Exemplar-Embed Complex Matrix Factorization for Facial Expression Recognition

Viet-Hang Duong1, Yuan-Shan Lee1, Jian-Jiun Ding2, Bach-Tung Pham1, Manh-Quan Bui1, Pham The Bao1, and Jia-Ching Wang1

1 Department of Computer Science and Information Engineering, National Central University, Jhongli, Taiwan
2 Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan
3 Faculty of Mathematics & Computer Sciences, University of Science, Ho Chi Minh City, Viet Nam

1. Major Contribution

✓ This paper presents an image representation approach, which is called exemplar-embed complex matrix factorization (EE-CMF) and is based on matrix factorization in the complex domain.
✓ The proposed EE-CMF approach effectively improve the performance of facial expression recognition.
✓ Wirtinger’s calculus was employed to determine derivatives and the gradient descent method was utilized to solve the complex optimization problems.

2. Existing Representation Methods

- Principal component analysis (PCA) based method
- Linear discriminant analysis (LDA)
- Nonnegative matrix factorization (NMF)
- Semi-NMF, Convex-NMF, Cluster NMF

Nonnegative Matrix Factorization

Given an $N \times M$ input data matrix $X = (x_1, x_2, \ldots, x_M)$, where $N$ is the number of facial images and each column $x_m$ corresponds to an image with size $N = a \times b$. The NMF problem is to find $W$ and $V$ that can minimize the following objective function:

$$\min_{W,V} \text{O}_{\text{nmf}}(W,V) = \frac{1}{2} \|X - WV\|^2_F$$

To relax the constraint of nonnegative data, EE-NMF imposes a constraint that the column vectors of $W$ must lie within the column space of $X$, i.e., $W = XA$.

The factors $V$ and $A$ are updated as follows:

$$A = A_{\text{nmf}} = \sqrt{V^*X}V + (X^*X)V^* = A_{\text{nmf}}(X^*X) + (X^*X)V^*$$
$$V = V_{\text{nmf}} = A^*X + A^*X^*AV$$

where $X^*X = (X^*X) - (X^*X)$

3. Wirtinger’s Calculus and Complex Optimization

✓ If $g(z, z^*) = f(x, y)$ where $z = x + jy$:

$$\frac{\partial g}{\partial z} = \frac{1}{2} \left( \frac{\partial f}{\partial x} + \frac{1}{i} \frac{\partial f}{\partial y} \right)$$
$$\frac{\partial g}{\partial z^*} = \frac{1}{2} \left( \frac{\partial f}{\partial x} - \frac{1}{i} \frac{\partial f}{\partial y} \right)$$

For the real-valued function $f(Z, Z^*)$, we have

$$\Delta f(Z, Z^*) \approx \langle \nabla_Z f, \Delta Z \rangle + \langle \nabla_{Z^*} f, \Delta Z \rangle$$

$$\Delta f(Z, Z^*) \approx \langle \nabla_Z f, \Delta Z \rangle + \langle \nabla_{Z^*} f, \Delta Z \rangle = \text{Re} \left\{ \langle \nabla_Z f, \Delta Z \rangle \right\}$$

Optimal Solution: Wirtinger’s calculus

First, fix $W$ and solve $V$ by

$$\min_V f(V) \text{ where } f(V) = \frac{1}{2} \|Z - ZWV\|^2_F$$

where $V$ be can solved iteratively

$$V_{\text{new}} = V_{\text{old}} - \beta (\nabla_v f(V))$$

$$\nabla_v f(V) = 2 \left( \frac{\partial f(V)}{\partial V} - \frac{\partial f(V)}{\partial (\text{Re } V)} + i \frac{\partial f(V)}{\partial (\text{Im } V)} \right)$$

$$\nabla_v f(V) = -W^*Z^*Z + W^*Z^*ZWV$$

Then, $W$ is updated based on the Moore-Penrose pseudoinverse, $\dagger$, and $W = Z(VZ)^\dagger$ with fixed $V$.

4. Proposed Method

EE-CMF aims at factorizing $Z$ into two matrices, $W \in \mathbb{C}^{N \times F}$ and $V \in \mathbb{C}^{F \times M}$, to satisfy the following objective function:

$$\min_{W,V} \text{O}_{\text{cmf}}(W,V) = \min_{W,V} \frac{1}{2} \|Z - ZWV\|^2_F$$

where $\|Z - ZWV\|^2_F = \text{Trace}(Z - ZWV)^*(Z - ZWV) = \text{Trace}(ZZ^* - V^*W^*Z^*Z + Z^*ZWV + V^*W^*Z^*ZV)$

Optimal Solution: Wirtinger’s calculus

First, fix $W$ and solve $V$ by

$$\min_V f(V) \text{ where } f(V) = \frac{1}{2} \|Z - ZWV\|^2_F$$

where $V$ be can solved iteratively

$$V_{\text{new}} = V_{\text{old}} - \beta (\nabla_v f(V))$$

$$\nabla_v f(V) = 2 \left( \frac{\partial f(V)}{\partial V} - \frac{\partial f(V)}{\partial (\text{Re } V)} + i \frac{\partial f(V)}{\partial (\text{Im } V)} \right)$$

$$\nabla_v f(V) = -W^*Z^*Z + W^*Z^*ZWV$$

Then, $W$ is updated based on the Moore-Penrose pseudoinverse, $\dagger$, and $W = Z(VZ)^\dagger$ with fixed $V$.

5. Simulation Results

Cohn–Kanade (CK) database

JFFA database

CK database training: set $= 1 : 4$

CK database training: set $= 2 : 3$

JFFA database training: set $= 1 : 4$

JFFA database training: set $= 2 : 3$