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# Toeplitz Matrix-based Transmit Covariance Matrix of Colocated MIMO Radar Waveforms for SINR Maximization

**Wei Xiong[1], Maria Greco[2], Fulvio Gini[2], Gong Zhang[1] and Zhenni Peng[1]**

1. Key Laboratory of Radar Imaging and Microwave Photonics, Ministry of Education
Nanjing University of Aeronautics and Astronautics, China
2. Dipartimento di Ingegneria dell' Informazione, University of Pisa, Italy

## 1. Introduction

Focusing on the signal-to interference-plus-noise ratio (SINR) maximization using the covariance matrix design of transmitted waveforms in colocated multiple-input multiple-output (MIMO) radars, we propose a kind of transmit covariance matrix (TCM) $\mathbf{R}_{pm}$ with the form of symmetrical Toeplitz matrix, whose full rank characteristic firstly can sufficiently exploit the waveform diversity advantage of MIMO radar and further suppress the maximum number of interfering sources. Meanwhile, the positive semi-definition characteristic of $\sin((\pi/2)\mathbf{R}_{pm})$ guarantees that these TCMs can be synthesized with binary phase shift keying (BPSK) waveforms in closed form. Furthermore, employing certain proposed TCM, higher SINR level can be yielded and lower sidelobe levels (SLLs) can be obtained for the unwanted sidelobe interference suppression, respectively. Simulation results validate the better performance of our proposed TCMs in comparison with the phased array, omnidirectional MIMO radar and the recently proposed TCMs.

## 2. Problem and Proposed TCM

Consider a colocated MIMO radar system equipped with a transmit uniform linear array (ULA) and a receive ULA. Each transmit element emits a distinct waveform. When there is a target located at $\phi_0$ and *Q* signal-dependent interfering sources at $\phi_i$ ($i=1,2,\cdots,Q$), denoting the transmit and receive steering vector with $\mathbf{a}(\phi_i)$ and $\mathbf{b}(\phi_i)$, then the received vector can be written as

$$\mathbf{y}(n)=\alpha_0\mathbf{b}(\phi_0)\mathbf{a}^T(\phi_0)\mathbf{s}(n)+\sum_{i=1}^{Q}\beta_i\mathbf{b}(\phi_i)\mathbf{a}^T(\phi_i)\mathbf{s}(n)+\mathbf{v}(n) \tag{1}$$

where $\alpha_0$ and $\beta_i$ are the complex reflection coefficients of the target and the *i*th interfering source, respectively, which obey the Swerling II model. $\mathbf{s}(n)$ is assumed as the vector of transmitted waveforms at the *n*th snapshot. $\mathbf{v}(n)\sim N(0,\sigma_v^2)$ denotes the zero-mean Gaussian noise term with covariance $\sigma_v^2$. After matched filtering, the filters' outputs stacking in one column vector is obtained as [15]

$$\mathbf{z}=\alpha_0\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)+\sum_{i=1}^{Q}\beta_i\mathbf{b}(\phi_i)\otimes\mathbf{Ra}(\phi_i)+\mathbf{v}_c \tag{2}$$

where $\mathbf{v}_c\sim N(0,\sigma_v^2(\mathbf{I}_{M_r}\otimes\mathbf{R}))$ stands for the colored Gaussian noise vector, $\otimes$ symbolizes the Kronecker product, $\mathbf{R}=(1/N)\sum_{n=1}^{N}\mathbf{s}(n)\mathbf{s}^H(n)\succeq 0$ refers to the TCM, which is positive semi-definite and is directly related with the sampled waveforms. *N* is the sample number.

