Feature LMS Algorithms

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1 Introduction

2 Feature LMS Algorithms

3 Results

4 Conclusions
Outline

1. Introduction
2. Feature LMS Algorithms
3. Results
4. Conclusions
Motivations

- When we have some *a priori knowledge* about the unknown system $w_o$:
  - We can exploit it for accelerating the convergence rate

- When we want to obtain an estimate of the unknown system $w_o$ such that a determined characteristic for the estimate is desirable:
  - Lowpass, highpass, linear phase
Feature LMS (F-LMS) algorithms ⇒ impose some structure on the adaptive filter’s coefficients ⇒ exploit hidden sparsity in system, such as sparsity in linear combination of coefficients

In this paper, we present the F-LMS algorithm for:

- Unknown systems with lowpass narrowband spectrum
- Unknown systems with highpass narrowband spectrum
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4 Conclusions
F-LMS Algorithm: Problem and Solution

- Problem:

\[
\xi_{F-LMS}(k) = \frac{1}{2} |e(k)|^2 + \alpha P(\mathbf{F}(k)\mathbf{w}(k)),
\]

where \( P(\cdot) \) is the sparsity promoting penalty function and \( \mathbf{F}(k) \) is the feature matrix that takes the unknown system to a sparse vector.

- For example, choose function \( P \) to be the \( l_1 \) norm and the feature matrix \( \mathbf{F}(k) \) to be time-invariant \( \mathbf{F} \)

\[
\xi_{F-LMS}(k) = \frac{1}{2} |e(k)|^2 + \alpha \|\mathbf{Fw}(k)\|_1
\]

- Solution:

\[
\mathbf{w}(k+1) = \mathbf{w}(k) + \mu e(k)\mathbf{x}(k) - \mu \alpha \mathbf{p}(k),
\]

where \( \mathbf{p}(k) \in \mathbb{R}^{N+1} \) is the gradient of function \( \|\mathbf{Fw}(k)\|_1 \).
Example I: F-LMS Algorithm for Lowpass Systems

- Unknown system has **lowpass** narrowband spectrum ⇒ its impulse response is **smooth** ⇒ the difference between adjacent coefficients is small
- Choose \( F \in \mathbb{R}^{N \times (N+1)} \) as
  \[
  F = \begin{bmatrix}
  1 & -1 & 0 & \cdots & 0 \\
  0 & 1 & -1 & \cdots & 0 \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  0 & 0 & \cdots & 1 & -1
  \end{bmatrix} \Rightarrow Fw(k) \text{ is a sparse vector}
  \]
- Therefore, \( p(k) = [p_0(k) \cdots p_N(k)]^T \) is given by
  \[
  \begin{cases}
  p_i(k) = \text{sgn}(w_0(k) - w_1(k)) & \text{if } i = 0, \\
  p_i(k) = -\text{sgn}(w_{i-1}(k) - w_i(k)) + \text{sgn}(w_i(k) - w_{i+1}(k)) & \text{if } i = 1, \cdots, N - 1, \\
  p_i(k) = -\text{sgn}(w_{N-1}(k) - w_N(k)) & \text{if } i = N.
  \end{cases}
  \]
Example II: F-LMS Algorithm for Highpass Systems

- Unknown system has highpass narrowband spectrum $\Rightarrow$ adjacent coefficients have similar absolute values, but with opposite signs
- Choose $F \in \mathbb{R}^{N \times (N+1)}$ as

$$
F = \begin{bmatrix}
1 & 1 & 0 & \cdots & 0 \\
0 & 1 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & 1
\end{bmatrix}
\Rightarrow Fw(k) \text{ is a sparse vector}
$$

- Therefore, $p(k) = [p_0(k) \cdots p_N(k)]^T$ is given by

$$
P_i(k) = \begin{cases} 
p_i(k) &= \text{sgn}(w_0(k) + w_1(k)) & \text{if } i = 0, \\
p_i(k) &= \text{sgn}(w_{i-1}(k) + w_i(k)) + \text{sgn}(w_i(k) + w_{i+1}(k)) & \text{if } i = 1, \cdots, N - 1, \\
p_i(k) &= \text{sgn}(w_{N-1}(k) + w_N(k)) & \text{if } i = N.
\end{cases}
$$
More Examples for Feature Matrix

- When unknown system is the result of **upsampling** a lowpass system by a factor of $L$ (e.g., $L = 2$)

\[
F = \begin{bmatrix}
1 & 0 & -1 & 0 & \cdots & 0 \\
0 & 1 & 0 & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & 0 & -1
\end{bmatrix}.
\]

- When unknown system is the result of **interpolating** a highpass system by a factor $L$ (e.g., $L = 2$)

\[
F = \begin{bmatrix}
1 & 0 & 1 & 0 & \cdots & 0 \\
0 & 1 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & 0 & 1
\end{bmatrix}.
\]
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Scenario: System Identification

- Algorithms tested: LMS and F-LMS algorithms
- Input signal: $\mathbf{x} \sim \mathcal{N}(0, 1)$
- Filter order: $N = 39$, i.e., 40 coefficients
- $\mathbf{w}(0) = [0, \cdots, 0]^T$
- $\alpha = 0.05$
- SNR: 20 dB
- Unknown lowpass system: $\mathbf{w}_{o,l} = [0.4, 0.4, \cdots, 0.4]^T$
- Unknown highpass system: $\mathbf{w}_{o,h} = [0.4, -0.4, 0.4 \cdots, -0.4]^T$
F-LMS Algorithm Identifying Unknown System with Lowpass Spectrum

- Unknown lowpass system: $w_{o,l} = [0.4, 0.4, \cdots, 0.4]^T$

Figure: Learning (MSE) curves
F-LMS Algorithm Identifying Unknown System with Highpass Spectrum

- Unknown highpass system: $w_{o,h} = [0.4, -0.4, 0.4, \ldots, -0.4]^T$

![Graphs showing MSE vs. Number of Iterations for LMS and F-LMS algorithms](a) $\mu_{LMS} = \mu_{F-LMS} = 0.03$

(b) $\mu_{LMS} = 0.01, \mu_{F-LMS} = 0.03$

**Figure:** Learning (MSE) curves
F-LMS Algorithm Identifying Unknown Block Sparse System

- Block sparse system with lowpass narrowband spectrum

![Graphs showing learning (MSE) curves](image)

(a) Unknown system

(b) $\mu_{\text{LMS}} = \mu_{\text{F-LMS}} = 0.03$

Figure: Learning (MSE) curves
F-LMS Algorithm Identifying Unknown Block Sparse System

- Block sparse system with highpass narrowband spectrum

Figure: Learning (MSE) curves
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Conclusions

- In this presentation:
  - We have proposed a family of algorithms called feature LMS algorithm
    - We have presented some examples of the F-LMS algorithms for exploiting the lowpass and highpass characteristics of unknown systems
    - Some other characteristics can be exploited (e.g., linear phase)
  - The F-LMS algorithms have some advantages such as higher convergence rate or lower steady-state MSE
  - The computational complexity of the proposed F-LMS algorithms are close to that of the LMS algorithm
Conclusions

Thank You!