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Restoration of Time-Varying Graph Signals Using Deep Algorithm Unrolling

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Contributions

- This paper proposes a restoration method of time-varying graph signals utilizing graph spatiotemporal smoothness.
- Our proposed network learns hyperparameters and filter coefficients thanks to Deep Algorithm Unrolling.
- Signal restoration accuracy improves in terms of RMSE.

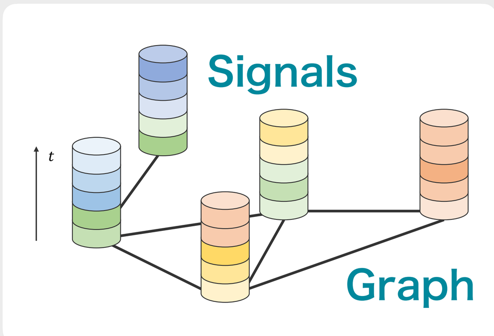
1. Introduction

Graph Signal Processing (GSP)[1]

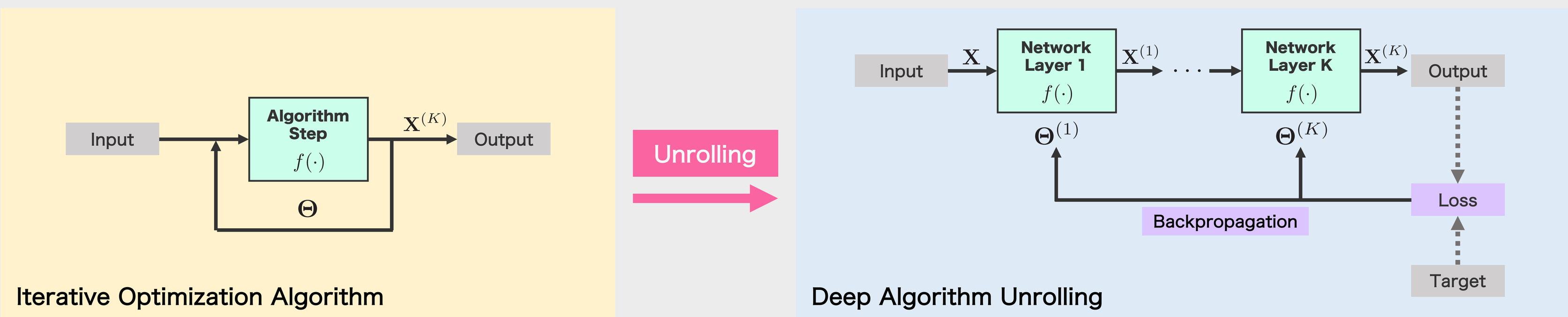
- Signals often have their underlying structures.
- GSP can consider the underlying structure of signals.
  e.g. Transportation network, bioinformatics.

Time-varying graph signals

- A series of time-varying signals defined on a graph is called time-varying graph signal.
  e.g. Human pose, social network.



Deep Algorithm Unrolling (DAU)[2]



- DAU is a method for learning the parameter(s) of the iterative algorithm using deep learning techniques.
- Original iterative algorithm (left) is unrolled as DAU (right) and the parameter(s) of each layer are trained by training data.

Our Purpose

Recover the original time-varying signal from the observed signal

$$\mathbf{Y} = \mathbf{J} \circ (\mathbf{X}^* + \mathbf{V})$$

Proposed Method — Estimation

$\mathbf{Y} \in \mathbb{R}^{(N \times T)}$: Observed time-varying graph signal

$\mathbf{J} \in \{0,1\}^{(N \times T)}$: Sampling operator

$\mathbf{X}^* \in \mathbb{R}^{(N \times T)}$: (Unknown) original time-varying graph signal

$\mathbf{V} \in \mathbb{R}^{(N \times T)}$: Noise term

N : # Nodes

T : # Times

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2. Proposed Method

Main Idea

- Time-varying graph signals are smooth on time and space.
- General iterative method needs to determine hyperparameters.

Design of optimization problem + Unrolled iterations

Design of Optimization Problem

$$\arg\min_{\tilde{\mathbf{X}}} \underbrace{\frac{1}{2}\|\mathbf{J} \circ \tilde{\mathbf{X}} - \mathbf{Y}\|_F^2}_{\text{Data fidelity}} + \underbrace{\frac{\alpha}{2}\mathrm{tr}((\tilde{\mathbf{X}}\mathbf{D}_L)^\top \mathbf{L}(\tilde{\mathbf{X}}\mathbf{D}_L))}_{\text{Spatiotemporal smoothness of L-steps graph signals}}$$

