DOA Estimation in Systems with Nonlinearities for mmWave Communication

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Overview

Towards Low Cost Low Complexity Hardware

Massive MIMO mmWave Communication Systems

- Widescale deployment - Systems with lower cost and complexity
- Efficient systems not limited by system nonlinearities
- Most methods overlook device characteristics for CSI estimation
- Nonlinearities often dealt with through linear approximation

Can massive MIMO systems help deal with device nonlinearities?

Multi-user mmWave Wireless Uplink Channel Model

System Model

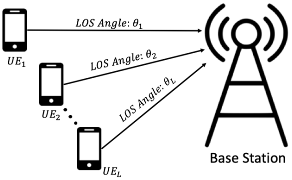


Fig: A multi-user wireless uplink transmission.

- Serving users efficiently requires analog steered beams to individual users
- Use of transmitted uplink pilots to geo-locate users (angles: $\{\theta_i\}$)
- Separate the received signal into the individual angular components

Received signal at BS with ULA

$$\mathbf{x}(t) = \sum_{l=1}^{L} \mathbf{a}(\theta_l) s_l(t) + \mathbf{w}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t)$$

where $\mathbf{a}(\theta_l) = \left[1, e^{j\frac{2\pi d}{\lambda}\sin(\theta_l)}, \ldots, e^{j\frac{(N_{\mathrm{rx}}-1)2\pi d}{\lambda}\sin(\theta_l)}\right]^T$ and $\mathbf{w}(t)$ is AWGN.

Aim: UE separation using Direction of Arrival (DOA) of each.

Nonlinearities in Received Signal

Base Station Model

- Rarely have access to the wireless signal $\mathbf{x}(t)$
- Different analog components like filters, ADCs, amplifiers, etc.
- Results in a nonlinear transformation of the signal

$$[\mathbf{y}(t)]_n = g(\Re[\mathbf{x}(t)]_n) + j\, g(\Im[\mathbf{x}(t)]_n)$$

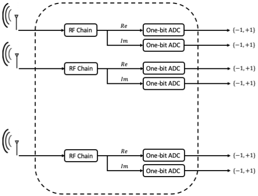


Fig: BS receiver with 1-bit ADC.

Specific Case: One-bit ADC

$$\text{sign}(\Re[\mathbf{x}(t)]_n) + j\, \text{sign}(\Im[\mathbf{x}(t)]_n)$$

- Lower cost power and complexity
- Analyze effect on DOA estimation

Signal analysis motivated by the use of low cost/simpler hardware

DOA Estimation using Autocorrelation

Considering DOA estimation without system nonlinearities

1. Pilot-aided case - Use signal cross correlation

$$\mathbf{R}_{\mathbf{xs}} = \mathbb{E}[\mathbf{x}(t)\mathbf{s}(t)^H] = \mathbf{A}\mathbb{E}[\mathbf{s}(t)\mathbf{s}(t)^H] \rightarrow \text{Spans signal space}$$

   DOA values can be estimated using the ESPRIT algorithm

2. Non pilot-aided case - Use signal cross correlation

$$\mathbf{R}_{\mathbf{xx}} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}(t)^H] = \mathbf{A}\mathbb{E}[\mathbf{s}(t)\mathbf{s}(t)^H]\mathbf{A}^H + \sigma_{\mathbf{w}}^2\mathbb{I}$$

   DOA values can be estimated using the MUSIC algorithm

In the presence of nonlinearities, we can analogously evaluate

1. Pilot-aided cross correlation: $\mathbf{R}_{\mathbf{ys}} = \mathbb{E}[\mathbf{y}(t)\mathbf{s}(t)^H]$.
2. Autocorrelation of the received signal: $\mathbf{R}_{\mathbf{yy}} = \mathbb{E}[\mathbf{y}(t)\mathbf{y}(t)^H]$.

Can $\mathbf{R}_{\mathbf{ys}}$ and $\mathbf{R}_{\mathbf{yy}}$ be written as functions of $\mathbf{R}_{\mathbf{xs}}$ and $\mathbf{R}_{\mathbf{xx}}$ respectively?

DOA Estimation for 1-bit Data

Closed form expression for the 1-bit ADC [Van Vleck,Middleton'66]

$$\left[\mathbf{R_{yy}}\right]_{n,m} = \frac{2}{\pi}\left(\sin^{-1}\left[\Re\left[\mathbf{R_{xx}}\right]_{n,m}\right] + j\,\sin^{-1}\left[\Im\left[\mathbf{R_{xx}}\right]_{n,m}\right]\right)$$

(Above expression valid only for Gaussian symbols)

Some prior work using this result

- Work by [Liu,Vaidyanathan'17] applied this to DOA estimation from 1-bit data, for sparse arrays
- Work by [Huang,Lao'19] use this for One-bit MUSIC algorithm

Our Contribution: Extending to General Nonlinearities

- Formulate the cross correlation for a general nonlinear transformation
- Pilot-aided and non-pilot aided operation for DOA estimation

Pilot-aided DOA Estimation: Theorem

Theorem: Pilot-aided cross correlation

If the nonlinearity $g(x)$ is odd-symmetric, the cross-correlation matrices $\mathbf{R_{ys}}$ and $\mathbf{R_{xs}}$ are proportional as $\mathbf{R_{ys}} = \gamma \mathbf{R_{xs}}$, where γ is a non-zero scalar constant of proportionality.

Proof: Application of Price's Theorem to Taylor series expansion of $g(x)$.

Corollary: If the nonlinearity $g(x)$ is even-symmetric, the cross matrix $\mathbf{R_{ys}} = 0$, & signal subspace (DOA angles) cannot be recovered.

