Hyperspectral imagery denoising via reweighed sparse low-rank nonnegative tensor factorization

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Introduction

In this paper, a reweighed sparse low-rank nonnegative tensor factorization (RSLRNTF) method is proposed to restore an HSI. It takes an HSI as a third-order tensor and factorizes it into the combination of a few component tensors where each one is the outer product of a low-rank matrix (coding matrix) and a vector (atom). A reweighed \(L_1\) norm is added to coding matrices to enforce their sparsity. The low-rankness in both spatial and spectral domain is in line with the spatial and spectral correlation in an HSI. Furthermore, we add nonnegativity constraint to both coding coefficients matrices and dictionary to learn parts-based representation of HSI, which facilitates preserving local structure information.

RSLRNTF

![Diagram of RSLRNTF](image)

Figure 1: Framework of proposed method.

\[
\min f(A, B, C) = \|Y - \sum_{r=1}^{R} A_r B_r^T c_r \|_F^2 + \frac{\delta}{2} \|A B^T - 1_{r \times r}\|_F^2
\]

\[
+ \lambda \sum_{r=1}^{R} \|W_r \odot (A_r B_r^T)\|_1 \quad \text{s.t.} \quad A, B, C \geq 0
\]

where \(A = [A_1 \cdots A_R] \), \(B = [B_1 \cdots B_R] \), \(C = [c_1 \cdots c_R] \) and \(\odot\) is element-wise product.

Update rules

\[
A \leftarrow A \ast \left( \frac{Y_r^r B_r + \mu A_r}{(A M^T M + \delta A B^T B + \mu A)} \right)
\]

\[
B \leftarrow B \ast \left( \frac{Y_r^r A_r + \mu V_r}{(B M^T M + \delta B A^T A + \mu B)} \right)
\]

\[
C \leftarrow C \ast \left( \frac{Y_r^r M_r}{(C M^T M)} \right)
\]

\[
U_r \leftarrow U_r + \frac{\mu A_r}{\|U_r + \lambda W_r V_r\|}
\]

\[
V_r \leftarrow V_r + \frac{\mu B_r}{\|A_r W_r^T U_r + \lambda V_r\|}
\]

\[
W_r \leftarrow 1 / (U_r V_r^T + \epsilon)
\]

Experimental results on simulated data

Visual comparison on simulated data

![Visual comparison of denoising results](image)

Figure 2: Denoising results of band 25 in simulated data when \(\sigma \in [0 - 0.1]\).

Experimental results on real-world data

![Experimental results on real-world data](image)

Figure 3: Denoising results of band 208 in the Urban data set.

Table 1: PSNR of Different Methods on Simulated Data.

<table>
<thead>
<tr>
<th>Noise variance (\sigma)</th>
<th>(\sigma \in [0, 0.05])</th>
<th>(\sigma \in [0, 0.1])</th>
<th>(\sigma \in [0, 0.15])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>34.49</td>
<td>26.89</td>
<td>24.84</td>
</tr>
<tr>
<td>LRMR [1]</td>
<td>43.12</td>
<td>39.12</td>
<td>36.63</td>
</tr>
<tr>
<td>LRTV [2]</td>
<td>43.04</td>
<td>37.63</td>
<td>36.26</td>
</tr>
<tr>
<td>PARAFAC [3]</td>
<td>38.58</td>
<td>37.51</td>
<td>35.99</td>
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<tr>
<td>LRTDTV [5]</td>
<td>42.63</td>
<td>39.14</td>
<td>36.94</td>
</tr>
<tr>
<td>RSLRNTF</td>
<td>43.87</td>
<td>40.01</td>
<td>37.21</td>
</tr>
</tbody>
</table>

Highlights

- \(A, B_r^T\): low-rank coding matrix derives spatial low-rankness.
- \(C\): low-rank dictionary represents spectral low-rankness.
- \(\|W_r \odot (A_r B_r^T)\|_1\): reweighed \(L_1\) norm makes coding matrix sparser.
- \(\frac{1}{2} \|A B^T - 1_{r \times r}\|_F^2\): avoid trivial solutions.
- Nonnegativity constraints promote local details preserving.
- Tensor model preserves all the information in an HSI.

References