The objective function in (3a) can be rewritten as

where $y_i = \mathbf{u}^T \mathbf{s}$.

Using the Cauchy-Schwarz inequality, we can obtain

Note that $\mathbf{A}^T \mathbf{Q}_u \mathbf{A} \mathbf{s}$, in (4) is singular. To improve the accuracy, as in [1], we also introduce a penalty term $\lambda \mathbf{D}^T \mathbf{D}$ into the objective function and add the second-order cone (SOC) constraints

In this cluster, let $\mathbf{s}$ be the reference node. The TDOA measurements are

Then the maximum likelihood estimator (MLE)

and the maximum likelihood estimate (MLE)

Next, (2c) can be written as

where $\mathbf{e} = \mathbf{A} \mathbf{s} + \mathbf{n}$.

The objective function in (3a) can be rewritten as

where $\mathbf{D} = \mathbf{D}_{\mathbf{s}} \mathbf{D}_{\mathbf{I}}^{-1} \mathbf{D}_{\mathbf{s}}$.

There are four clusters TDOA measurements, and each cluster has two nodes. The positions of the sensor node are $[0.5,0.1], [0.1,0.5], [0.0,0.9], [0.9,0.0], [0.9,0.9]$. We set $K = 2, \mathbf{n}_0 = 10^{-3}, \mathbf{y}_0 = 10^{-6}, \gamma = 10^{-4}, \delta = 10^{-4}$ for the computation of (8) and (13). The proposed SDP algorithm is implemented by CVX toolbox, using 500Dolphins as a solver, and the precision is set to be 1e-6.

REFERENCES


ROBUST LOCALIZATION ALGORITHM FOR NODES' POSITION ERRORS

In the previous discussion, the sensor positions are accurate. However, in practical cases, there exist the sensor position errors [2]. The obtained but erroneous sensor position can be expressed as $\mathbf{s} = \mathbf{s}^* + \delta \mathbf{s}$, where $\delta \mathbf{s}$ is the sensor position error, which is modeled as Gaussian white noise with covariance matrix $\mathbf{C}_\delta$.

Under the condition of independent noises $\mathbf{D}_{\mathbf{s}}$ and $\mathbf{n}_0$, the MLE problem can be written as

The above formulation can be rephrased as

Let $Y = X \mathbf{X}$. Similar to the derivation of (8), we give the robust SDP localization algorithm

where $\mathbf{D}_{\mathbf{s}}$ is the sensor position error, which is modeled as Gaussian white noise with covariance matrix $\mathbf{C}_\delta$.

Let $Y = X \mathbf{X}$. Similar to the derivation of (8), we give the robust SDP localization algorithm

$$
\min_{\mathbf{D}_{\mathbf{s}}, \mathbf{X}} \mathbf{D}_{\mathbf{s}}^T \mathbf{D}_{\mathbf{s}} + \| \mathbf{X} - \mathbf{X}^* \|_F^2 + \lambda \mathbf{X}^T \mathbf{Y} \mathbf{X}
$$

Let $Y = X \mathbf{X}$. Similar to the derivation of (8), we give the robust SDP localization algorithm

$$
\min_{\mathbf{D}_{\mathbf{s}}, \mathbf{X}} \mathbf{D}_{\mathbf{s}}^T \mathbf{D}_{\mathbf{s}} + \| \mathbf{X} - \mathbf{X}^* \|_F^2 + \lambda \mathbf{X}^T \mathbf{Y} \mathbf{X}
$$

$\mathbf{X}^T \mathbf{Y} \mathbf{X} \preceq 0$

SIMULATIONS

There are four clusters TDOA measurements, and each cluster has two nodes. The positions of the sensor node are $[0.5,0.1], [0.1,0.5], [0.0,0.9], [0.9,0.0], [0.9,0.9]$. We set $K = 2, \mathbf{n}_0 = 10^{-3}, \mathbf{y}_0 = 10^{-6}, \gamma = 10^{-4}, \delta = 10^{-4}$ for the computation of (8) and (13). The proposed SDP algorithm is implemented by CVX toolbox, using 500Dolphins as a solver, and the precision is set to be 1e-6.