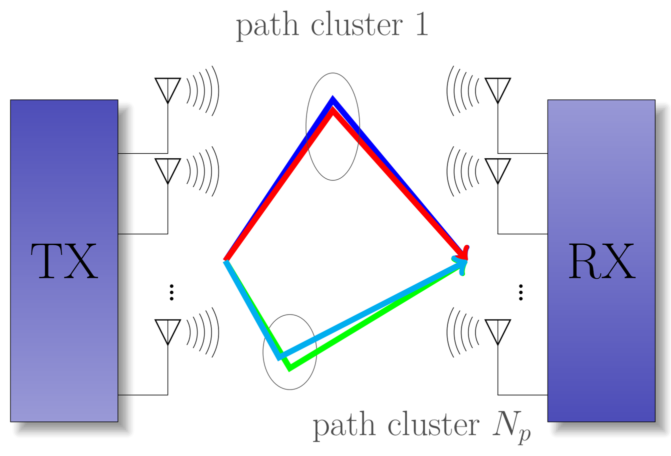
- Millimeter wave (mmWave) channels are characterized by **high variability** that severely challenges their recovery over **short training periods**.
- Large antenna sizes require **large numbers of training symbols** for satisfactory performance.
- Current channel estimation techniques exploit either the **channel sparsity** in the beamspace domain [2] or its **low rank** property in the angular domain [3].

II. Background

We consider a $N_{\mathrm{R}} \times N_{\mathrm{T}}$ massive MIMO system operating over quasi-static mmWave channel with **small number of scatterers** N_p.

Geometric decomposition

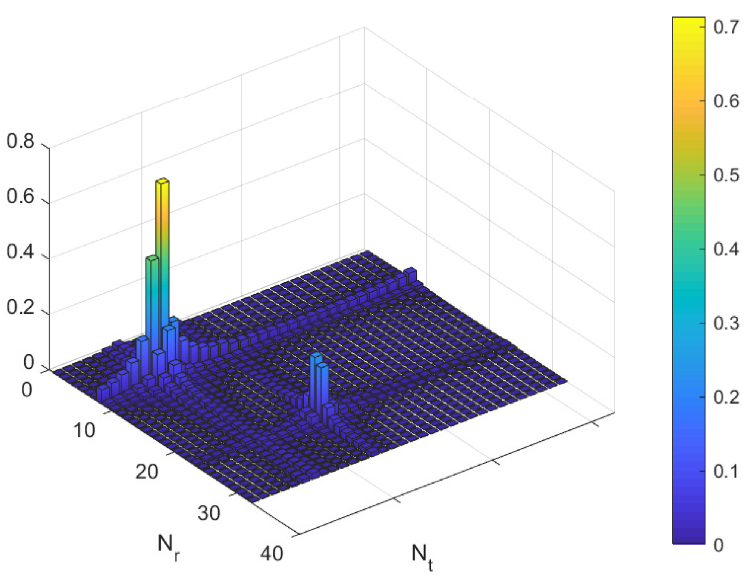
$$\mathbf{H} = \sum_{k=1}^{N_p} \underbrace{\alpha_k}_{\text{gain}} \underbrace{\mathbf{a}_{\mathrm{R}}(\phi_{\mathrm{R}}^{(k)})\mathbf{a}_{\mathrm{T}}^H(\phi_{\mathrm{T}}^{(k)})}_{\text{steering vectors}}$$



- The channel is decomposed into a sum of N_p rank-1 matrices. Hence, the **rank of the channel** is at most N_p.

