1. Introduction

Focusing on the signal-to interference-plus-noise ratio (SINR) maximization using the covariance matrix design of transmitted waveforms in colocated multiple-input multiple-output (MIMO) radars, we propose a kind of transmit covariance matrix (TCM) \( R_u \), with the form of symmetrical Toeplitz matrix, whose full rank characteristic firstly can sufficiently exploit the waveform diversity advantage of MIMO radar and further suppress the maximum number of interfering sources. Meanwhile, the positive semi-definition characteristic of \( \text{det}(\mathbf{\Sigma}_R R_u) \) guarantees that these TCMs can be synthesized with binary phase shift keying (BPSK) waveforms in closed form. Furthermore, employing certain proposed TCM, higher SINR level can be yielded and lower side-lobe levels (SLLs) can be obtained for the unwanted side-lobe interference suppression, respectively. Simulation results validate the better performance of our proposed TCMs in comparison with the phased array, omnidirectional MIMO radar and the recently proposed TCMs.

2. Problem and Proposed TCM

Consider a colocated MIMO radar system equipped with a transmit uniform linear array (ULA) and a receive ULA. Each transmit element emits a distinct waveform. When there is a target located at \( \phi_0 \) and Q signal-dependent interfering sources at \( \phi_j \ (j = 1, 2, \ldots, Q) \), denote the transmit and receive steering vector with \( a(\phi) \) and \( b(\phi) \), then the received vector can be written as

\[
y(\phi) = c_{\theta}(\phi) a(\phi) M \phi + \sum_{j=1}^{Q} \beta_j b(\phi) M \phi + v(\phi)
\]

where \( c_{\theta}(\phi) \) and \( \beta_j \) are the complex reflection coefficients of the target and the \( j \)th interfering source, respectively, which obey the Swerling II model. \( x(\phi) \) is assumed as the vector of transmitted waveforms at the nth snapshot, \( \text{diag}(\mathbf{\Sigma}) \) denotes the zero-mean Gaussian noise term with covariance \( \mathbf{\Sigma} \). After matched filtering, the filters’ outputs stacking in one column vector is obtained as

\[
z = \text{diag}(\mathbf{\Sigma}) \otimes R_u \mathbf{a}^H R_u \mathbf{b} \otimes R_u + v
\]

where \( \text{diag}(\mathbf{\Sigma}) \otimes R_u \) stands for the colored Gaussian noise vector, \( \otimes \) symbolizes the Kronecker product, \( R_u = (1/N) \sum_{n=0}^{N-1} \mathbf{a}^H(\phi_n) \mathbf{a}(\phi_n) \) refers to the TCM, which is positive semi-definite and is directly related with the sampled waveforms. \( N \) is the sample number.

In view of the monotonic relation between the detection probability and SNR/SINR [15], the design criterion of \( R_u \) is to maximize the SINR via minimizing the interference-to-noise ratio (INR) of each target. Meanwhile, we have that the interference-to-noise ratio of the \( i \)th target can be derived as (5) and (6), respectively. It is seen that \( R_u \) can be optimized for the MIMO radar waveform design and SINR improvement.

\[
\text{SNIR}_i = p_i R_u \mathbf{a}^H R_u \mathbf{b} \otimes R_u
\]

To improve the SINR and consider the SINR level of phased array as an upper limit, a kind of more general symmetrical Toeplitz matrix \( R_u \) is proposed as the TCM for the waveform design of colocated MIMO radars

\[
R_u = \begin{bmatrix}
1 & M^{-1} & M^{-2} & \cdots & M^{-\text{M-1}} \\
M^{-1} & 1 & M^{-1} & \cdots & M^{-2} \\
M^{-2} & M^{-1} & 1 & \cdots & M^{-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
M^{-\text{M-1}} & M^{-2} & M^{-1} & \cdots & 1
\end{bmatrix}
\]

where \( 0 < M < 3(M_i + 1) \) is the control parameter to generate difference TCM. Especially, when \( M = 1 \), \( R_u = R_{\text{NM}} \), when \( M = 2 \), \( R_u = R_{\text{NM}} \) in [10], and \( \sin((\pi/2) R_u) = R_{\text{NM}} \) in [9]. Then following characteristics can be proved mathematically:

- **Full Rank**: It is obvious that \( R_u \) can be characterized by its first row or column. Let \( \text{det}(\mathbf{\Sigma}) \mathbf{R}_{\text{NM}} = \text{det}(\mathbf{\Sigma} - \mathbf{R}_{\text{NM}} \mathbf{R}_{\text{NM}}^H) \), denote the elements in the first row and assume \( \text{det}(\mathbf{\Sigma}) = \text{det}(\mathbf{\Sigma} - \mathbf{R}_{\text{NM}} \mathbf{R}_{\text{NM}}^H) \), then we can prove that \( R_u \) is full-rank based on the lemma in [16]. The detailed proof is not shown here.

- **Maximum SINR for only noise**: For the only noise case without interference, we have \( R_u = R_{\text{NM}} \) and the maximum SINR employing \( R_u \) can be formulated as (8) when the target is assumed to be located at \( \phi_0 = 0 \).

\[
\text{SNIR}_{\text{NM}} = \frac{p R_u \mathbf{a}^H R_u \mathbf{b} \otimes R_u}{\mathbf{R}_u \mathbf{a}^H R_u \mathbf{a}(\phi_0) + \mathbf{R}_u \mathbf{b} \otimes R_u}
\]

\[
= \frac{p M \sum_{m=-M}^{M} \sum_{n=-M}^{M} M_{m,n} M_{n,m} + M}{M \sum_{m=-M}^{M} \sum_{n=-M}^{M} M_{m,n}^2 - 1} = \frac{p M}{M - 1/4}
\]

where \( M \leq 3(M_i + 1) \).

3. Numerical Results

In our simulation, to evaluate the performance of the representative TCMs \( R_u \), where \( m \) is selected from 0.5 to 2.5 with the interval 0.5, two examples are presented in comparison with the phased array, omnidirectional MIMO radar, and the scheme using \( R_{\text{NM}} \) in [9]. The target is located at \( \phi_m = 0 \), where the power is expected to be cohered.

- **Case 1**: it is assumed that the transmit and receive element number are 10, and there are two signal-dependent interfering sources located at \( \phi_1 = -15^\circ \) and \( \phi_2 = 25^\circ \) with the interference-to-noise ratio (INR) 30 dB. Fig. 1 depicts the obtained SINR levels of the compared schemes with different SINR and the receive beamformers are shown in Fig. 2. Higher SINR level can be obtained by employing \( R_u \) with smaller \( m \) and \( R_{\text{NM}} \) outperforms the other MIMO radar scheme. \( R_u \) has comparable low SLLs with \( R_{\text{NM}} \) in Fig. 2. The SLLs using \( R_u \) are also low.

4. Representative References