1. Overview

Self-reset analog-to-digital converter (SR-ADC):

An illustration of the transfer characteristics of a clipping-ADC and a self-reset-ADC with the thresholds at ±λ.

Output: 
\[ y(t) = M_λ(x(t)) = \text{mod}(x(t) + \lambda, 2\lambda) - \lambda = x(t) - z(t), \]
where \( z(t) = \sum \alpha_k I_{[t_k, t_{k+1})}(t) \) is a piecewise-constant signal.

2. Problem Formulation

1. Given \( y(t) = M_λ(x(t)) \), reconstruct \( x(t) \).
2. Given \( y(nT) = M_λ(x(nT)) \), reconstruct \( x(nT) \).

3. Continuous-Domain Analysis

Definition 1 (Vanishing moments)
A wavelet \( \psi \) has \( p+1 \) vanishing moments if \( \int t^k \psi(t) \, dt = 0 \), for \( k \in [0, p] \).

Key idea: Use wavelets to annihilate the smooth parts of \( y(t) \).

Lemma 1 Let \( x(t) \) be a polynomial of degree \( p \), \( \psi(t) \) be a wavelet with \( p+1 \) vanishing moments, and \( y(t) = M_λ(x(t)) \). Then, \( y_0(t) := (y * \psi)(t) = \sum \alpha_k \xi(t-t_k) \), where \( \xi(t) = -\int_{-\infty}^{\infty} \psi(\tau) \, d\tau \).

- Estimation of \( \alpha_k \)'s: by matched filtering.
- Sufficient condition for exact reconstruction: \( t_k - t_{k-1} > (2p - 1) \) \( \Rightarrow \frac{2\lambda}{T} < (2p - 1) \).

4. Discrete-Domain Analysis

\( y(nT) = M_λ(x(nT)) = x(nT) - z(nT) \)
Annihilation of a sampled polynomial of degree \( p \): \( \sum_{n \in Z} n^k g[n] = 0 \), for \( k \in [0, p] \).

Compactly supported \( g[n] \): Daubechies filter of order \( p \) with support \([0, 2p-1] \).

5. Main Result: WAVE-BUS

WAVE-BUS: WAVElet-Based Unlimited Sampling

Lemma 3 Let \( y(t) = M_λ(x(t)) \), where \( x(t) \) is a Lipschitz-continuous signal that satisfies \( |x(a) - x(b)| \leq L|b - a| \). Then, we have \( T_f := \frac{2\lambda}{T} \leq T_f \).

Proof: From Lipschitz continuity, \( |x(t + T_f) - x(t)| \leq L T_f < 2\lambda, \forall t \).
Hence, at any \( t_k \), no folding happens in the interval \([t_k, t_k + T_f] \). Thus, \( T_f \leq T_f \).

- Sufficient condition for exact reconstruction: Sampling interval \( T \leq \frac{2\lambda}{T_f} < \frac{1}{\gamma^2} \).

6. Reconstruction Algorithm

Algorithm 1: WAVE-BUS

1. Input: \( y(nT) = M_λ(x(nT)) \), \( L \), \( \lambda \), \( p \), \( T \)
2. Output: \( \hat{x}(nT) \)
3. Method:
   1. Wavelet filtering: \( y_g[n] := (y * g)[n] = \sum \beta_k m[n - k] \) with \( \beta_k = (\alpha_k - \alpha_{k-1}) \)
   and \( m[n] := - \sum_{k = -\infty}^{\infty} g[k] \).
   Computing \( \{m_k\} := \text{LASSO: arg min}_{\|h\|_1} \|y_g - A h\|_2^2 + \gamma\|h\|_1 \)
   - A – convolutional dictionary of time-shifted versions of \( m[n] \)
   - \( h \) – sparse vector with \( h[n_k] = \beta_k \) and \( \text{supp}(h) \) = cardinality of \( \{m_k\} \)
   Selection of sampling interval \( (T) \):
      - Let \( T_f := \min(n_k - n_{k-1}) \) be the minimum sampling interval.
      - For no overlap: \( T_f > \text{supp}\{m[n]\} \).
      - Support of Daubechies wavelet filter of order \( p \) is \( 2p \).
      - A sufficient condition on \( T \): \( T < \frac{T_f}{T_f} \).

7. Simulation Results

- Sum of sinusoids of frequency 4 Hz and 7 Hz with \( T = 2 \) ms.

References


Acknowledgements

- The work has been funded by the Science and Engineering Research Board (SERB), Government of India through the project “Sub-Nyquist Sampling.”
- Conference travel funded by Indian Institute of Science, Bangalore and SIPCOT 2018 travel grant.

Confidence and Formatting

- The text is formatted with clear headings, paragraphs, and list items.
- Equations and formulas are properly aligned and numbered.
- Key ideas and definitions are highlighted.
- The main results are clearly stated with proofs and algorithms.
- The references are appropriately cited.
- The overall layout is readable and concise, with key points emphasized.