MIMO Radar Target Detection Using Low-Complexity Receiver

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**Background**

- A large number of transmitters are often available.
- Processing additional transmitters requires additional hardware or software complexity at the receivers.
- It is necessary to design the receivers wisely to control the complexity cost.

**Received Signal Model**

- The received target-present signal at the \(n\)-th receiver:
  \[ r_n(t) = \sum_{m=1}^{M} \gamma_{mn} s_m(t) + w(t), \]
- \(\gamma_{mn}\) is a complex white Gaussian variable.
- \(s_m(t)\) is the output of the \(m\)-th transmitter.
- \(w(t)\) is the zero-mean Gaussian temporally white clutter-plus-noise such that \(E\{w(t)w^*(t)\} = N_0\delta(t - \tau)\).

**Detection Problem**

- Hypothesis Testing Problem
  \[ H_0: r_n(t) = w(t) \]
  \[ H_1: r_n(t) = \sum_{m=1}^{M} \gamma_{mn} s_m(t) + w(t), \]
- Optimal Test Statistic (TS)
  \[ T = r_n(t)^* (\Sigma^{-1}_n - \Sigma^{-1}) r_n(t) \]
  where
  \[ \Sigma_n = N_0 I + \mathbb{E}\{\Psi^* \Psi\} \]

**Transmitter selection**

- Special Case
  - Spatial white reflection coefficients and clutter-plus-noise
    \[ \Lambda = \mathbb{E}\{\xi(\xi^*)\} = \mathbb{E}\{\xi(\xi^*)\} = \mathbb{E}\{\xi(\xi^*)\} = \mathbb{E}\{\xi(\xi^*)\} \]
  - Orthogonal waveforms
    \[ \int_{-T/2}^{T/2} s_m(t) s_n(t) dt = 0 \quad \text{for} \ m \neq n. \]

**MSCNR-based Selection**

- Define the SCNR of the \((m,n)\)-th path as
  \[ \psi_{mn} = \frac{E\{s_m(t)\}^2}{N_0 R_{mn} R_{nn}} \]
  The TS can be rewritten as
  \[ T_n = \sum_{m=1}^{M} \psi_{mn} J_m(t) \]
  where \(\psi_{mn} = \frac{E\{s_m(t)\}^2}{N_0 R_{mn} R_{nn}}, \quad J_m(t) = \sum_{n=1}^{N} \gamma_{mn} s_n(t) + w(t). \]
- Lemma: Denote by \(p_{mn}^{(1)}, p_{mn}^{(2)}\) the decreasing sequence of nonnegative \(p_{mn}^{(1)} \rightarrow p_{mn}^{(2)} \rightarrow \cdots \rightarrow p_{mn}^{(M)}\), where \(R_{mn} = N_0 \Delta f \equiv \rho_0 + \rho_1 \beta \) for \( \rho_0, \rho_1 \in \mathbb{R}^{+}\) if \( \rho_{mn}^{(1)} \geq \rho_{mn}^{(2)} \) then
  \[ P_{mn}(\rho_{mn}^{(i)}) \geq 2 P_{mn}(\rho_{mn}^{(i+1)}). \]

**Numerical Examples**

- Example 1: Special case
  - Transmitters: \((x_3, y_3) = ((-1, 0) \text{ km}, (0, 2) \text{ km}), \text{ and } (x_2, y_2) = (0, 2) \text{ km}.\)
  - Receivers: \((x_1, y_1) = ((1, 0) \text{ km})\text{ and } (x_2, y_2) = ((0, 1) \text{ km}).\)
  - Frequency vector \(f = [1, 1, 2, 1] \text{ and } F_y = 10^{-2}. \)

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**Conclusion**

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