Beamspace representation

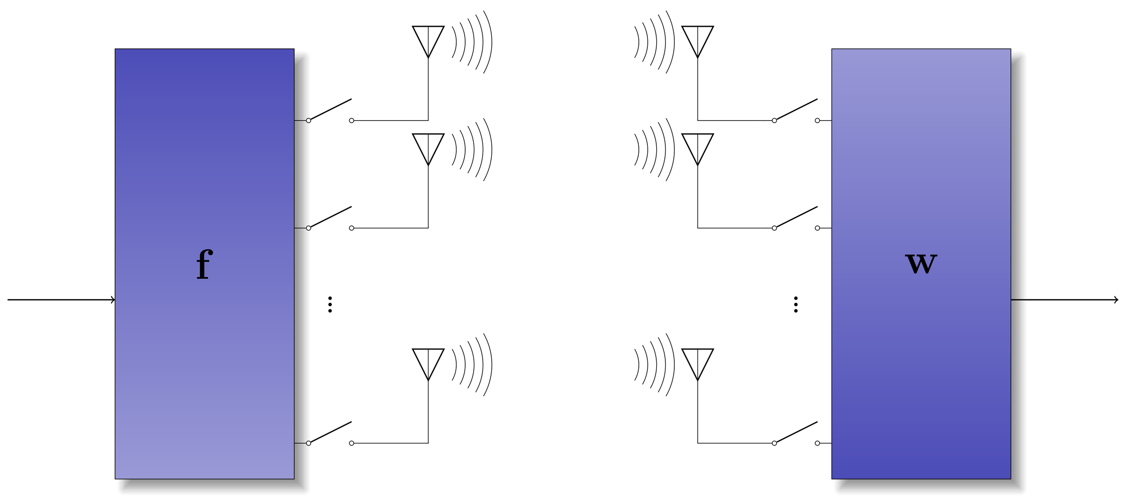
$$\mathbf{H} = \underbrace{\mathbf{F}}_{\substack{\text{DFT}\\\text{matrix}}} \; \underbrace{\mathbf{S}}_{\substack{\text{sparse}\\\text{matrix}}} \; \underbrace{\mathbf{F}^H}_{\substack{\text{DFT}\\\text{matrix}}}$$



- The amplitude of the beamspace channel $\|\mathbf{S}\|$ has at most N_p high amplitute entries. However, there are several entries with lower amplitudes. This phenomenon is called the **power leakage effect**.

II. Proposed System Design

- To exploit both properties we introduce a *joint optimization formulation* which extends the standard **matrix completion**.
- Matrix completion requires a **sub-sampled** version of the channel matrix $\mathbf{H}_\Omega$.
- We adopt **analog BF with switches** for the transmiter (TX) and the receiver (RX), i.e., $\mathbf{f} \in \{0,1\}^{N_{\mathrm{T}}}$, $\mathbf{w} \in \{0,1\}^{N_{\mathrm{R}}}$ are the combining and precoding vectors.



- At t-th training instance, the post-processed received signal at the N_{R}-element RX is

$$r[t] \triangleq \sqrt{P_t}\mathbf{w}^T\mathbf{H}\mathbf{f} + n[t]$$

where P_t is the Transmitter (TX) power and $n[t]$ is the AWGN with variance σ_n^2.

- The *mapping* of the training symbols to the sub-sampled channel matrix $\mathbf{H}_\Omega$ is captured by the binary matrix $\mathbf{\Omega} \in \{0,1\}^{N_{\mathrm{R}} \times N_{\mathrm{T}}}$, with $\|\mathbf{\Omega}\|_0 = M$.
- To estimate the (i,j)-th non-zero element of $\mathbf{H}_\Omega$ at the t-th training instance, we set $\mathbf{w} = \mathbf{e}_i$ and $\mathbf{f} = \mathbf{e}_j$ as the RX combining and TX precoding vectors.

Joint Optimization Problem

$$\min_{\mathbf{H},\mathbf{S}} \underbrace{\tau_H\|\mathbf{H}\|_*}_{\substack{\text{low}\\\text{rank}}} + \underbrace{\tau_S\|\mathbf{S}\|_1}_{\substack{\text{high}\\\text{sparsity}}} \quad \text{s.t.} \; \underbrace{\mathbf{\Omega} \circ \mathbf{H} = \mathbf{H}_\Omega}_{\substack{\text{training}\\\text{symbols}}} \; \text{and} \; \underbrace{\mathbf{H} = \mathbf{D}_{\mathrm{R}}\mathbf{S}\mathbf{D}_{\mathrm{T}}^H}_{\substack{\text{beamspace}\\\text{representation}}}$$

III. Proposed Solution via Alternating Minimization

- To tackle the joint optimization problem, the cost function is *decomposed as the sum* of four unknown variables.
- Then, the solution is obtained via a *machine learning technique*, the *Alternating Direction Method of Multipliers* (ADMM).
- The general procedure for obtaining the solution follows the next steps:

Introduce the two auxiliary matrix variables
$\mathbf{Y} \triangleq \mathbf{H}$ and $\mathbf{C} \triangleq \mathbf{Y} - \mathbf{D}_{\mathrm{R}}\mathbf{S}\mathbf{D}_{\mathrm{T}}^H$

Replace the constraints with
$\|\mathbf{\Omega} \circ \mathbf{H} - \mathbf{H}_\Omega\|_F^2$ and $\|\mathbf{Y} - \mathbf{D}_{\mathrm{R}}\mathbf{S}\mathbf{D}_{\mathrm{T}}^H\|_F^2$

Solve the augmented problem
$$\min_{\mathbf{H},\mathbf{Y},\mathbf{S},\mathbf{C}} \tau_H\|\mathbf{H}\|_* + \tau_S\|\mathbf{S}\|_1 + \frac{1}{2}\|\mathbf{C}\|_F^2 + \frac{1}{2}\|\mathbf{\Omega} \circ \mathbf{Y} - \mathbf{H}_\Omega\|_F^2$$
s.t. $\mathbf{H} = \mathbf{Y}$ and $\mathbf{C} = \mathbf{Y} - \mathbf{D}_{\mathrm{R}}\mathbf{S}\mathbf{D}_{\mathrm{T}}^H$

- The ℓ-th algorithmic iteration with $\ell = 0, 1, \ldots$ the following separate sub-problems need to be solved:

$$\mathbf{H}^{(\ell+1)} = \arg\min_{\mathbf{H}} \mathcal{L}_1(\mathbf{H}, \mathbf{Y}^{(\ell)}, \mathbf{S}^{(\ell)}, \mathbf{C}^{(\ell)}, \mathbf{Z}_1^{(\ell)}, \mathbf{Z}_2^{(\ell)}), \quad (1)$$
$$\mathbf{Y}^{(\ell+1)} = \arg\min_{\mathbf{Y}} \mathcal{L}_1(\mathbf{H}^{(\ell+1)}, \mathbf{Y}, \mathbf{S}^{(\ell)}, \mathbf{C}^{(\ell)}, \mathbf{Z}_1^{(\ell)}, \mathbf{Z}_2^{(\ell)}), \quad (2)$$
$$\mathbf{S}^{(\ell+1)} = \arg\min_{\mathbf{S}} \mathcal{L}_1(\mathbf{H}^{(\ell+1)}, \mathbf{Y}^{(\ell+1)}, \mathbf{S}, \mathbf{C}^{(\ell)}, \mathbf{Z}_1^{(\ell)}, \mathbf{Z}_2^{(\ell)}), \quad (3)$$
$$\mathbf{C}^{(\ell+1)} = \arg\min_{\mathbf{C}} \mathcal{L}_1(\mathbf{H}^{(\ell+1)}, \mathbf{Y}^{(\ell+1)}, \mathbf{S}^{(\ell+1)}, \mathbf{C}, \mathbf{Z}_1^{(\ell)}, \mathbf{Z}_2^{(\ell)}), \quad (4)$$
$$\mathbf{Z}_1^{(\ell+1)} = \mathbf{Z}_1^{(\ell)} + \rho(\mathbf{H}^{(\ell+1)} - \mathbf{Y}^{(\ell+1)}), \quad (5)$$
$$\mathbf{Z}_2^{(\ell+1)} = \mathbf{Z}_2^{(\ell)} + \rho(\mathbf{Y}^{(\ell+1)} - \mathbf{D}_{\mathrm{R}}\mathbf{S}^{(\ell+1)}\mathbf{D}_{\mathrm{T}}^H - \mathbf{C}^{(\ell+1)}). \quad (6)$$

where $\mathcal{L}_1$ is the augmented Lagrangian, ρ is the stepsize, and for $\ell = 0$: $\mathbf{H}^{(0)} = \mathbf{Z}_1^{(0)} = \mathbf{Z}_2^{(0)} = \mathbf{0}$.

