NON-SEPARABLE QUADRUPLE LIFTING STRUCTURE FOR FOUR-DIMENSIONAL INTEGER WAVELET TRANSFORM WITH REDUCED ROUNDING NOISE

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Two types of data compression

Lossless Data Compression in JPEG 2000

- Original Image
- 5/3 Double Lifting of Integer Wavelet Transform
- Encoding
- Decoding
- Inverse Integer Wavelet Transform
- Reconstructed image = Original Image

Lossy Data Compression in JPEG 2000

- Original Image
- 9/7 Quadruple Lifting of Integer Wavelet Transform
- Encoding part
- Quantization
- Decoding part
- Dequantization
- Inverse Integer Wavelet Transform
- Compressed Image
- Reconstructed Image

- Low compression ratio
- High compression ratio

Two types of data compression

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Rounding Noise in Lifting Structure of Integer Wavelet Transform

(example in double type)

"\(\downarrow 2\)" denotes "rounding" operator
\(z\) : delay
\(\downarrow 2\) : downsampling by 2
\(A_1, A_2\) : coefficients of filter bank

Rounding operators are reduced in this research
Rounding Noise

• The lower the rounding noise, the higher the compression performance

How?
• By introducing the ‘non-separable’ structure

Effect

Rounding noise = y - \hat{y}
Two Lifting Structure
(example in 2D for double type)

<table>
<thead>
<tr>
<th></th>
<th>Separable</th>
<th>Non-separable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounding Operators</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Reduced!</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Quadruple 1D IWT

$A_1 A_2 A_3 A_4$

$X \cdot 2^F \downarrow 2 \rightarrow X_0$

$\downarrow 2 \rightarrow X_1$

$A_1 \quad A_2 \quad A_3 \quad A_4$

$Y_L \quad Y_H$

$Z \quad \downarrow 2$

$\diamond$: rounding operator

$z$: delay

$\downarrow 2$: downsampling by 2

$A_1, A_2, A_3, A_4$: coefficients of filter bank
Quadruple Separable 4D

\[ A_1A_2A_3A_4 \quad B_1B_2B_3B_4 \quad C_1C_2C_3C_4 \quad D_1D_2D_3D_4 \]
Quadruple Separable 4D

\[ A_1A_2A_3A_4 \quad B_1B_2B_3B_4 \quad C_1C_2C_3C_4 \quad D_1D_2D_3D_4 \]
Quadruple Separable 4D

\[ A_1A_2A_3A_4 \quad B_1B_2B_3B_4 \quad C_1C_2C_3C_4 \quad D_1D_2D_3D_4 \]
Possible combinations of the structures

<table>
<thead>
<tr>
<th>x-dimension</th>
<th>y-dimension</th>
<th>z-dimension</th>
<th>t-dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1A_2A_3A_4$</td>
<td>$B_1B_2B_3B_4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1A_2(A_3A_4B_1B_2)_{2D}$</td>
<td>$B_3B_4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(A_1A_2B_1B_2)_{2D}$</td>
<td>$(A_3A_4B_3B_4)_{2D}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vdots$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

24 Possible Combinations

<table>
<thead>
<tr>
<th>x-dimension</th>
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<tr>
<td>$A_1A_2A_3A_4$</td>
<td>$B_1B_2B_3B_4$</td>
<td>$C_1C_2C_3C_4$</td>
<td></td>
</tr>
<tr>
<td>$(A_1A_2B_1B_2C_1C_2)_{3D}$</td>
<td>$(A_3A_4B_3B_4C_3C_4)_{3D}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1A_2(A_3A_4B_1B_2)_{2D}$</td>
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40320 Possible Combinations

<table>
<thead>
<tr>
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<td>$D_1D_2D_3D_4$</td>
</tr>
<tr>
<td>$A_1A_2A_3A_4(B_1B_2C_1C_2D_1D_2)_{3D}$</td>
<td>$(B_3B_4C_3C_4D_3D_4)_{3D}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_1A_2(A_3A_4B_1B_2)_{2D}$</td>
<td>$(B_3B_4C_1C_2)_{2D}$</td>
<td>$(C_3C_4D_1D_2)_{2D}$</td>
<td>$D_3D_4$</td>
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<tr>
<td>$\vdots$</td>
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</table>

$2.092 \times 10^{13}$ Possible Combinations
How to find the best structure from the $2.092 \times 10^{13}$ structures?

