Distributed TDOA-based indoor source localisation

Wangyang Yu, Nikolay D. Gaubitch, Richard Heusdens
Outline

• Background
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• Pruning out erroneous TOA measurements
• TDOA-based localization
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Background

• Localisation techniques
  – In the hospital
  – In museums

• Outdoor environment: GPS

• Indoor environment: TOA, TDOA, RSS, AOA
Problem Formulation

- Time difference of arrival techniques
- \( M \) receivers with known location
  \( r_1, r_2, \ldots, r_M \)
- \( N \) transmitters whose locations are to be estimated
  \( s_1, s_2, \ldots, s_N \)
- The time of arrival (TOA) information

\[
t_{ij} = c^{-1} ||r_i - s_j|| + n_{ij}
\]
Pruning out erroneous TOA measurements

- Erroneous TOA measurements
- TOA measurements

\[ t_{ij} = \| \mathbf{r}_i - \mathbf{s}_j \| \]

\[ -(\mathbf{r}_i - \mathbf{r}_1)^T (\mathbf{s}_j - \mathbf{s}_1) = 0.5(t_{ij}^2 - t_{1j}^2 - t_{i1}^2 + t_{11}^2) \]

\[ \mathbf{R}^T \mathbf{S} = \mathbf{T} \]
Pruning out erroneous TOA measurements

• The set of all $N - 1$ unique combinations of the set of TOA measurements

\[ U_{N-1} = \binom{S_N}{N - 1} \]

• For a specific combination, construct $T_u$, and compute $e_u$

\[ e_u = \|T_u\|_F^2 = \sum_{i=1}^{N_r} \sigma_i^2(T_u) \]
Pruning out erroneous TOA measurements

**Algorithm 1** Pruning incorrect TOA measurements

1. For \( n = 0, 1, N - N_{\text{min}} \)
2. Generate the set of all possible combinations of the set \( S_{N-n} \)
   \[
   U_{N-n+1} = \binom{S_{N-n}}{N-n+1}
   \]
3. For each \( u \in U_{N-n+1} \), construct \( T_u \) and compute \( e_u \).
4. Update the best TOA sets,
   \[
   S_{N-n+1} = \begin{cases} 
   \arg \min_s e_u & \text{if } \min e_u / \max e_u < \alpha \\
   S_{N-n} & \text{otherwise}
   \end{cases}
   \]
5. End if \( S_{N-n+1} = S_{N-n} \)
TDOA-based localization
—— Centralised localisation

- TDOA measurements

\[ \Delta t_{1j} = t_1 - t_j = \frac{||s - r_1||}{c} - \frac{||s - r_j||}{c} + n_{1j} \]

- Define

\[ d_j^2 = (x_j - x_0)^2 + (y_j - y_0)^2 \]

\[ K_j^2 = x_j^2 + y_j^2 \]

- For receiver \( j \):

\[ -x_j x_0 - y_j y_0 = d_{j1} d_1 + \frac{1}{2}(d_{j1}^2 - K_j^2) \]
TDOA-based localization
—— Centralised localisation

• Vector form:

\[ A_1 s = d_1 b_1 + c_1 \]

• Least-square problem:

\[ \min_{s} \| A_1 s - (d_1 b_1 + c_1) \|_2^2 \]

• Solution:

\[ \hat{s} = (A_1^T A_1)^{-1} A_1^T (d_1 b_1 + c_1) \]
TDOA-based localization
—— Centralised localisation

• Least-square problem:

\[
\min_{\mathbf{s}_k} \left\| \mathbf{A}_k \mathbf{s}_k - (d_k \mathbf{b}_k + \mathbf{c}_k) \right\|_2^2
\]

• Solution:

\[
\hat{\mathbf{s}}_k = \left( \mathbf{A}_k^T \mathbf{A}_k \right)^{-1} \mathbf{A}_k^T (d_k \mathbf{b}_k + \mathbf{c}_k)
\]

• Average location estimation:

\[
\hat{\mathbf{s}} = \frac{1}{M} \sum_{k=1}^{M} \hat{\mathbf{s}}_k
\]
TDOA-based localization
—— Distributed localisation

• Distributed localisation:

\[
\min_{s_k} \sum_{k=1}^{M_k} \left\| A_k s_k - (d_k b_k + c_k) \right\|^2_2,
\]

subject to \( s_k = s_j, \forall (k, j) \in E \)

• Solution: standard solvers.
Experiments

—— Experimental Setup

- Room: 6×5×4 m
- 8 receivers, 30 transmitters
- A hit: an erroneous TOA measurement being detected
- A false alarm: a correct TOA measurement being classified as erroneous
Experiments
Experiments

— Experimental Setup

- Room: $10 \times 10$ m

- 8 receivers uniformly placed on the boundary

- Two receivers can communicate when the distance is less than 10 m

- Propagation speed: $c = 340$ m/s
Experiments
Conclusions

- An algorithm to prune out erroneous TOA measurements.

- TDOA-based localisation
  - Centralised localisation
  - Distributed localisation

- Accessing to neighbouring information doesn’t decrease the accuracy.