Deep Residual Learning for Model-Based Iterative CT Reconstruction using Plug-and-Play Framework

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Model-Based Iterative Reconstruction (MBIR)

- Computed Tomography (CT) Reconstruction
  - Diagnostic Radiology
  - Additive Manufacturing Inspection

- MBIR Flowchart

```
CT Scanner

Object → Measured Sinogram

Prior Model

Corrected Image

Transformed Image

Object being imaged

Reconstructed Image

yes

Converge?

no

Forward Model

Calculated Sinogram

Difference

Error Sinogram

Physical CT system
```
Advantage of MBIR over Filtered-Back Projection (FBP)

- Superior Image Quality: Low Noise and High Resolution

<table>
<thead>
<tr>
<th></th>
<th>FBP</th>
<th>MBIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-dose</td>
<td><img src="image1" alt="FBP Low-dose" /></td>
<td><img src="image2" alt="MBIR Low-dose" /></td>
</tr>
<tr>
<td>(quarter dose)</td>
<td><img src="image3" alt="FBP Low-dose" /></td>
<td><img src="image4" alt="MBIR Low-dose" /></td>
</tr>
<tr>
<td>Sparse-view</td>
<td><img src="image5" alt="FBP Sparse-view" /></td>
<td><img src="image6" alt="MBIR Sparse-view" /></td>
</tr>
<tr>
<td>(16 views)</td>
<td><img src="image7" alt="FBP Sparse-view" /></td>
<td><img src="image8" alt="MBIR Sparse-view" /></td>
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</tbody>
</table>
Prior Model in MBIR

- Accurate prior modeling is critical to the image quality of MBIR.

- Typical Prior Model: MRF
  - Penalize intensity fluctuation in the neighborhood
  - Challenge: Noise-induced fluctuation vs. underlying object

- Solution: Prior Model from an Image Database

![Diagram showing Image DB/Prior Knowledge linked to Deep Residual Learning, which in turn is connected to MBIR Prior Model]
MBIR Optimization

- MAP Estimation

\[
\hat{x} = \arg \min_x \left[ \frac{1}{2} \| A x - y \|_W^2 + \frac{1}{2\sigma^2} \Phi(x) \right]
\]

\(x \in \mathbb{R}^N\): reconstructed image, \(y \in \mathbb{R}^M\): measured CT scan
\(A \in \mathbb{R}^{M\times N}\): system matrix for CT scan, \(W \in \mathbb{R}^{M\times M}\): measurement noise variance
\(\sigma \in \mathbb{R}\): regularization parameter, \(\Phi: \mathbb{R}^N \to \mathbb{R}\): prior model

- First-order iterative optimization
  - Iterative Coordinate Descent (ICD) / Ordered Subset (OS)
  - Prior model should be first-order differentiable.

Not flexible for Data-driven Prior
Plug-and-Play (PnP) Framework

- Variable Splitting

\[(\hat{x}, \hat{v}) = \arg \min_{x,v} \left[ \frac{1}{2} \|Ax - y\|_w^2 + \frac{1}{2\sigma^2} \Phi(v) \right] \text{ s.t. } x = v\]

- Alternating Direction Method of Multipliers (ADMM)

\[L_\lambda(x, v, u) = \frac{1}{2} \|Ax - y\|_w^2 + \frac{1}{2\sigma^2} \Phi(v) + \frac{1}{2\lambda^2} \|x - v + u\|_2^2\]

Step 1: Reconstruction Module

\[\hat{x} = \arg \min_x L_\lambda(x, \hat{v}, u) = \arg \min_x \left[ \frac{1}{2} \|Ax - y\|_w^2 + \frac{1}{2\lambda^2} \|x - \hat{v} + u\|_2^2 \right]\]

Step 2: De-noising Module

\[\hat{v} = \arg \min_v L_\lambda(\hat{x}, v, u) = \arg \min_v \left[ \frac{1}{2\sigma^2} \Phi(v) + \frac{1}{2\lambda^2} \|\hat{x} + u - v\|_2^2 \right]\]

Step 3: Update Dual Variable

\[u \leftarrow u + (\hat{x} - \hat{v})\]

Independent Module for De-noising
Deep Residual Learning for De-noising

- Deep Neural Network
  - Powerful performance for vision tasks such as de-noising
  - Weights of a neural network learned on large training dataset
  - Challenge: Long training time

- Deep Residual Learning for Efficient Training
  - Bypassing low-freq. image
Deep Residual Learning: Training

- **Cost**
  \[
  l(\Theta) = \frac{1}{2K} \sum_{k=1}^{K} \left\| R(\tilde{\nu}_{k}^{tr}; \Theta) - (\hat{\nu}_{k}^{tr} - \tilde{\nu}_{k}^{tr}) \right\|_2^2
  \]

  \(\{\tilde{\nu}_{k}^{tr}, \hat{\nu}_{k}^{tr}\}\): noisy and clean image pairs, \(R(\bullet; \Theta)\): Residual neural network

- **Training Database**
  - 40x40 patches for all slices, Data augmentation (flip, rotation)
  - Randomly selected 256000 patches, mini-batch size: 128

Database

- FBP
- Ground-Truth MBIR (20 iter.)
Deep Residual Learning: Testing

<table>
<thead>
<tr>
<th></th>
<th>FBP</th>
<th>Deep Residual Learning De-noising</th>
<th>Ground-Truth MBIR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Testing Scan #1</strong></td>
<td>![FBP Image]</td>
<td>![Deep Residual Learning De-noising Image]</td>
<td>![Ground-Truth MBIR Image]</td>
</tr>
<tr>
<td><strong>Testing Scan #2</strong></td>
<td>![FBP Image]</td>
<td>![Deep Residual Learning De-noising Image]</td>
<td>![Ground-Truth MBIR Image]</td>
</tr>
</tbody>
</table>
# MBIR Result: Qualitative @ 1 iter.

<table>
<thead>
<tr>
<th>Method</th>
<th>Image 1</th>
<th>Image 2</th>
<th>Image 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard MBIR with MRF Prior</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td>PnP MBIR with Deep Learning Prior</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td>Ground-Truth MBIR</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
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Reconstructed Image

Difference Image

[800,1200] [800,1200] [800,1200]

[0,50] [0,50] [0,50]
MBIR Result: RMSD

Significant Speed-up with Faster Convergence
Computational Time

- Deep Learning Training Time
  - 4 NVIDIA Titan X GPU (12GB memory)
  - Google Tensor Flow
  - 65 minutes / 50 epochs

- Deep Learning Testing Time
  - ~10ms/slice

- Standard MBIR and PnP MBIR require similar amount of recon. time per iteration.
Conclusion

- Summary
  1. Image prior modeling from FBP/MBIR database
     - Deep Residual Learning for Image De-noising
  2. Incorporating the prior model from a database into MBIR
     - Plug-and-Play Optimization Framework

- Deep Residual Learning is effective in reducing the noise and enhancing the resolution in FBP.

- PnP MBIR with deep learning prior significantly improves the image quality compared with standard MBIR.