Multidimensional nonseparable oversampled lapped transforms (NSOLTs) are proposed.

- NSOLTs are MD redundant filter banks.
- The filters are nonseparable, symmetric, real, overlapped, and compact-supported.
- They are based on lattice structures and tree decomposition is available.
- The redundancy is flexilly controlled by $\gamma$ of Chs. $P$ and downsampling ratio $M$.
- NSOLTs are capable of constructing Parseval tight frames in ANY $\gamma$ of dimensions.
- Example-based design is available through the dictionary learning approach.
- Comparison with separable systems shows the significance in terms of non-linear Approx.

### III. Lattice Structures

Type-I and Type-II lattice structures are proposed.

\[
\begin{align*}
D \hat{y} &= \arg \min_{y} \|x - Dy\|_2^2, \\
\text{s.t. } &\|y\|_0 \leq K
\end{align*}
\]

#### Sparse Approximation

\[
\hat{y} = \arg \min_{y} \|x - Dy\|_2^2, \\
\text{s.t. } &\|y\|_0 \leq K
\]

#### Dictionary Update

\[
\Theta = \arg \min_{\Theta} \|x - D_{\Theta} y\|_2^2
\]

### IV. Example-based Design

Design Examples

\[\text{Training image, part of barbara, that is 64} \times 64\text{ pixels in 8-bit grayscale.}
\]

Spars approximation results with 2-D NSOLT dictionaries through IHT for training image, where $M = \text{dia}((M_1, M_2))$ and the other construction parameters are summarized in Tab. 1.

\[\Theta = \text{dia}((M_1, M_2))\]

Impulse responses of synthesis filters designed through SPLQ and NSOLT framework for training image, where $M = \text{dia}(M_1, M_2)$ and the other construction parameters are summarized in Tab. 1.

\[\Theta = \text{dia}(M_1, M_2)\]

Impulse responses of synthesis filters designed through NSOLT framework for the training data.

### V. Performance Evaluation

### VI. Conclusion

- The theory of the 1-D OLPPRFBs in [Gan et al., (2003)] is extended to the MD case.
- Parseval tight frame with non-separable symmetric atomic images can be constructed.
- NSOLTs yield a structured framework of MD convolution (linear) networks.
- Its applications include image/volumetric data restoration and pattern recognition.

### Design Examples

\[\Theta = \text{dia}((M_1, M_2))\]

Impulse responses of synthesis filters designed through SPLQ and NSOLT framework for training image, where $M = \text{dia}(M_1, M_2)$ and the other construction parameters are summarized in Tab. 1.

\[\Theta = \text{dia}(M_1, M_2)\]

Impulse responses of synthesis filters designed through NSOLT framework for the training data.

### Related Topics Beyond the Paper