- L-tap FIR filter captures long-term variations.
- Laplacian quadratic form forces signal smoothness over graph.
- Model needs pre-determined hyperparameters

$$\mathbf{D}_L = \begin{bmatrix} -d_1 & & & & \\ -d_2 & -d_1 & & & \\ \vdots & \vdots & & & \\ -d_L & -d_{L-1} & & & \\ 1 & -d_L & \ddots & \vdots & \\ & 1 & \ddots & -d_{L-1} & \\ & & \ddots & -d_L & \\ & & & 1 & \end{bmatrix} \in \mathbb{R}^{T \times (T-L)}$$

$d_1, \ldots, d_L \in \mathbb{R}_{\geq 0} \quad \sum_{l=1}^{L} d_l = 1$

$\mathbf{L} \in \mathbb{R}^{(N \times N)}$: Graph Laplacian

$\Theta = \{\alpha, d_1, \ldots, d_L\}$: Hyperparameters

Unrolled Iterations

Change Θ to $\boldsymbol{\Theta} = \{\Theta^{(1)}, \ldots, \Theta^{(K)}\}$ $\quad K$: # Iterations

- Parameters are learned through supervised learning thanks to DAU.
- Each network layer can use its own parameters.

Algorithm 1 Time-Varying Graph Signal Restoration with DAU

Input: $\mathbf{Y}$: Degraded signal
$\mathbf{J}$: Binary operator defined in (2)
$\mathbf{L}$: Graph Laplacian
K: Number of layers
$\boldsymbol{\Theta} = \{\Theta^{(1)}, \ldots, \Theta^{(K)}\}$: Trainable parameters
Output: $\mathbf{X}^{(k)}$: Restored signals

for $k = 1$ to K **do**
1) Stepsize decision
$\tau = -\frac{g(\Delta\mathbf{X}^{(k-1)}, \nabla\mathbf{X}^{(k-1)})}{g(\Delta\mathbf{X}^{(k-1)}, \nabla\mathbf{X}^{(k-1)} + \mathbf{Y})}$
2) Search direction update
$\mathbf{X}^{(k)} = \mathbf{X}^{(k-1)} + \tau\Delta\mathbf{X}^{(k-1)}$
$\gamma = \frac{\|\nabla\mathbf{X}^{(k)}\|_F^2}{\|\nabla\mathbf{X}^{(k-1)}\|_F^2}$
$\Delta\mathbf{X}^{(k)} = -\nabla\mathbf{X}^{(k)} + \gamma\Delta\mathbf{X}^{(k-1)}$
end for
3) Updating trainable parameter $\boldsymbol{\Theta}$

$\Delta\mathbf{X}^{(k)} = -\nabla\mathbf{X}^{(k)} + \gamma\Delta\mathbf{X}^{(k-1)}$

$\nabla\mathbf{X}^{(k)} = \mathbf{J} \circ \mathbf{X}^{(k)} - \mathbf{Y} + \alpha^{(k)}\mathbf{L}\mathbf{X}^{(k)}\mathbf{D}_L^{(k)}\mathbf{D}_L^{(k)\top}$

▼

improvement of accuracy and convergence speed

References

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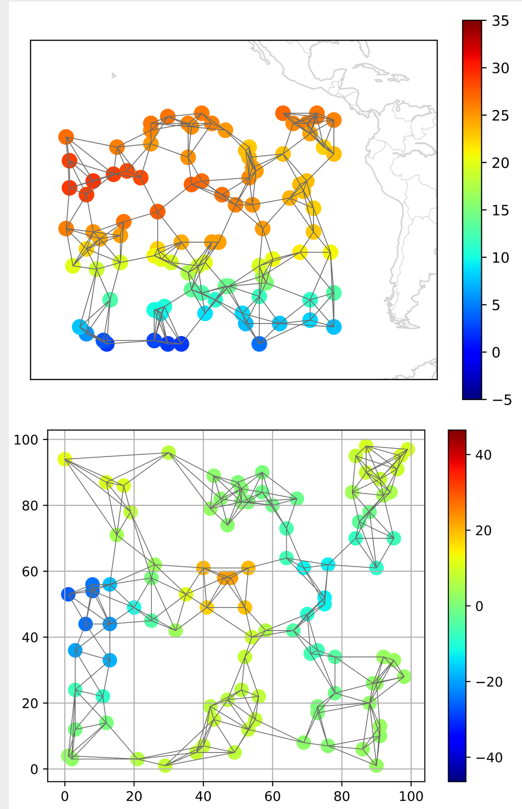
3. Experimental Results

Datasets

- Sea Surface Temperature Dataset
  $N = 100$, k-NN ($k = 5$), $T = 216$
- Synthetic Dataset
  $N = 100$, k-NN ($k = 5$), $T = 600$



