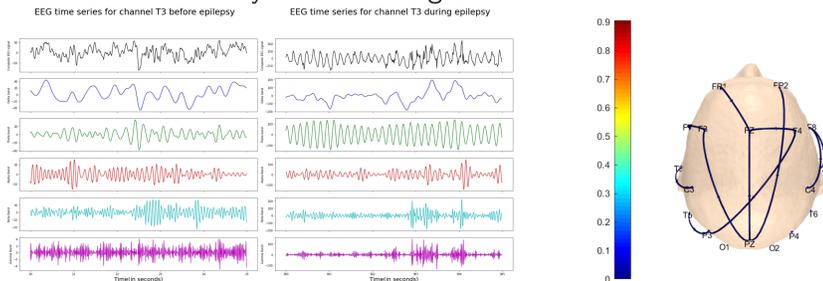


Motivation

Background:

- ▶ Estimation of effective connectivity networks between different channels of brain time series data is of key importance
- ▶ Most of the current methods (like Granger Causality) use linear modelling of the data which may lead to wrong estimation of effective connectivity



Aim of our research:

1. To formulate an effective connectivity measure that can estimate non-linear directed connections in multi-channel time series data
2. To propose simple extensions for detecting frequency specific and time evolving effective connectivity patterns

Contribution

- ▶ Developed an algorithm that can estimate the non-linear Granger causality connections (NLGC). Using Butterworth filters we also estimated frequency specific NLGC connections (Spec NLGC)
- ▶ Analyzed effect of degrading SNR, which is crucial for brain time series
- ▶ Application to an actual seizure EEG data gives insightful results regarding connectivity changes in different bands during epilepsy

Problem Formulation

- ▶ VAR(K) model of time series data is given as:

$$X(t) = \sum_{n=1}^K A^{(n)} X(t-n) + \epsilon(t)$$

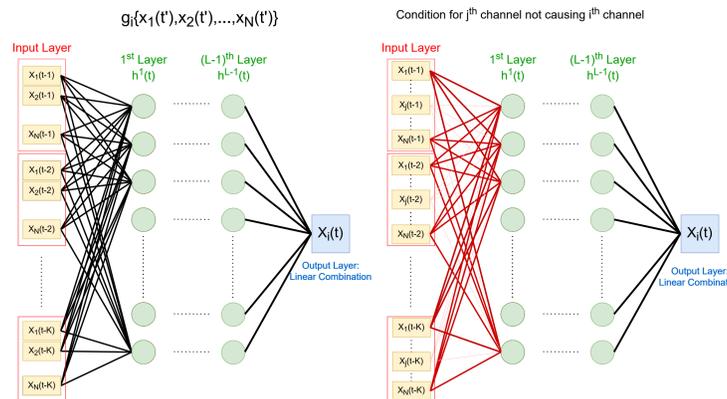
- ▶ A generalization to the classical VAR(K) model would be to model the current values $X(t)$ using some non-linear function $g(\cdot)$ such that:

$$X(t) = g(X_1(t'), X_2(t'), \dots, X_N(t')) + \epsilon(t)$$

- ▶ We induce non-linearity via the use of neural networks (NNs). In order to make the NNs more interpretable and be able to infer the Granger Causal connectivity, individual NNs are used for every channel:

$$X_i(t) = g_i(X_1(t'), X_2(t'), \dots, X_N(t')) + \epsilon_i(t)$$

Network Architectures



- ▶ To implement each of the $g_i(\cdot)$, cMLPs of single hidden layer $h^1(t) \in \mathbb{R}^H$ with H neurons is used:

$$\text{Hidden layer : } h^1(t) = \sigma \left[\sum_{n=1}^K W^{1n} X(t-n) + b^1 \right]$$

$$\text{Output layer : } X_i(t) = W^2 h^1(t) + b^2$$

- ▶ In order to estimate the presence or absence of a Granger causal connection from any of the N channels to the i^{th} , we train the network weights: W^{1n} for all lags n , and the weights: b^1, W^2, b^2
- ▶ To reduce down the cases of false positives, we need to regularize the network training, this is achieved via the use of hierarchical penalty
- ▶ The final optimization equation with regularization is given as:

$$\min_{W^1, W^2, b^1, b^2} \sum_{t=1}^T [X_i(t) - g_i(X(t-1), \dots, X(t-K))]^2 + \lambda \sum_{j=1}^N \sum_{n=1}^K \|(W_{:j}^{1n}, \dots, W_{:j}^{1K})\|_2$$

Simulation Results

Proposed method is tested in two simulation settings:

- ▶ First to analyze the performance for overall connectivity using NLGC
- ▶ Secondly to analyze the performance for frequency band specific connectivity using Spec NLGC method
- ▶ Non-linear directed connectivity detection capability of VAR-LASSLE and VAR-LASSLE based PDC is compared with NLGC and Spec NLGC respectively, the AUPR scores are:

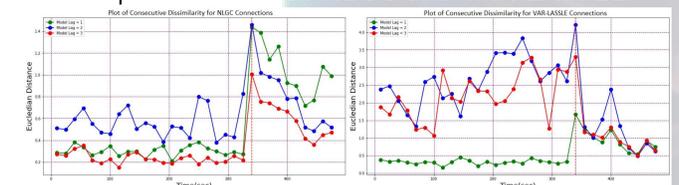
Method	2 dB	5 dB	10 dB	15 dB	20 dB
VAR-LASSLE(1)	0.42 ± 0.01	0.42 ± 0.02	0.41 ± 0.01	0.41 ± 0.01	0.41 ± 0.01
NLGC(1)	0.62 ± 0.03	0.72 ± 0.05	0.84 ± 0.02	0.82 ± 0.02	0.8 ± 0.03
PDC(1)	0.26 ± 0.03	0.23 ± 0.00	0.25 ± 0.03	0.23 ± 0.00	0.24 ± 0.00
Spec NLGC(1)	0.79 ± 0.02	0.82 ± 0.01	0.9 ± 0.01	0.91 ± 0.02	0.81 ± 0.02
VAR-LASSLE(2)	0.41 ± 0.03	0.41 ± 0.02	0.41 ± 0.01	0.42 ± 0.03	0.44 ± 0.02
NLGC(2)	0.45 ± 0.03	0.45 ± 0.06	0.71 ± 0.05	0.87 ± 0.02	0.9 ± 0.05
PDC(2)	0.32 ± 0.03	0.26 ± 0.05	0.24 ± 0.06	0.14 ± 0.03	0.11 ± 0.00
Spec NLGC(2)	0.63 ± 0.035	0.68 ± 0.07	0.8 ± 0.02	0.9 ± 0.03	0.92 ± 0.03
VAR-LASSLE(3)	0.4 ± 0.02	0.4 ± 0.03	0.44 ± 0.02	0.43 ± 0.02	0.43 ± 0.02
NLGC(3)	0.36 ± 0.06	0.42 ± 0.03	0.75 ± 0.04	0.88 ± 0.03	0.93 ± 0.00
PDC(3)	0.26 ± 0.05	0.27 ± 0.05	0.29 ± 0.03	0.17 ± 0.04	0.28 ± 0.10
Spec NLGC(3)	0.65 ± 0.07	0.74 ± 0.06	0.91 ± 0.03	0.98 ± 0.01	0.99 ± 0.00

Application to EEG Data

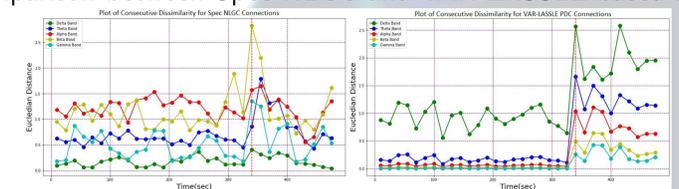
- ▶ We apply the NLGC and Spec NLGC method on a 18-channel seizure EEG data with 50,000 time-points, having a sample rate of 100 Hz
- ▶ The EEG data is divided up into time windows of 2000 time points with 50% overlap, and time evolving GC matrices were evaluated for each of them
- ▶ In order to understand the network dynamics and visualize the amount of change in the GC connectivity network, we plotted the *Euclidean Distance* (ED(t)) between consecutive GC matrices:

$$ED(t) = \sqrt{\sum_{\text{all } i,j} |[GC(t)]_{i,j} - [GC(t-1)]_{i,j}|^2}$$

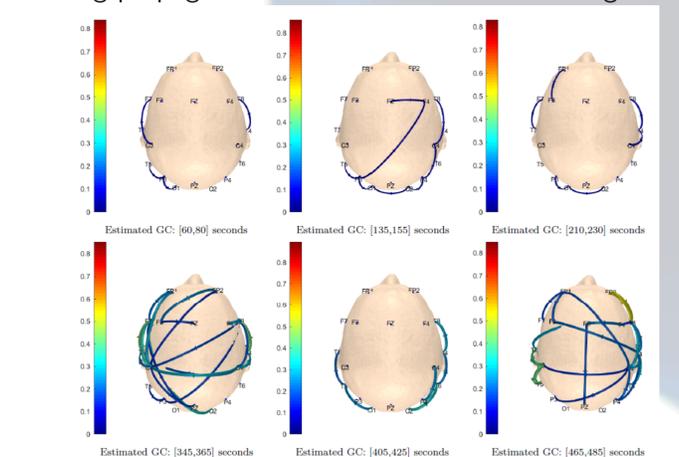
- ▶ Comparison between NLGC and VAR-LASSLE:



- ▶ Comparison between Spec NLGC and VAR-LASSLE based PDC:



- ▶ Visualizing propagation of NLGC connections during seizure:



Conclusions

- ▶ Novel framework for frequency specific non-linear GC connections
- ▶ Experiments on simulated data exhibited huge performance boost and implementation on epileptic EEG data provided new insights.

Future Work: Integration of Spec NLGC with sophisticated approaches to deal with non-stationarity can be explored in future studies