IV. Evaluation

- Orthogonal matching pursuit (OMP) and vector approximate message passing (VAMP) exploit only the sparsity of the channel matrix.
- Singular value thresholding (SVT) capitalizes only on its low rank property.
- TSSR [3] exploits both properties but in a sequencial manner.
- Normalized Mean-Square-Error (NMSE) was evaluated as

$$\mathrm{NMSE} = \mathcal{E}\{10\log_{10}\|\hat{\mathbf{H}} - \mathbf{H}\|_F^2/\|\mathbf{H}\|_F^2\}$$

- Achievable Spectral Efficiency (ASE) was evaluated as

$$\mathrm{ASE} = \mathcal{E}\{\log_2\det(\mathbf{I}_{N_{\mathrm{R}}} + (N_{\mathrm{T}}N_{\mathrm{R}}(\sigma_n^2 + \mathrm{NMSE}))^{-1}\mathbf{H}\mathbf{H}^H)\}$$

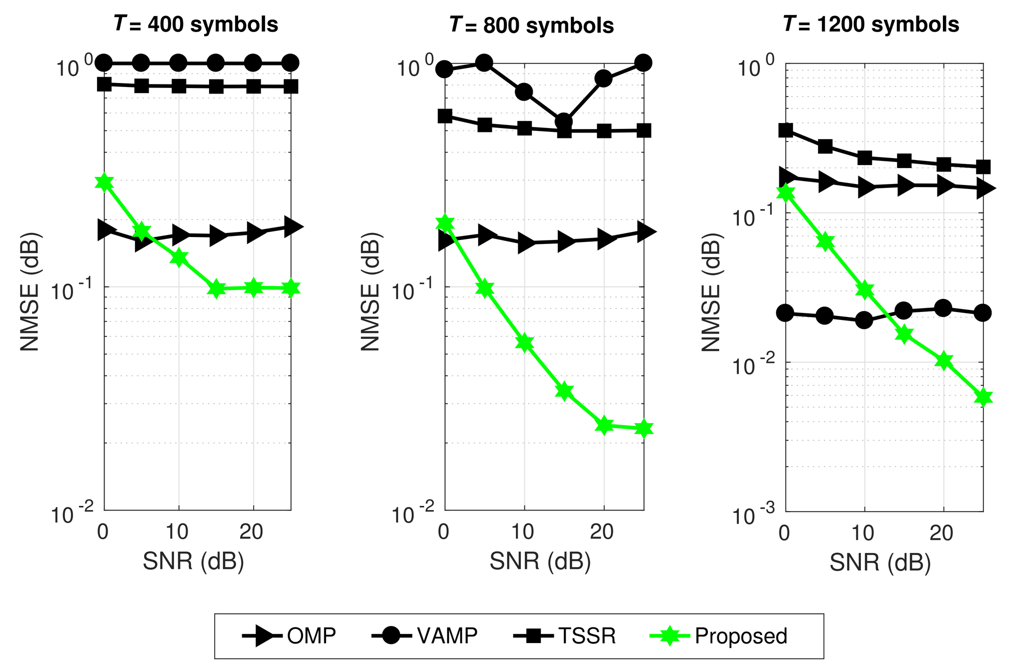


Figure 1: NMSE w.r.t. transmit SNR for a 64×64 MIMO channel with $N_p = 2$ and different T values.