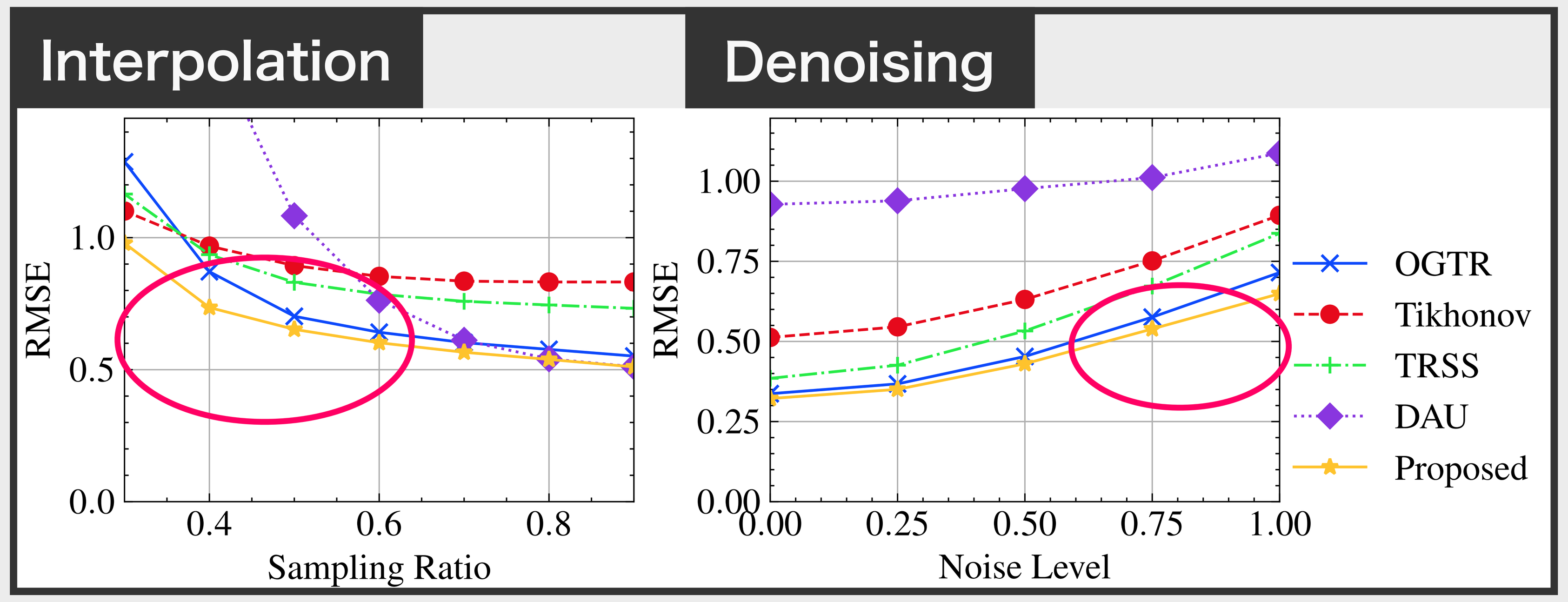
Evaluation Measure

$$loss_{\mathrm{RMSE}} = \sqrt{\frac{1}{NT}\sum_{n,t}(\tilde{X}_{n,t} - X^*_{n,t})^2}$$

Existing Methods

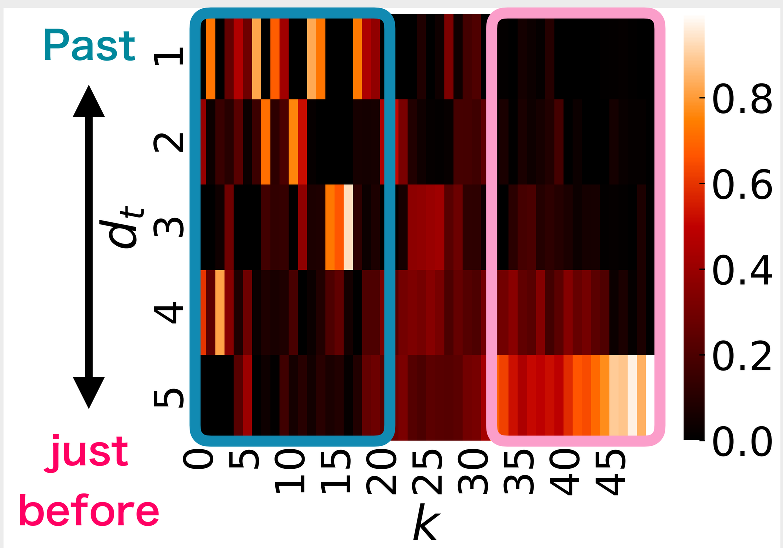
- **Tikhonov** regularization[3]
- Sobolev smoothness (**TRSS**)[4]
- Unsupervised **DAU**[5]
- Graph temporal difference construction (**OGTR**)[6]

Restoration Result (SST dataset)



High restoration performance in Low sampling ratio / High noise level

Consideration of Trained Parameters



Extensive use of past signals
→Captures global characteristics

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Extensive use of just before signals
→Brush up using detailed features

4. Conclusion

- Our proposed method improves the accuracy of restoring time-varying graph signals compared to conventional restoration methods