By maintaining the original lifting structure, which is

$$A_1 A_2 A_3 A_4 B_1 B_2 B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4$$

and turn it into the non-separable structure

$$A_1 A_2 (A_3 A_4 B_1 B_2)_{2D} (B_3 B_4 C_1 C_2)_{2D} (C_3 C_4 D_1 D_2)_{2D} D_3 D_4$$
Comparison of the structures

4D DWT (9,7) Quadruple Lifting Structure:

Separable 4D (Existing I):

\[ A_1 A_2 A_3 A_4 B_1 B_2 B_3 B_4 C_1 C_2 C_3 C_4 D_1 D_2 D_3 D_4 \]

Non-separable 1D and 3D for 4D (Existing II):

\[ A_1 A_2 A_3 A_4 (B_1 B_2 C_1 C_2 D_1 D_2)_{3D} \]
\[ (B_3 B_4 C_3 C_4 D_3 D_4)_{3D} \]

Non-separable 1D and 2D for 4D (Proposed):

\[ A_1 A_2 (A_3 A_4 B_1 B_2)_{2D} (B_3 B_4 C_1 C_2)_{2D} (C_3 C_4 D_1 D_2)_{2D} D_3 D_4 \]

<table>
<thead>
<tr>
<th>Structure</th>
<th>Rounding Operators</th>
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</thead>
<tbody>
<tr>
<td>Separable 4D (Existing I)</td>
<td>192</td>
</tr>
<tr>
<td>Non-separable 3D (Existing II)</td>
<td>96</td>
</tr>
<tr>
<td>Non-separable 2D (Proposed)</td>
<td>96</td>
</tr>
</tbody>
</table>
Separable 4D (Existing I)

Cascade of 1D

A_1 A_2 A_3 A_4  B_1 B_2 B_3 B_4  C_1 C_2 C_3 C_4  D_1 D_2 D_3 D_4

192 rounding operators
4D Wavelet composed of 1D and Non-separable 3D (Existing II)

Cascade of 1D and two NonSep 3Ds

\[ A_1 A_2 A_3 A_4 \]

\[ (B_1 B_2 C_1 C_2 D_1 D_2)_{3D} \]

\[ (B_3 B_4 C_3 C_4 D_3 D_4)_{3D} \]

96 rounding operators
4D Wavelet Composed of 1D and Non-separable 2D (Proposed)

$A_1 A_2 \quad (A_3 A_4 B_1 B_2)_{2D} (B_3 B_4 C_1 C_2)_{2D} (C_3 C_4 D_1 D_2)_{2D} D_3 D_4$

Cascade of 1D and three NonSep 2Ds

96 rounding operators
Input Data

MRI data
Size: 50 x 224 x 224 x 16

Random signal
Size: 128 x 128 x 32 x 16

Auto Regressive Model (AR)
Size: 256 x 256 x 32 x 16
Evaluation: Average variance of rounding noise in each frequency band

The lower the better

Random Signal
AR Model
4D MRI

Type of data

Variance of Rounding Error

Existing I  Existing II  Proposed

0 2 4 6 8 10 12

The lower the better
Evaluation: Rounding noise in each frequency band

(a) 4D Random Signal

(b) 4D AR Model

(c) 4D MRI
Evaluation: Coding performance in lossy mode

The higher the better the quality of image

Test data: 4D AR Model
### Conclusion

<table>
<thead>
<tr>
<th>Structure</th>
<th>Rounding Noise</th>
<th>Coding Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separable 4D (Existing I)</td>
<td>★ ★</td>
<td>★ ★ ★</td>
</tr>
<tr>
<td>Non-separable 3D (Existing II)</td>
<td>★</td>
<td>★ ★</td>
</tr>
<tr>
<td>Non-separable 2D (Proposed)</td>
<td>★ ★ ★</td>
<td>★ ★ ★ ★</td>
</tr>
</tbody>
</table>

*Between the Existing I and Proposed methods,*

- Number of rounding operators: 50 [%] \(\downarrow\)
- Variance of rounding noise: 16.09 [%] \(\downarrow\)
- Coding performance: 2.57[dB] \(\uparrow\)
How to derive the non-separable?
Derivation of the ‘non-separable’ structure: Basic properties for modification

Property I

Property II
Derivation of the ‘non-separable’ structure: Derivation process for 2D

\[ A_1 A_2 B_1 B_2 \]

\[ (A_1 A_2 B_1 B_2)_{2D} \]

Rearranging

Applying property I and II

Same process is applied to 3D and 4D
How to find the best structure from the $2.092 \times 10^{13}$ structures?

