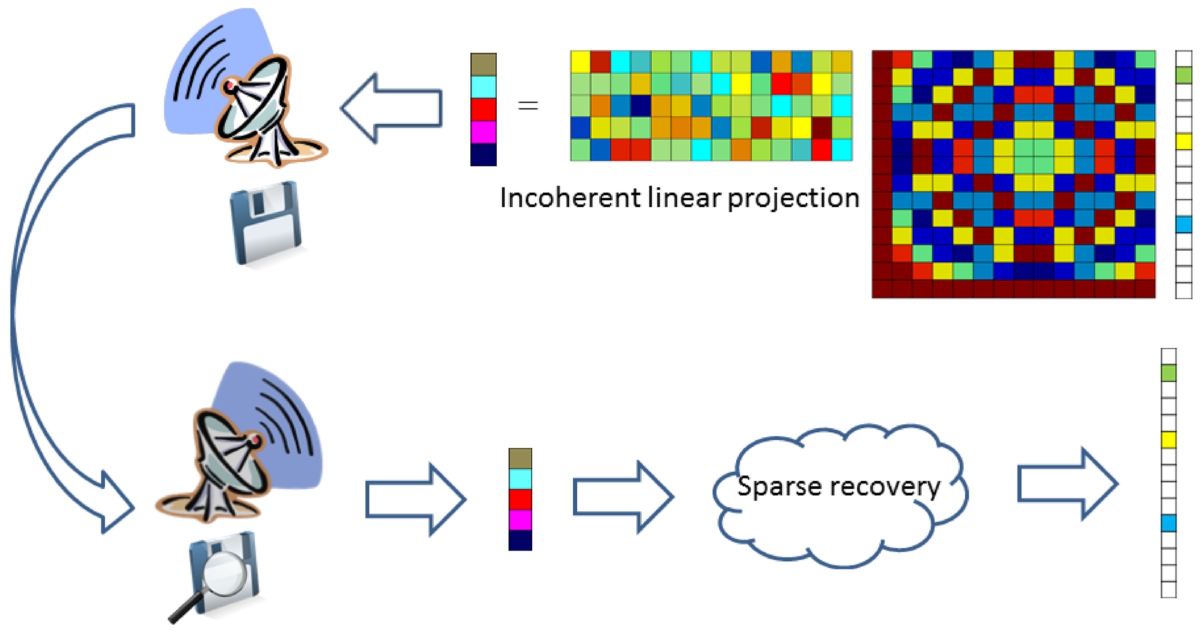
BAYESIAN SPARSE SIGNAL DETECTION EXPLOITING LAPLACE PRIOR

{ SWATANTRA KAFLE, THAKSHILA WIMALAJEEWA AND PRAMOD K. VARSHNEY } SYRACUSE UNIVERSITY

MOTIVATION

- Most of the real signals are sparse



- Compressed Sensing allows for compression of sparse signals before transmission/storage and reconstruction when required.

- Many signal processing applications deal with drawing an inference from received data such as detection and classification of signals, and estimation of signal parameters.

- Applications in sensor networks, cognitive radio networks, and radar networks.

PROBLEM FORMULATION

- Consider a distributed network with P nodes that observe the sparse signals

- The observation model at the p-th node

$$\mathcal{H}_1: \ \boldsymbol{z}_p = \boldsymbol{x}_p + \boldsymbol{\eta}_p$$
$$\mathcal{H}_0: \ \boldsymbol{z}_p = \boldsymbol{\eta}_p$$

- The compressed observation matrix at the FC can be represented as

$$\boldsymbol{Y} = \Phi \boldsymbol{Z} + \boldsymbol{W}$$

- The detection problem with compressed observation reduces to

$$\mathcal{H}_1: \boldsymbol{Y} = \Phi \boldsymbol{X} + \boldsymbol{N}$$
$$\mathcal{H}_0: \boldsymbol{Y} = \boldsymbol{N}$$

- Each $\mathbf{x}_p$ is modeled as a random signal.

SPARSE SIGNAL DETECTION

Likelihood Ratio Based Detection

- Direct Laplace prior on $\mathbf{x}_p$

- With $L = \frac{p(\boldsymbol{Y}|\boldsymbol{X},\lambda,\mathcal{H}_1)}{p(\boldsymbol{Y}|\mathcal{H}_0)}$,

$$\Lambda_{MMV}(\lambda) = \int L \, p(\boldsymbol{X}|\lambda)\, d\boldsymbol{X} = \beta(\lambda) \prod_{p=1}^{P} \prod_{n=1}^{N} I_{p,n},$$

where $I_{p,n}$ is given by,

$$I_{p,n} = \sqrt{2\pi\sigma_0^2}\exp\left(\frac{(v_{p,n}+\sigma_0^2\lambda)^2}{2\sigma_0^2 C}\right) Q\left(\frac{(v_{p,n}+\sigma_0^2\lambda)}{\sqrt{C}\sigma_0}\right) + \sqrt{2\pi\sigma_0^2}\exp\left(\frac{(v_{p,n}-\sigma_0^2\lambda)^2}{2\sigma_0^2 C}\right)\left(1 - Q\left(\frac{(v_{p,n}-\sigma_0^2\lambda)}{\sqrt{C}\sigma_0}\right)\right).$$

Partial Estimate based Detection

- **Idea:** Signal Detection does not require complete signal reconstruction.

