PLUG-IN MEASURE-TRANSFORMED QUASI-LIKELIHOOD RATIO TEST FOR RANDOM SIGNAL DETECTION

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PROBLEM FORMULATION
Detection of a random signal that lies on a known rank-one subspace:

\[ H_0: \begin{cases} X_n = W_n, & n = 1, \ldots, N \\ Y_m = W_m, & m = 1, \ldots, M \end{cases} \]

\[ H_1: X_n = S_n \alpha + W_n, \quad n = 1, \ldots, N \\ Y_m = W_m, \quad m = 1, \ldots, M \]

GAUSS-GAUSS DETECTOR
- GLRT detector that assumes jointly Gaussian signal and noise.
- Advantages: simple implementation, ease of performance analysis.
- Disadvantage: sensitive to model mismatch.

PLUG-IN NSDD-GLRT
- Conditional GLRT detector that assumes a compound-Gaussian noise.
- The scatter matrix is replaced by noise-only secondary data ML estimate.
- Advantages: robust against heavy-tailed noise outliers.
- Disadvantage: computationally demanding in high-dimensions, does not reject large-norm outliers.

MEASURE TRANSFORMED (MT) GQLRT: BASIC IDEA
- Selects a Gaussian probability model that best empirically fits a transformed probability measure of the data.
- By proper choice of the transform the MT-GQLRT can gain enhanced robustness to outliers.
- Have the computational and implementation advantages of the GGD.

PROBABILITY MEASURE TRANSFORM
Let \( X \sim \mathcal{C}^d \). Define the measure space \((X, \mathcal{S}, P_X)\). Given a non-negative function \( u: \mathbb{C}^d \rightarrow \mathbb{R}_+ \), satisfying \(0 < E[u(X); P_X] < \infty\). A transform \( T_u: P_X \rightarrow Q_u \) is defined as:

\[ T_u[P_X](A) = Q_u(A) = \int_A P_X(X) \, d\phi_u(x), \]

where \( A \in \mathcal{S}, \) and \( \phi_u(x) \Delta \frac{u(x)}{E[u(X); P_X]} \)

The function \( u(\cdot) \) is called the MT-function.

THE MEASURE-TRANSFORMED MEAN AND COVARIANCE
- MT-mean: \( \mu_u^T \Delta E[X \phi_u(X); P_X] \)
- MT-covariance: \( \Sigma_u^{\phi} \Delta E[XX^H \phi_u(X); P_X] - \mu_u^T \mu_u^{\phi} \)

THE MEASURE-TRANSFORMED MEAN AND COVARIANCE
- The mean and covariance of \( Q_u \) can be estimated using samples from \( P_X \),

\[ \mu_u = \frac{1}{N} \sum_{n=1}^N X_n \phi(X_n) \quad \text{and} \quad \Sigma_u = \frac{1}{N} \sum_{n=1}^N X_n X_n^H \phi(X_n) - \mu_u \mu_u^H \]

where \( \phi(X_n) \Delta u(X_n)/\sum_{n=1}^N u(X_n) \)

ROBUSTNESS TO OUTLIERS
- Define the \( r \)-contaminated probability measure:

\[ R_r \Delta (1 - r) P_X + r \delta_y \]

where \( 0 \leq r \leq 1, \ y \in \mathbb{C}^d, \) and \( \delta_y \) is the Dirac probability measure at \( y \).
- The influence function (IF) \([1]\) of an estimator with statistical functional \( H \):

\[ IF_H[y; P_X] \Delta \lim_{r \rightarrow 0} \frac{H[P_r] - H[P_X]}{r} \]

- Describes the effect on the estimator of an infinitesimal contamination at \( y \).
- An estimator is said to be B-robust if its IF is bounded.

Proposition (Boundedness of the influence function)
The influence functions of the empirical MT-mean and MT-covariance are bounded if the MT-function \( u(\cdot) \) and the product \( u(X)u(Y) \) are bounded.

MEASURE-TRANSFORMED (MT) GQLRT: DERIVATION OF THE TEST
- Compares the empirical KL-div between \( Q_u \) and two normal distributions that are characterized by the MT-mean and MT-covariance under each hypothesis:

\[ T_u[P_X](A) - Q_u(A) \sim \int_A P_X(X) \phi_u(x) \]

- Equivalent test-statistic under any MT-function \( u(x) \rightarrow u(P_X^H x), \ \nu: \mathbb{C}^d \rightarrow \mathbb{R}_+ \)

\[ T_u \sim \frac{\sigma^2}{\Sigma_u^{-1}} + \frac{\mu_u^T \Sigma_u^{-1} \mu_u}{\sigma^2} \]

where \( \Sigma_u \Delta \Sigma_u^{-1} + \mu_u \mu_u^H \)

PLUG-IN MEASURE-TRANSFORMED GLRT
- Replace \( \Sigma_u \) by its empirical estimate obtained from the secondary data:

\[ T_u \sim \frac{a^T \Sigma_u^{-1} a}{a^T \Sigma_u^{-1} a} \]

- Under some mild regularity conditions \( T_u \) is asymptotically normal.
- To induce outlier resilience, choose the Gaussian MT-function:

\[ u_2(x, \omega) = \exp \left(-\frac{1}{2} \|P_X^H x\|^2/\omega^2 \right), \ \omega \in \mathbb{R}_+ \]

SELECTION OF THE GAUSSIAN MT-FUNCTION WIDTH PARAMETER
- Principle: control the asymptotic local power sensitivity to change in the signal variance relative to the omniscent LRT under Gaussian data.

Selection Rule:

\[ \omega^* = \inf \left\{ \omega \in \mathbb{R}_+ : R(\omega) = r \right\} \]

\[ R(\omega) \Delta \frac{d R(\omega)}{d \omega} \frac{d R(\omega)}{d \omega} \bigg|_{\omega = 0} = \frac{N_{\max} \left(0, \det \left( \frac{1}{\sqrt{N} + Q^{-1}(\omega)} \right) \right) + Q^{-1}(\omega)}{\sqrt{N}} \]

- Constant false alarm rate (CFAR) w.r.t. the noise power.

EXAMPLES
- Parameters: 8-element ULA, \( N = M = 500, \ P_{fa} = 10^{-3}, \ r = 0.9, \) BPSK signal.

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REFERENCES