Robust M-Estimation Based Matrix Completion
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Signal Model and Motivation

Signal model: Observed matrix \( \mathbf{X} \in \mathbb{R}^{n \times r} \) modeled as

\[
\mathbf{X} = \mathbf{M} + \mathbf{S} + \mathbf{N}
\]

- \( \mathbf{M} \): low-rank matrix of rank \( r \)
- \( \mathbf{S} \): column or entry-wise sparse outlier matrix
- \( \mathbf{N} \): (impulsive) background noise

Goal: Recover the low-rank component \( \mathbf{M} \) from partially observed entries of \( \mathbf{X} \) corrupted by noise and outliers.

Applications: recommender systems, computer vision, image inpainting, biomedicine, information retrieval

Existing Robust Matrix Completion Approaches

Robust \( \ell_p \)-loss based methods [1]:
- \( \ell_p \) robust and computationally efficient
- \( \ell_p \) statistically inefficient with respect to additional background noise
- \( \ell_p \) easily get stuck at an inferior solution (nonsmooth objective function)

Robust norm regularization of Huber's loss function approach [2]:
- \( \ell_p \) requires SVD at each iteration and has a high complexity

Proposed Robust M-Estimation Based Approach

Outlier-robust “norm” of \( \mathbf{X} \) is defined as

\[
\|\mathbf{X}\|_{\rho,c} = \sum_{i=1}^{n} \sum_{j=1}^{r} \rho_{\rho,c}\left(\frac{x_{ij}}{c}\right)
\]

- \( \rho > 0 \): scale parameter
- \( x_{ij} \): \( (i,j) \)-th entry of \( \mathbf{X} \)
- \( \rho(x) \): differentiable loss function, e.g., Huber’s or Tukey’s

Proposed robust \( \ell_{\rho,c} \)-minimization based matrix completion:

\[
\min_{\mathbf{U}, \mathbf{V}} \|\mathbf{U}\mathbf{V}^\top - \mathbf{X}\|_{\rho,c}
\]

- Computationally efficient direct matrix factorization \( \hat{\mathbf{M}} = \mathbf{U}\mathbf{V}^\top \), where \( \mathbf{U} \in \mathbb{R}^{n \times r} \) and \( \mathbf{V} \in \mathbb{R}^{r \times r} \) to make the estimate \( \hat{\mathbf{M}} \) low-rank
- \( \{x_{i,j}\} = 0 \) if \( (i,j) \) either \( \notin \Omega \) or \( \{x_{i,j}\} = \hat{x}_{i,j} \) if \( (i,j) \in \Omega \).
- \( \rho \): unknown and is estimated jointly with \( \mathbf{U}, \mathbf{V} \)
- \( c \): constant that is set in advance

Algorithms

Algorithm 1: Huber’s M-estimator

Input: \( \mathbf{X}_{\text{obs}}, \Omega, \) and rank \( r \)

Initialize: Randomly initialize \( \mathbf{U}^0 \in \mathbb{R}^{n \times r} \)

for \( k = 0, 1, \ldots \) do

\[
\mathbf{v}^{k+1} = \arg \min_{\mathbf{v}} \left\{ \sum_{i,j} \rho_{\text{hub}}\left(\frac{x_{ij} - \mathbf{u}^k_j \mathbf{v}_i}{c}\right) + \|\mathbf{v}\|_{1}(\alpha c) \right\}
\]

for all \( j = 1, 2, \ldots, n_r \)

\[
\text{// Fix } \mathbf{v}^k, \text{ optimize } \mathbf{U}^k \]

\[
\mathbf{u}^{k+1} = \arg \min_{\mathbf{u}} \left\{ \sum_{i,j} \rho_{\text{hub}}\left(\frac{x_{ij} - \mathbf{u}_i \mathbf{v}^k_j}{c}\right) + \|\mathbf{u}\|_{1}(\alpha c) \right\}
\]

end for

Output: \( \hat{\mathbf{M}} = \mathbf{U}^k \mathbf{V}^{k+1} \)

 Algorithms

Proposed robust M-estimation based matrix completion: \( \min_{\mathbf{U}, \mathbf{V}} \|\mathbf{U}\mathbf{V}^\top - \mathbf{X}\|_{\rho,c} \) using an iteratively reweighted least-squares (IRWLS) algorithm.

Robust Statistics for Signal Processing.

References