- Three stage hierarchical prior on $\mathbf{x_p}$ which impose Laplace prior on signal coefficients.

- Estimate a fraction of signal.

- Algorithm

  Inputs : $\boldsymbol{\Phi}$, $\boldsymbol{Y} = [\boldsymbol{y}_1, \cdots, \boldsymbol{y}_P]$

  Initialize $\zeta_j = 0, \forall j$. Set $k = 0$.

  While $k \le R$

  Select a particular ζ_j^k out of $\zeta^k = [\zeta_1^k, \cdots, \zeta_N^k]$.

  Update $\boldsymbol{\mu}_p = \boldsymbol{\Sigma}_p \boldsymbol{\Phi}^T \boldsymbol{y}_p, \boldsymbol{\Sigma}_p = [\boldsymbol{\Phi}^T\boldsymbol{\Phi} + \boldsymbol{Z}]^{-1}$

  Update algorithm parameters, and $k = k+1$.

  end While

  Detection decision:
  If $\Lambda_{MT} = \frac{1}{RP}\sum_{r=1}^{R}\sum_{p=1}^{P}\mu_{p,r}^2 \ge \theta, \boldsymbol{\mathcal{H}}_1$ is true, otherwise $\boldsymbol{\mathcal{H}}_0$ is true where θ is the threshold.

SPARSE SIGNAL DETECTION (CONTD...)

- Can we further reduce computational complexity?

- Energy of signal on the most likely support as detection parameter.

- Algorithm

  Inputs : $\boldsymbol{\Phi}$, $\boldsymbol{Y} = [\boldsymbol{y}_1, \cdots, \boldsymbol{y}_P]$

  Outputs : Decision statistic Λ_{prj}, Detection Decision

  Solve for ζ_j such that $\frac{\partial \mathcal{L}(\zeta)}{\partial \zeta_j} = 0, \forall j$

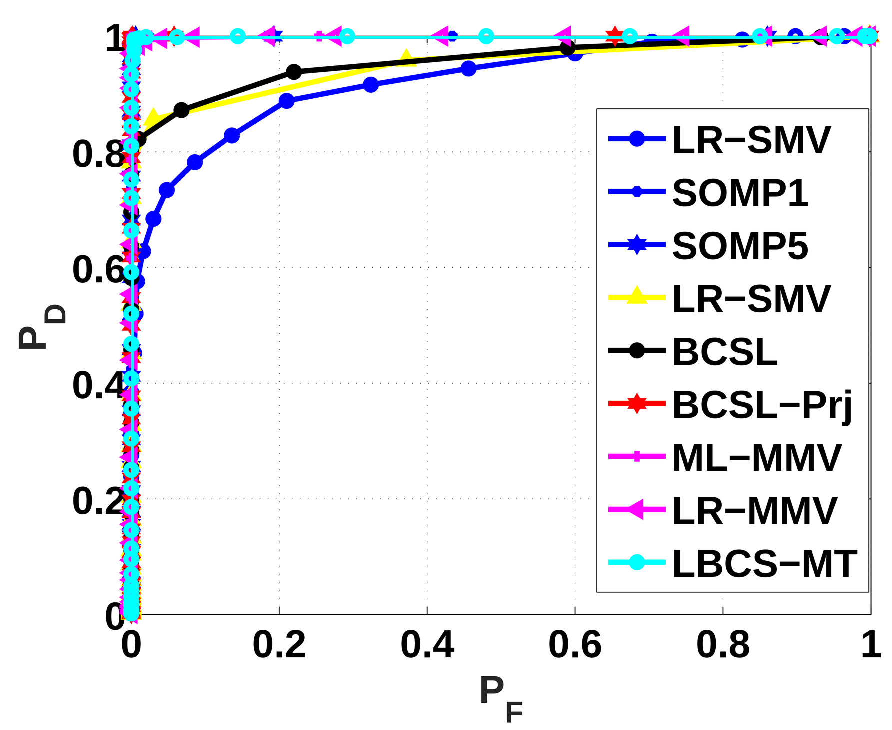
  Evaluate $l(\zeta_j), \forall \zeta_j$.