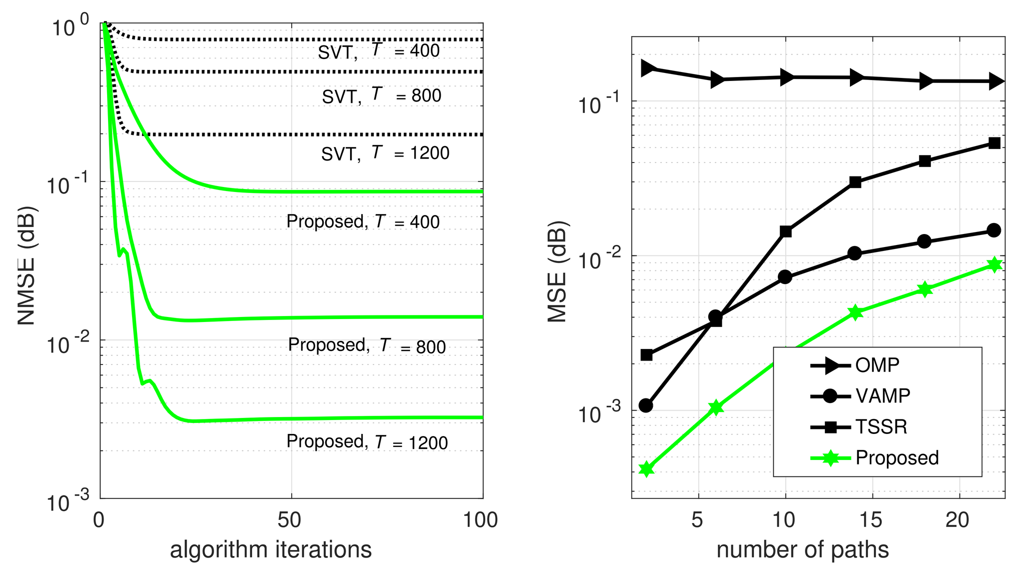


Figure 2: NMSE for a 64×64 MIMO channel and 30dB transmit SNR w.r.t. (i) algorithmic iterations and different T; and (ii) N_p for $T = 2000$.

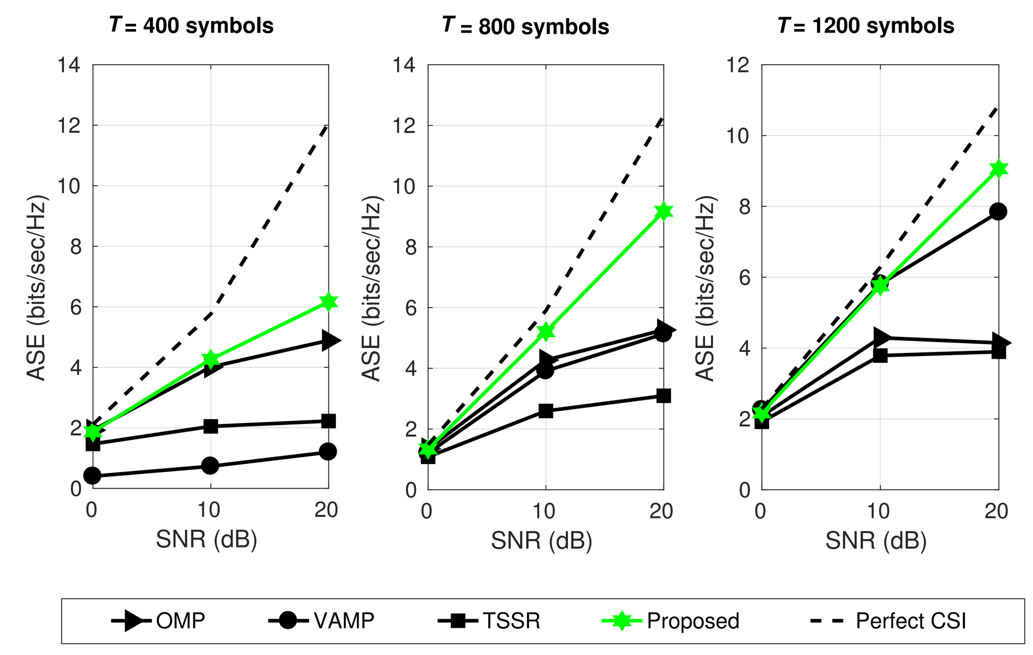


Figure 3: ASE w.r.t. transmit SNR for a 32×32 MIMO channel with $N_p = 2$ and different T values.

Conclusions

The proposed technique

- exploits the properties from **low-rank and sparsity domains jointly**,
- combats effectively **power leakage effect**,
- exhibits **improved performance in terms of NMSE** for channel estimation with **short beam training length**.
- Future work will extend the proposed framework for the **wideband channel model**.

Key References

[1] E. Vlachos, G. C. Alexandropoulos, and J. Thompson, "Massive MIMO channel estimation for millimeter wave systems via matrix completion," *IEEE Signal Processing Letters*, vol. 25, no. 11, pp. 1675–1679, Nov 2018.

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[3] X. Li, J. Fang, H. Li, and P. Wang, "Millimeter wave channel estimation via exploiting joint sparse and low-rank structures," *IEEE Trans. Wireless Commun.*, vol. 17, no. 2, pp. 1123–1133, Feb. 2018.

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