TOEPLITZ MATRIX COMPLETION FOR DIRECTION FINDING USING A MODIFIED NESTED LINEAR ARRAY

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Motivation

Background
- Recently, a modified nested linear array (MNLA) has been reported for a potential in increasing the degree-of-freedom.
- However, there exist some “holes” in its difference co-array, which results in missing “lags” and limited performance of direction-of-arrival (DOA) estimation.
- The interest of this paper is to tackle the problem caused by the missing lags in MNLA.

Our Approach
- Construct a covariance matrix with Toeplitz Hermitian structure.
- Derive and solve a semidefinite program.
- Construct a covariance matrix with Toeplitz matrix.
- Perform DOA estimation with the obtained Toeplitz covariance matrix.

Signal Model

The MNLA with total sensor number \( L = M + N \), is depicted in Figure 1. The observation data using MNLA can be formed as

\[
\mathbf{x}(t) = \mathbf{B}(t) \mathbf{s}(t) + \mathbf{n}(t)
\]

where \( \mathbf{B} \) is the steering matrix, \( \mathbf{s}(t) \) and \( \mathbf{n}(t) \) denote the signal and noise, respectively. The data covariance matrix is defined as follows

\[
\mathbf{R}_x = E(\mathbf{x}(t)\mathbf{x}(t)^H) = \mathbf{BB}^H + \sigma_n \mathbf{I}
\]

**Remark 1:** In the case of uniform linear array (ULA), the resulting covariance matrix is a Toeplitz Hermitian matrix.

**Remark 2:** Here, the array configuration under consideration is MNLA rather than ULA. Thus, \( \mathbf{R}_x \) is not a Toeplitz matrix, and the DOA estimation performance would be limited whenever it is directly used based on traditional direction finding techniques.

**Remark 3:** To this end, a Toeplitz matrix completion procedure is applied to transform the MNLA covariance matrix to a ULA counterpart before performing DOA estimation.

Methodology

**Progress of Toeplitz Matrix Completion**

- **A four-step scheme:**

  **Step 1:** Construct a Toeplitz matrix \( \mathbf{T}_0 \).
  **Step 2:** Formulate a low-rank matrix recovery problem.
  **Step 3:** Reform the problem into a semidefinite program.
  **Step 4:** Solve the problem.

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**An example:** \( K = 1 \) signal and \( L = 5 \) sensors. The sensor locations are \( \{0, 1, 3, 7, 12\} \).

The corresponding covariance matrix is

\[
\mathbf{R}^{(5)}_x = \begin{bmatrix}
\mathbf{R}_0 & \mathbf{R}_1 & \mathbf{R}_2 & \mathbf{R}_3 & \mathbf{R}_4
\end{bmatrix}
\]

It can be seen that some lags in \( \mathbf{R}^{(5)}_x \), including \( \beta_0(\theta_0), \beta_1(\theta_0), \beta_2(\theta_0), \beta_3(\theta_0), \beta_4(\theta_0) \), are missing. The Toeplitz covariance matrix is constructed as \( \mathbf{T}_0 = \text{toepl}(\mathbf{c}, \mathbf{r}) \), where

\[
\mathbf{c} = [\beta_0(\theta_0), \beta_1(\theta_0), \beta_2(\theta_0), \beta_3(\theta_0), \beta_4(\theta_0), \beta_5(\theta_0), \beta_6(\theta_0), \beta_7(\theta_0), \beta_8(\theta_0), \beta_9(\theta_0), \beta_{10}(\theta_0), \beta_{11}(\theta_0)]
\]

\[
\mathbf{r} = [\beta_0(\theta_0), \beta_1(\theta_0), \beta_2(\theta_0), \beta_3(\theta_0), \beta_4(\theta_0), \beta_5(\theta_0), \beta_6(\theta_0), \beta_7(\theta_0), \beta_8(\theta_0), \beta_9(\theta_0), \beta_{10}(\theta_0), \beta_{11}(\theta_0)]
\]

DOA Estimation with Toeplitz Covariance Matrix

Once the Toeplitz covariance matrix is obtained, classical approaches like multiple signal classification (MUSIC) algorithm can be adopted for DOA estimation.

Results

**Angle Resolution Comparison**

- \( K = 2 \) signals, \( L = 5 \) sensors.
- Angle resolution comparison: SNR is set to \(-5 \text{ dB}, 0 \text{ dB}, \text{ and } 5 \text{ dB (from row 1 to row 3)}, \) and the angle separation \( \Delta_\theta \) between two sources is set to \( 2^\circ, 3^\circ, 6^\circ, \text{ and } 9^\circ \) (from column 1 to column 4).
- It is seen that, the proposed method outperforms the others in any experimental situations.

**DOA RMSE Comparison**

- \( K = 2 \) signals, \( L = 5 \) sensors, 500 snapshots, SNR ranges from \(-4 \text{ dB} \) to \(12 \text{ dB} \).
- \( K = 2 \) signals, \( L = 5 \) sensors, SNR is \(10 \text{ dB} \), the number of snapshots varies from 10 to 500.