  Arrange $l(\zeta_j)$ in descending order and choose K' indices j for the first K' largest $l(\zeta_j)$. Let $\hat{\mathcal{U}}$ be the set containing these indices

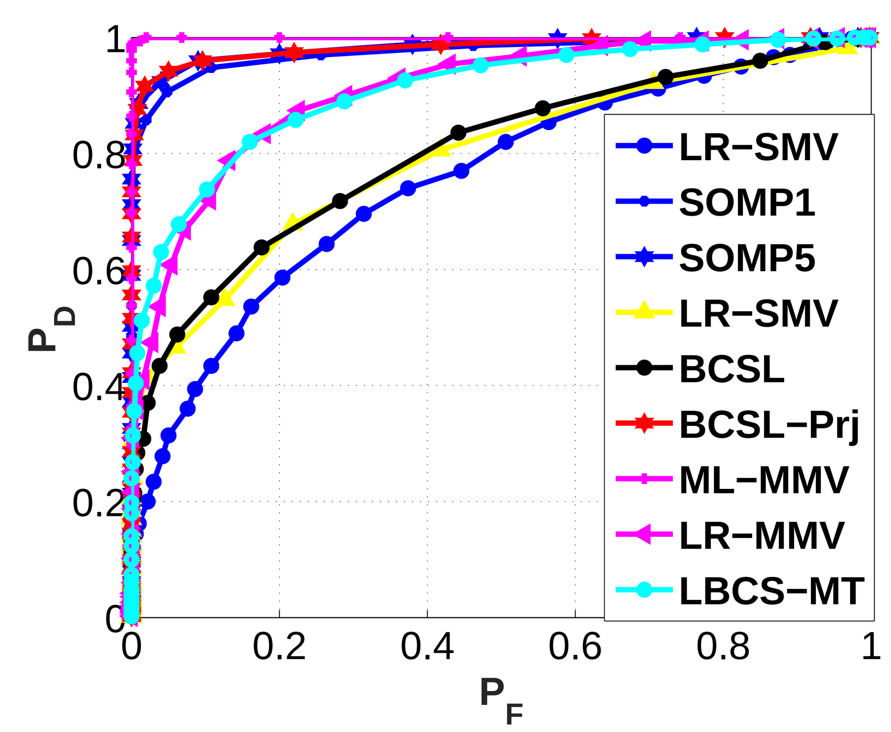
  Detection decision:
  If $\Lambda_{prj} = \sum_{p=1}^{P}\|\boldsymbol{\Omega}\boldsymbol{y}_p\|_2^2 \ge \theta, \boldsymbol{\mathcal{H}}_1$ is true, otherwise $\boldsymbol{\mathcal{H}}_0$ is true where θ is the threshold.

RESULTS

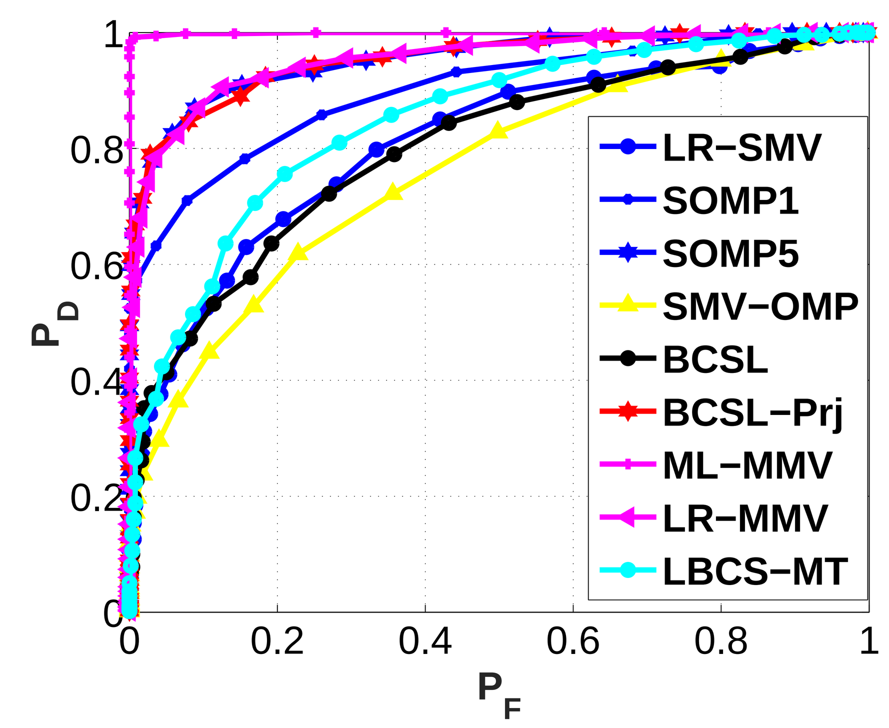
1. $M/N \approx 0.49, \eta = 1.76$ dB



2. $M/N = 0.49, \eta = 13.98$ dB



3. $M/N = 0.1, \eta = 1.76$ dB



4. Run Time Comparison

Run times when $N = 512$ and $K = 5$

$M/N \rightarrow$	0.60	0.70	0.80	0.90	1.00
LR-MMV	0.70	0.76	0.89	0.96	0.93
LBCS-MT	0.82	1.43	2.01	1.60	1.22
BCSL-Prj	0.17	2.37	2.92	2.24	3.01
SOMP5	17.26	16.25	20.25	20.86	23.19

CONCLUSION

- Algorithms developed for sparse signal detection without signal reconstruction

- Reduction in computational complexity compared to the state-of-the-art algorithm.