In view of the monotonic relation between the detection probability and SNR/SINR [15], the design criterion of $\mathbf{R}$ and receive filter always is the SINR maximization. By employing the principle of minimum variance distortionless response (MVDR), the optimum receive filter and the maximum (optimum) SINR can be derived as (3) and (4), respectively. It is seen that $\mathbf{R}$ can be optimized for the MIMO radar waveform design and SINR improvement.

$$\boldsymbol{\omega}=\frac{\mathbf{R}_{in}^{-1}\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)}{\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}^H\mathbf{R}_{in}^{-1}\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)} \tag{3}$$

$$SINR_{opt}=\rho\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}^H\mathbf{R}_{in}^{-1}\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0) \tag{4}$$

To improve the SINR and consider the SINR level of phased array as an upper limit, a kind of more general symmetrical Toeplitz matrix $\mathbf{R}_{pm}$ is proposed as the TCM for the waveform design of colocated MIMO radars

$$\mathbf{R}_{pm}=\begin{bmatrix} 1 & \frac{M_t-m}{M_t} & \cdots & \frac{M_t-(M_t-1)m}{M_t} \\ \frac{M_t-m}{M_t} & 1 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \frac{M_t-m}{M_t} \\ \frac{M_t-(M_t-1)m}{M_t} & \cdots & \frac{M_t-m}{M_t} & 1 \end{bmatrix} \tag{5}$$

where $0<m<3M_t/(M_t+1)$ is the control parameter to generate difference TCM. Especially, when $m=1$, $\mathbf{R}_{pm}=\mathbf{R}_{p1}$, when $m=2$, $\mathbf{R}_{pm}=\mathbf{R}_{p2}$ in [10], and $\sin((\pi/2)\mathbf{R}_{p1})=\mathbf{R}_{2x}$ in [9]. Then following characteristics can be proved mathematically.

### ◆ Full Rank

It is obvious that $\mathbf{R}_{pm}$ can be characterized by its first row or column. Let $\{h(d)\}_{d=0}^{M_t-1}=\{(M_t-dm)/M_t\}_{d=0}^{M_t-1}$ denote the elements in the first row and assume $\{H(k)\}_{k=0}^{M_t-1}$ are the frequency domain samples for $\{h(d)\}_{d=0}^{M_t-1}$, then we can proved that $\mathbf{R}_{pm}$ is full-ranked based on the lemma in [16]. The detailed proof is not shown here.

### ◆ Maximum SINR for only noise

For the only noise case without interferences, we have $\mathbf{R}_{in}^{-1}=(\mathbf{I}_{M_r}\otimes\mathbf{R}_{pm})^{-1}=\mathbf{I}_{M_r}\otimes\mathbf{R}_{pm}^{-1}$ and the maximum SINR employing $\mathbf{R}_{pm}$ can be formulated as (6), when the target is assumed to located at $\phi_0=0$

$$\begin{aligned}SINR_{\mathbf{R}_{pm}}&=\rho\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}_{pm}^H(\mathbf{I}_{M_r}\otimes\mathbf{R}_{pm}^{-1})\mathbf{b}(\phi_0)\otimes\mathbf{R}_{pm}\mathbf{a}(\phi_0)=\rho M_r(\mathbf{a}^H(\phi_0)\mathbf{R}_{pm}^H\mathbf{a}(\phi_0))\\&=\rho M_r\sum_{k=1}^{M_t}\left(\left(M_t-k+1-\sum_{i=0}^{M_t-k}\left(\frac{im}{M_t}\right)\right)+k-\sum_{i=0}^{k-1}\left(\frac{im}{M_t}\right)-1\right)=\rho M_r\left(\left(1-\frac{m}{3}\right)M_t^2+\frac{m}{3}\right)\end{aligned} \tag{6}$$

Note that for $0<m<3M_t/(M_t+1)$, we can arrive at $M_t<\left(1-\frac{m}{3}\right)M_t^2+\frac{m}{3}<M_t^2$. Therefore, we can conclude the maximum SINR using $\mathbf{R}_{pm}$ outperforms the one of the omnidirectional MIMO radar and can get close to that of phased array radar.

### ◆ Synthesis with BPSK waveforms

In the finite alphabet correlated waveform design [7][8], for the realization of $\mathbf{R}_{pm}$ using BPSK waveforms in closed form, therefore $\sin((\pi/2)\mathbf{R}_{pm})$ has to be positive semi-definite. Fortunately, we can derive (7) as

$$\det\left(\sin((\pi/2)\mathbf{R}_{pm})-\lambda\mathbf{I}_{M_t}\right)=0\Rightarrow\lambda^{M_t-2}\left(\lambda^2-M_t\lambda+\sum_{i=1}^{M_t-1}(M_t-i)\sin^2\left(\frac{i\pi}{(2M_t)/m}\right)\right)=0 \tag{7}$$

and we conclude that $\sin((\pi/2)\mathbf{R}_{pm})$ has $M_t-2$ zero eigenvalues and two eigenvalues larger than 0.

