Introduction and Motivation

- **Goal:** state estimation from sequential measurements
- **A Motivating Example:**

**Figure 1: Autonomous Cars**

Reproduced from [1]

- **States:** positions and velocities of cars
- **Measurements:** LIDAR and/or short-range RADAR
- **Major challenges:**
  - Higher dimensional states, informative measurements
  - Multimodality in process and measurement noises

Problem Formulation

**Dynamic Model:**

\[
p(x_k|x_{k-1}) = \sum_{n=1}^{M} P(x_k|x_{k-1}, z_k) \]

**Measurement Model:**

\[
p(z_k|x_k) = \sum_{n=1}^{N} P(z_k|x_k, c_k)\]

Particle Filter

- Employs sequential importance sampling and resampling.
- Poor representation of posterior distribution for high dimensionality of state and/or informative measurements.

Particle Flow

- Particle flow [1] involves solution of a differential equation to migrate particles form the prior distribution to the posterior distribution.

Particle Flow Particle Filter (PFPPF)

PFPPF of [2] constructs its proposal distribution based on a modified deterministic particle flow applied to the samples from the prior.

The deterministic mapping \( \eta_i^k = T'(\eta_i^k, x_{k-1}^i, z_k) \) is invertible, so the proposal can be evaluated as:

\[
q(x_i^k|z_{i-1}, x_k) = \frac{p(x_i^k|z_{i-1}, x_k)}{\det(D(T'(\eta_i^k, x_{k-1}^i, z_k)))}
\]

where, \( T' \) is the Jacobian function of the mapping \( T \).

Proposed Algorithm (PFPPF-GMM)

Our design of joint proposal distribution of \((x_i, d_i, c_i)\):

\[
q(x_i^k, d_i, c_i|z_{i-1}, d_{i-1}, c_{i-1}, z_k) = P(d_i)P(c_i|q(x_i|z_{i-1}))
\]

Conditioned on the auxiliary variables \((d_i, c_i)\), invertible particle flow of [2] is used to construct \(q(x_i^k|x_{k-1}^i, d_i, c_i, z_k)\).

Importance weights for the joint posterior:

\[
\omega_i^k = \frac{p(x_i^k, d_i, c_i|z_k)}{q(x_i^k|d_i, c_i, z_k)} = \frac{p(x_i^k|x_{k-1}^i, d_i, c_i) p(z_i|d_i, c_i)}{q(x_i^k|x_{k-1}^i, d_i, c_i, z_k)}
\]

Estimation via importance sampling:

\[
p(x_i|z_k) \approx \sum_{i=1}^{N} w_i^k \delta(x_i - x_i^k)
\]

Numerical Experiments and Results

We compare the novel PFPPF-GMM algorithm with:

- Kalman filter type algorithm for unimodal posterior (UKF)
- Particle flow algorithms for unimodal posterior (LEDH, EDH)
- Particle flow particle filters for unimodal posterior (PFPPF (LEDH, PFPPF (EDH))
- Bootstrap Particle Filter (BPF)
- Filtering algorithms for multimodal posteriors (EKF-GMM, PF-GMM, GSPF)

Linear Dynamic and Measurement Models:

- Dimension of state, \( d = 64 \).
- Dynamic model:
  \[
g_{ij}(x_{j-1}) = 0.5x_{j-1} + 8 \cos(1.2(j - 1))
\]
- Measurement model:
  \[
h_{ij}(x_j) = \sum_{c=1}^{d} \xi_{ij}^c, 1 \leq c \leq d.
\]

We presented a novel particle filter for Gaussian mixture noise models.

- Successfully tracks multiple modes of the posterior distribution.
- The proposed filter offers impressive performance in higher dimensions and in settings with low measurement noise.

**Figure 2:** Particle filtering

**Figure 3:** Particle flow

**Figure 4:** Average MSE vs Execution time

**Figure 5:** Average MSE vs Execution time

**References**