Price's Theorem[1]

For $(x, y) \sim \mathcal{N}(x, y)$ with covariance ρ_{XY} & function $g(x, y)$,

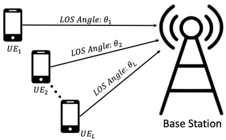
$$\frac{\partial}{\partial \rho_{XY}} \mathbb{E}\left[g(x, y)\right] = \mathbb{E}\left[\frac{\partial^2}{\partial x \partial y} g(x, y)\right].$$

For odd-symmetric nonlinearities, $\mathbf{R_{ys}}$ can be used for DOA estimation without any loss of information or post processing.

[1] R.Price, "A useful theorem for nonlinear devices having gaussian inputs," IRE Transactions on Information Theory, 1958.

Pilot-aided DOA Estimation: Simulation

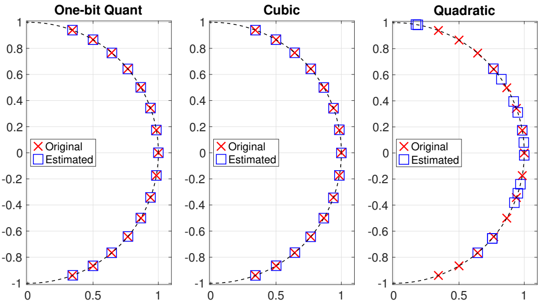
Simulation Setup



Receiver Nonlinearities

1. 1-bit Quantization
2. Cube Nonlinearity
3. Square Nonlinearity

Simulation Results



(1) (2) (3)

Comparison of polar plots for 15 sources distributed in $[-70^\circ, 70^\circ]$; $N_{rx} = 64$ and angle recovery with ESPIRIT, taking 1000 snapshots, at SNR $= 0$ dB

Perfect recovery for odd and no recovery for even nonlinearities.

DOA Estimation without Pilot Data: Theorem

Theorem: Autocorrelation transformation

For an odd symmetric nonlinearity $g(x)$, there exists an element-wise functional mapping between autocorrelation matrices, $\mathbf{R_{yy}}$ & $\mathbf{R_{xx}}$, as

$$\mathbf{R_{yy}} = g_1 \mathbf{R_{xx}} + g_2 \sum_{k=0}^{\infty} \beta_k \mathbf{a}(\hat{\theta}_k)\mathbf{a}(\hat{\theta}_k)^H.$$

The set $\{\hat{\theta}_k\}$ is a function of original angles, g_1, g_2 and $\{\beta_k\}$ are scalars.

Proof: Using properties of higher order moments for Gaussian variables.

Implications of this analysis

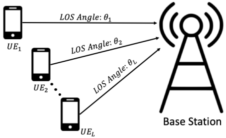
- Original subspace $\mathbf{R_{xx}}$ embedded in transformed space
- $g_1 >> g_2 \implies \mathbf{R_{xx}}$ is the dominant subspace of the $\mathbf{R_{yy}}$ space
- Such embedding is not possible for even transformation
- Massive MIMO can accommodate increased angular subspace

Effect of nonlinearity→Element-wise mapping of autocorrelation; i.e.,

$$\left[\mathbf{R_{yy}}\right]_{n,m} = f\left(\left[\mathbf{R_{xx}}\right]_{n,m}\right)$$

DOA Estimation without Pilot Data: Simulation Results

Simulation Setup



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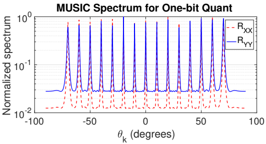
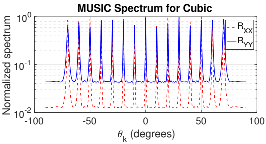


(1)



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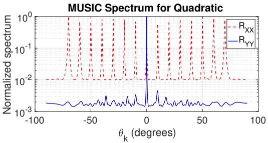


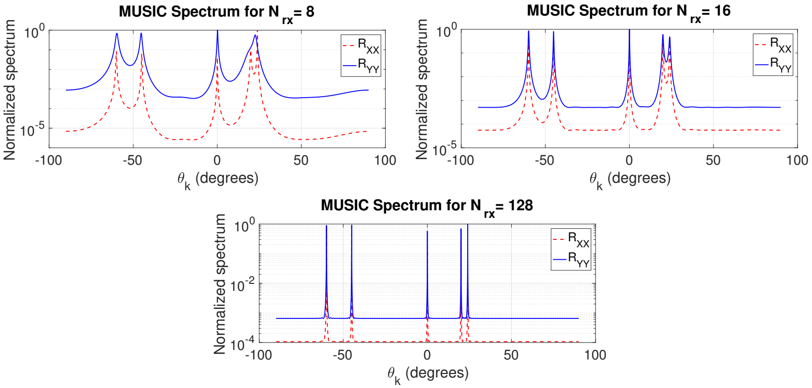
(3)

MUSIC Spectrum for 15 sources distributed in $[-70^\circ, 70^\circ]$; $N_{rx} = 64$ and angle recovery with MUSIC, taking 5000 snapshots, at $SNR = 0dB$

DOA subspace preserved for odd nonlinearity; not possible for even case.

DOA Estimation without Pilot Data: Massive MIMO

Effect of Massive MIMO for 1-bit Quantization



Increasing antennas can accommodate nonlinear transformed DOA subspace without additional post-processing.

Conclusions and Future Work

DOA estimation in systems with nonlinearities

- Generalized to a broad class of nonlinearities
- Shown the power of Massive MIMO for dealing with nonlinearities
- This can simplify channel estimation hardware cost and complexity

Future work ahead: Receiver for data detection

- Analysis for general circular processes: M-QAM constellation
- Joint DOA estimation and data decoding with nonlinearities
- Learning the autocorrelation correlation mapping beyond 1-bit ADCs

Thank you! Are there any questions?