Goal: Create sound fields from directional sources in the listening areas of focused sources with linear loudspeaker array.

Question: How can we introduce directivity to virtual sound sources between a loudspeaker array and audience seats?

Proposed: 1. Create a cluster of focused sources to form multipoles in the listening area.
2. Analytical conversion of circular harmonics into weights for each multipole.

Results: Reproduced directivity on the basis of multipoles comprising multiple focused sources.

Introduction

- Virtual sound sources closing in on the audience seats for live viewings and theaters
- Focused source method [1]: Creating sound fields from monopole sources between audience seats and a linear loudspeaker array
- Sound field and driving function

\[ P(r) = \int_\mathbb{D} D(x_0, x) G_{zd}(r - x) dx_0 \]

\[ D(x_0, x) = \frac{-jk}{2} c_0 \nabla_1 \nabla_2 |x_0 - x| H_0^{(2)}(k|x_0 - x|) \]

- Limited listening areas due to the nature of sound focusing
- Accurate reproduction of sound directivities help the audience feel more realistic sensations [2]
- Our goal: Creating sound fields from directional sources in the listening areas of focused sources

Proposed method

- Basic idea: The analytical conversion of circular harmonic coefficients into weights for each multipole
  - Circular harmonic decomposition of arbitrary sound fields in 2D

\[ S(x) = \sum_{n,m} S_n^{(2)} H_n^{(2)}(kr) e^{j(n \alpha + m \beta x)} = \sum_{n,m} S_n^{(2)} H_n^{(2)}(kr)(\cos \alpha + j \sin \alpha)^n \]

- Euler’s formula

By applying binomial expansion

\[ S(x) = \sum_{n,m} S_n^{(2)} H_n^{(2)}(kr) \cdot \left( \begin{array}{c} N \ \text{m} \\ m \end{array} \right) \cdot (\cos \alpha - j \sin \alpha)^n \]

- Taylor’s expansion at the origin of the coordinate

\[ S(x) = \sum_{n,m} \frac{\partial^n S(x)}{\partial x^n \partial y^m} |_0 \cdot \frac{x^n \cdot y^m}{m!} \cdot \sum_{n,m} \frac{\partial^n S(x)}{\partial x^n \partial y^m} |_0 \cdot \frac{\cos \alpha \cdot \sin \alpha}{m!} \]

- On the unit circle

- By comparing coefficients of the term \( \cos^n \alpha \cdot \sin^m \alpha \)

\[ \frac{\partial^n S(x)}{\partial x^n \partial y^m} |_0 = f'(m+n) \left\{ S_n^{(2)} H_n^{(2)} + (-1)^m S_{n-m}^{(2)} H_n^{(2)} \right\} \]

- The driving functions for sound fields from a sound source based on superposition of multipoles

\[ D(x_0) = \sum_{n,m} \frac{\partial^n S(x)}{\partial x^n \partial y^m} |_0 \cdot \frac{g^{(2)}_{n,m}(x_0, x^n)}{m! \cdot n!} \cdot D(x_0 \cdot x^n \cdot y^m) \]

- On the unit circle

\[ X^{n,m} = \left\{ (x^n, y^m) | x^n = x^{n-1} \pm \Delta, y^m = y^m \pm \Delta \right\} \]

\[ G^{n,m} = g^{(2)} \cdot g^{(2)} | x^{n-1} = \pm 1 \cdot g^{n-1} \]

Computer simulations

- Sound field reproduction of monopoles and dipoles comprising focused sources (sine waves)
  - 41-ch linear array with 0.1 m intervals, \( f = 1 \) kHz
  - virtual sound sources at \((x, y) = (0, 1)\)

- Directional source on the basis of superposition of multipoles comprising a cluster of focused sources
  - Simulation setup: same as the above cases
  - The order of circular harmonics and multipole: \( N = 4 \)
  - Original sound field: modeled by randomly generated \( S_n^{(2)} \)

Selected references