Create six "Rules"

to exclude Unnecessary structures

focus on only 7 candidates

find the Best structure

which has the minimum rounding noise
The six “Rules”

1. \(B_1B_2A_3A_4A_1A_2B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4\) - Excluded
   - Same rounding noise

2. \(A_2A_1A_3A_4B_1B_2B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4\)
   - Not \((9,7)\) quadruple wavelet

3. \(A_1A_2B_1B_2C_1C_2D_1D_2A_3A_4B_3B_4C_3C_4D_3D_4\)
   - Large rounding noise

Permitted

4. \((A_1A_2)(A_3A_4)(B_1B_2)(B_3B_4)(C_1C_2)(C_3C_4)(D_1D_2)(D_3D_4)\)

5. \(A_1A_2(A_3A_4B_1B_2)_{2D}B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4\)

6. \(A_1A_2(A_3A_4B_1B_2C_1C_2)_{3D}B_3B_4C_3C_4D_1D_2D_3D_4\)
The rounding noise is experimentally investigated
Autoregressive model

AR model is created to make spectrum random input to become the same as image spectrum

AR Model

\[ \frac{W(z)}{1 - \rho z^{-1}} = H(z) \]

\[ H(z): \rho = 0.99 \]

\[ \rho = 0 \]

\[ \rho = 0.9 \]

AR Model Spectrum = Image Spectrum
Analysis on Experimental Results

Effect of One Rounding Operator

The lower the better

Rounding noise is amplified

variance of Rounding Error

Rounding Number

Existing I  Existing II  Proposed
Tested data: MRI [R-D Curve]

- PSNR [dB]
- Bits per pixel [bpp]

- Existing I
- Existing II
- Proposed
Why Wavelet?
International Standard of Image Compression

JPEG
• Discrete Cosine Transform (DCT)

JPEG2000
• Discrete Wavelet Transform (DWT)

All basis have the same length

Long in low frequency, Short in high frequency

Data can be compressed

- long gradation
- small edges

Existing 4D Data Compression Methods

- fMRI Image
  - Method: Motion Compensation [V. Sanchez et al, 2009]
- fMRI Image
- 4D Remote Sensing
- 4D Geometry
## Proposal

<table>
<thead>
<tr>
<th></th>
<th>Quadruple (9,7)</th>
</tr>
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<tbody>
<tr>
<td><strong>2D</strong></td>
<td></td>
</tr>
<tr>
<td>Separable</td>
<td>JPEG 2000</td>
</tr>
<tr>
<td>Non-separable</td>
<td>ICIP ’09</td>
</tr>
<tr>
<td><strong>3D</strong></td>
<td></td>
</tr>
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</tr>
<tr>
<td>Non-separable</td>
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</tr>
<tr>
<td><strong>4D</strong></td>
<td></td>
</tr>
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<td>Non-separable</td>
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</tbody>
</table>

Implementation of Integer Wavelet Transform in JPEG 2000 with Lifting Structure

<table>
<thead>
<tr>
<th>Advantage</th>
<th>Problem</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Perfect reconstruction</td>
<td>• Rounding noise existed in the lifting structure</td>
<td>• Lossless =&gt; Double (5,3)</td>
</tr>
<tr>
<td>• Signal is recovered without loss</td>
<td></td>
<td>• Lossy =&gt; Quadruple (9,7)</td>
</tr>
</tbody>
</table>
Data trends: The increased of data dimensions

1D
- Text
- Sound

2D
- Still Image

3D
- Video
- Medical Image
- Light Field Image
- Hyperspectral Image

4D
- Medical Image
- 3D stereo video
- Hyperspectral image

Large File Size
Compression is necessary