High-dimensional Embedding Denoising Autoencoding Prior for Color Image Restoration

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Outline

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2. Denoising autoencoding (DAE)

Proposed M²DAEP: 1. Motivation
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2. Image deblurring

Conclusions: An enhanced DAEP for color IR!
Part I – Background:

Image Restoration (IR) Model

Denoising Autoencoding (DAE)

Background 1: Image restoration (IR) model

\[ \hat{u} = \arg \min_u \left\| Hu - f \right\|^2 + \lambda \varphi(u) \]

Regularization term:
Exploits characteristics of a natural image

Data-fidelity term:
Ensures that the solution conforms to the degradation process

Background 2: Denoising autoencoding (DAE)

The autoencoder error $A_{\sigma_{\eta}}(u) - u$ is proportional to the gradient of the log likelihood of the smoothed density:

$$A_{\sigma_{\eta}}(u) - u = \sigma_{\eta}^2 \nabla \log [g_{\sigma_{\eta}} * q](u)$$

where the data distribution is $\text{Probability}(u) = \int q(u + \eta)d\eta$.

Part II – Proposed M²DAEP
Motivation 1: Patch-based methodology (image similarity)

Refs: Buades et al. 2005; Elad et al. 2006; Dabov et al. 2006; Milanfar et al., 2007; Zhang et al., 2010; Dong et al., 2012.
Motivation 2: Image patch aggregation strategy

The patch matching procedure enables multi-patches with similar structural patterns to be found and grouped.
Idea: Multi-model implementation

Inspired by image patch similarity and aggregation strategy, we adopt a multi-models and 6-dimensional version of DAEP for color IR tasks.

Convolution operator in multi-channel image features at iterative procedure.

Network and prior learning: 6-D implementation

The network architecture used for learning a DAE in this work is the residual encoder-decoder network (RED-Net).

The network consists of 10 convolutional and 10 deconvolutional layers symmetrically arranged. Shortcuts connect matching convolutional and deconvolutional layers.

\[ A_{\sigma_\eta} (I(u)) \]

M^2DAEP for SISR

Flowchart of employing the learned M^2EDAP to SISR application.

\[
\min_u \left\| Hu - f \right\|^2 + \frac{\lambda}{N} \sum_{i=1}^{N} \left\| I(u) - A_{\sigma_{\eta_i}}(I(u)) \right\|^2
\]

Considering the 6D and multi-models ($N=2$), the general mathematical model for color IR can be derived as follows:

$$\min_u \|Hu - f\|^2 + \frac{\lambda}{N} \sum_{i=1}^{N} \|I(u) - A_{\sigma, \eta_i}(I(u))\|^2$$

- $I(u) = [u, u_1]$
- $N$ stands for the number of M$^2$DAEP model
- The first term is the data-fidelity term
- The second term consists of the network-driven prior information
Proposed IR solver

Due to the nonlinearity of the model, we apply the proximal gradient method to tackle it. The model is approximated by standard least square (LS) minimization:

$$\min_u \|Hu - f\|^2 + \frac{\lambda}{\beta N} \sum_{i=1}^{N} \|I - (I^k - \beta \nabla G_i(I^k))\|^2$$

- $G_i(I) = \|I - A_{\sigma_i}(I)\|^2$
- $\nabla G_i(I) = [1 - \nabla I A_{\sigma_i}^T(I)] [I - A_{\sigma_i}(I)]$
- The function $G(I)$ is $1/\beta$-Lipschitz smooth
- $\|\nabla G(I') - \nabla G(I'')\|_2 \leq \|I' - I''\|_2 / \beta$ denotes the index number of iterations

Proposed IR solver

Given $\beta=1$, the above formula can be solved by calculating the gradient as follows:

$$H^T (Hu - f) + \lambda \left\{ I + \frac{1}{N} \sum_{i=1}^{N} [\nabla_I A_{\sigma_{ni}}^T (I^k) (A_{\sigma_{ni}} (I^k) - I^k) - A_{\sigma_{ni}} (I^k)] \right\} = 0$$

$$u^{k+1} = \frac{H^T f + \frac{\lambda}{N} \sum_{i=1}^{N} R\left\{ A_{\sigma_{ni}} (I^k) - \nabla_I A_{\sigma_{ni}}^T (I^k) [A_{\sigma_{ni}} (I^k) - I^k] \right\}}{(H^T H + \lambda)}$$

$R$ stands for the mean operator employed on the six channels.

Proposed IR solver

- $A_{\sigma_{\eta}}(I^k)$ is the forward output with the input $I^k + \sigma_{\eta}$.
- $A_{\sigma_{\eta}}(\circ)$ are already learned at the network training stage.
- $\nabla_I A_{\sigma_{\eta}}^T(I^k)[A_{\sigma_{\eta}}(I^k) - I^k]$ is the backward network output with the input $A_{\sigma_{\eta}}(I^k) - I^k$.
- Update the solution $u^k$ by alternately updating the network estimation $A_{\sigma_{\eta}}(I^k)$, $\nabla_I A_{\sigma_{\eta}}^T(I^k)$, and LS solver until the $u$ value convergences.
- The mathematical model is tackled by the proximal gradient and alternative optimization.

Algorithm: $M^2$DEAP

Training stage

- Training images: 6-dimensional dataset $\{I \mid I(u) = [u, u_1]\}$
- Noisy levels: $\delta_{\eta_1}$ and $\delta_{\eta_2}$
- Network: 6-channel DAE network
- Outputs: Trained network $A_{\sigma_{\eta_1}}(\circ)$ and $A_{\sigma_{\eta_2}}(\circ)$

Testing stage

- Initialization: $u^0 = H^Tf$; $K$; $N = 2$
- For $k = 1, 2, \ldots, K$
  - Update the auxiliary variable: $I^k = [u^k, u_1^k]$
  - Calculate the prior gradient components: $A_{\sigma_{\eta_i}}(I^k)$, $\nabla_I A_{\sigma_{\eta_i}}^T(I^k)[A_{\sigma_{\eta_i}}(I^k) - I^k]$; $i = 1, 2, \ldots, N$
  - Update the solution via solving the LS problem:
    $$u^{k+1} = \frac{H^Tf + \frac{\lambda}{N} \sum_{i=1}^{N} R\{A_{\sigma_{\eta_i}}(I^k) - \nabla_I A_{\sigma_{\eta_i}}^T(I^k)[A_{\sigma_{\eta_i}}(I^k) - I^k]\}}{(H^TH + \lambda)}$$

End
Part III – Experimental Results:

Single Image Super-Resolution

Image Deblurring

Experimental Results: SISR

From top to bottom and left to right:

SRCNN, DnCNN-3, IRCNN, DMSP, SRMD, DAEP and M^2DAEP.

Experimental Results: Image deblurring

M²DAEP achieves the highest values for almost images.

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<th>DMSP</th>
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M²DAEP produces cleaner and sharper image edges and textures than other competing methods.

Part III – Conclusions
Conclusions: Enhanced DAEP for color IR

- Presented a **6-channel denoising autoencoder prior**, which built on the assumption that an optimal denoising autoencoder is a local mean of the correct data density.

- **Auxiliary variables technique** was applied to integrate higher-dimensional structural information.

- This work paved a new way to incorporate **higher-dimensional prior** information into color IR applications.

Thanks all!

Code:  https://github.com/yqx7150/M2DAEP

Code:  http://www.escience.cn/people/liuqiegen