## 3. Numerical Results

In our simulation, to evaluate the performance of the representative TCMs $\mathbf{R}_{pm}$, where *m* is selected from 0.5 to 2.5 with the interval 0.5, two examples are presented in comparison with the phased array, conventional omnidirectional MIMO radar, and the scheme using $\mathbf{R}_{2x}$ in [9]. The target is located at $\phi_0=0$, where the power is expected to be cohered.

***Case 1:*** It is assumed that the transmit and receive element number are 10, and there are two signal-dependent interfering sources located at $\phi_1=-15^o$ and $\phi_2=25^o$ with the interference-to-noise ratio (INR) 30 *dB*. Fig. 1 depicts the obtained SINR levels of the compared schemes with different SNR and the receive beampatterns are shown in Fig. 2. Higher SINR level can be obtained by employing $\mathbf{R}_{pm}$ with smaller *m*, and $\mathbf{R}_{p0.5}$ outperforms the other MIMO radar scheme. $\mathbf{R}_{p2}$ has comparable low SLLs with $\mathbf{R}_{2x}$ in Fig. 2. The SLLs using $\mathbf{R}_{p1.5}$ are also low.

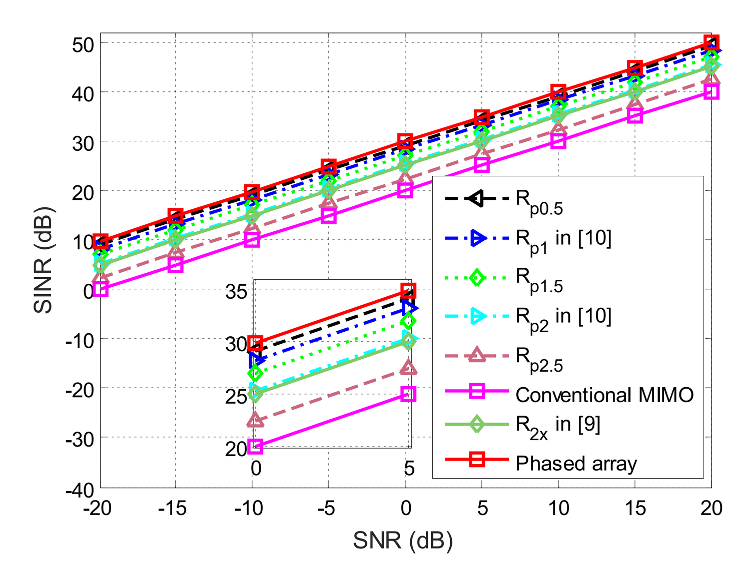


Fig.1. SINR versus SNR, where the transmit and receive element number are 10

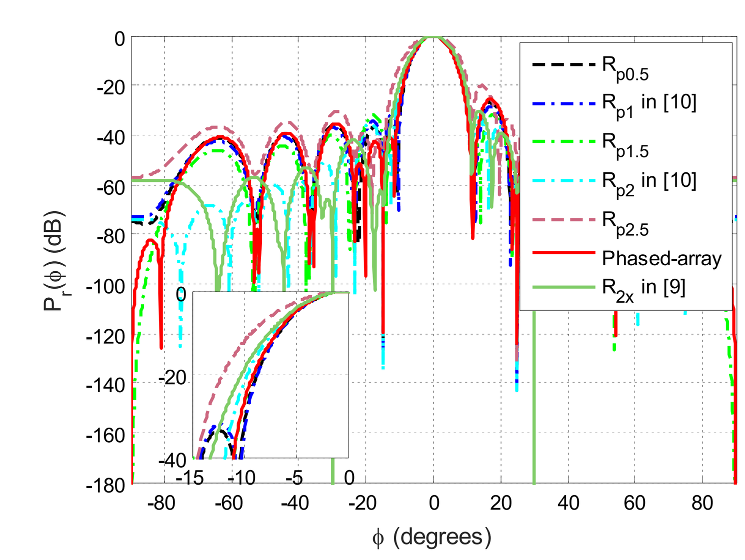


Fig.2. Receive beampatterns, where the transmit and receive element number are 10

***Case 2:*** To show the maximum capability of interference suppression of all schemes, in Fig.3, we assume that the transmit and receive element number are 6, and there are 12 interfering sources. Since $\mathbf{R}_{pm}$ are full-ranked, the colocated MIMO radar using $\mathbf{R}_{pm}$ can suppress 10 interfering sources. While the phased array only can suppress 5 interfering sources and the scheme using $\mathbf{R}_{2x}$ can suppress 7 interfering sources. From Fig. 3, it is seen that when the interfering source number $N_i>5$, the SINR level of phased-array radar drops obviously, when $N_i>7$, the one of scheme using $\mathbf{R}_{2x}$ also degrades. In contrast, the schemes using $\mathbf{R}_{pm}$ can suppress the interferences effectively with the output SINR>32*dB* and the SINR level using $\mathbf{R}_{p0.5}$ outperforms the others.

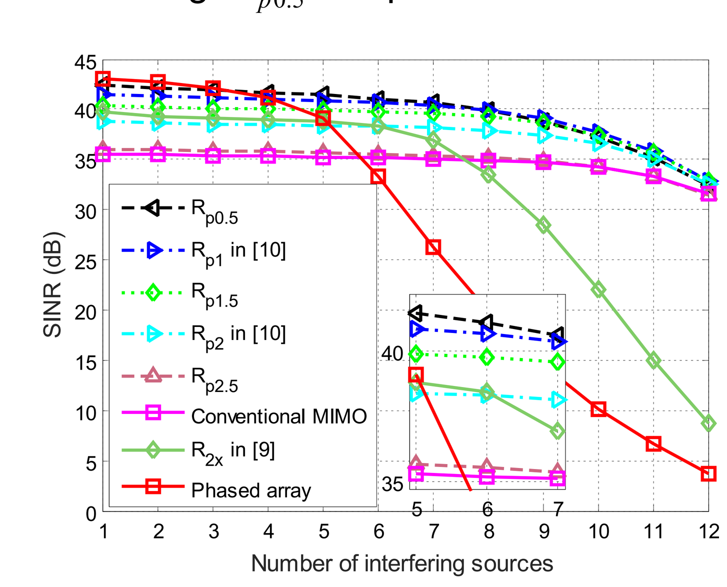


Fig.3. Average SINR level comparison for different number of interfering sources, where the transmit and receive element number are 6.

## 4. Representative References

[7] S. Ahmed, J. Thompson, Y. Petillot, and B. Mulgrew, "Finite alphabet constant-envelope waveform design for MIMO radar," IEEE Trans. Signal Process., vol. 59, no. 11, pp. 5326-5337, Nov. 2011.

[8] S. Jardak, S. Ahmed, and M. Alouini, "Generation of correlated finite alphabet waveforms using Gaussian random variables," IEEE Trans. Signal Process., vol. 62, no. 17, pp. 4587-4596, Sept. 2014.

[9] S. Ahmed and M.-S. Alouini, "MIMO-radar waveform covariance matrix for high SINR and low side-lobe levels," IEEE Trans. Signal Process., vol. 62, no. 8, pp. 2056-2065, Apr. 2014.

[10] S. Imani and S. A. Ghorashi, "SINR maximization in colocated MIMO radars using transmit covariance matrix," Signal Process., vol. 119, pp. 128-135, Feb. 2016.

[15] M. Haghnegahdar, S. Imani, S. A. Ghorashi and E. Mehrshahi, "SINR enhancement in colocated MIMO radar using transmit covariance matrix optimization," IEEE Signal Process. Lett., vol. 24, no.3, pp. 339-343, Mar. 2017.

[16] J. Lipor, S. Ahmed and M. S. Alouini, "Fourier-based transmit beampattern design using MIMO radar," IEEE Trans. Signal Process., vol. 62, no. 9, pp. 2226–2235, May 2010.

E-mail: wxiong_nuaa@sina.com; m.greco@iet.unipi.it; f.gini@ing.unipi.it; gzhang@nuaa.edu.cn